Financial Structure and Product Qualities

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Abstract: We examine the interaction between financial and microeconomic decisions in a differentiated duopoly where additional willingness-to-pay for high quality is uncertain. Product specification is *endogenous*. We consider two three-stage games, according to the order of moves: qualities-financial structure-prices and financial structure-qualities-prices. Once debt is contracted, the manager maximizes equity instead of total value. We find that in both games debt a) increases both prices and qualities but most likely reduces product differentiation due to rival quality response; b) reduces the value of the levered high quality firm because it increases the low quality. Moreover, c) the cost of debt is higher for the second game, implying that it is higher for projects using debt to finance a product's development-cum-commercialization compared to those financing only the commercialization stage. The results turn out to be robust to alternative specifications of quality and market size uncertainty.

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I. Introduction

Unless its purpose is to replace an existing product, a new firm introducing a new product version always prefers sufficient consumer heterogeneity, for otherwise a price war with the seller of the basic product is unavoidable. When introducing an improved version, therefore, part of the seller's concern is how "wide" the market is, or, alternatively, how far the willingness-to-pay for high quality goes.[4] Assuming for simplicity a uniform taste distribution, this corresponds to uncertainty over the exact position of the high end of that distribution, and ultimately, over the position of the demand function. The presence of such uncertainty may affect the decision on how to finance the project, which in turn may have effects on prices and product specification. The purpose of this paper is to study the interaction between financial structure and product and price choices.

Although most studies in both economics and finance adopt the principle of separation of financial structure choice from decisions on investment, pricing and output, it is well known in fact that in the presence of uncertainty these dual sets of decisions interact with each other. Jensen and Meckling (JM, 1976) were the first to point out that the presence of debt in the financial structure of a firm may induce the equity owners, who are assumed to control the operations of the firm, to undertake investments with even negative contributions to the total value of the firm, provided that they are associated with sufficiently higher risk. This loss in firm value is known as the *agency cost* of debt (J&M effect) and is due to the fact that, under limited liability, a risk-neutral owner-manager undervalues the losses debt holders incur in states of bankruptcy, thus preferring riskier projects with even lower value.

In an oligopoly with demand uncertainty, however, besides the agency cost of debt this behavior may also create strategic effects that can enhance the value of the levered firm, as Brander and Lewis (1986) were the first to show. This happens since debt induces the owner-manager to overstate good demand states, and therefore sell higher quantities. This corresponds to a more aggressive behavior which, in a Cournot duopoly increases profits of the levered firm at the expense of its all-equity rival. In a Bertrand oligopoly debt induces a softer behavior, thus increasing both rivals'

[4] The willingness-to-pay for quality increments of the consumers with low taste for quality is easier to estimate since a) the price of the existing basic product is close to their willingness-to-pay for that quality, and b) their willingness-to-pay for higher qualities represent small increments over that for the basic product.

profits.[5] Thus, despite the agency cost in oligopolies, some amount of debt raises firm value (BLS effect).

In this paper we examine the interactions between financial and microeconomic decisions in a differentiated oligopoly in which product specification is *endogenous*. More specifically, we analyze how leverage may affect the market outcome when consumer demand for the basic product is already well-known, but the additional willingness-to-pay for high quality is uncertain. Consider, for instance, a firm providing dial-up connection (low quality) to the internet and another one ready to introduce wireless connection (high quality). Both dial-up and wireless are technologically available at many quality levels, but each firm is to offer only a single product. While the willingness-to-pay for quality improvements of the dial-up connection is known with certainty, the corresponding willingness-to-pay for improvements in the wireless is uncertain.

Using a vertically differentiated duopoly as in Shaked and Sutton (1983) we analyze two three-stage games differing with respect to the decision sequence. Each game corresponds to the financing of a different type of venture. In game Q quality is chosen before financial structure and in game S financial structure is chosen before quality. At the last stage of both these games firms choose their prices (Bertrand competition). Hence, the choice of financial structure concerns in game Q the decision on how to finance the commercialization of the product after the design has been completed, while in game S it concerns the financing of a venture comprising both product development and commercialization. While quality choices are endogenous in both games, the high quality in game Q is chosen by total-value maximization since no debt has been contracted at the moment quality choices are made. In game S it is chosen by equity maximization, implying that its choice is subject to a J&M-type agency distortion, exactly like the price decision.

We ask three sets of questions. First, how are prices, qualities and product differentiation affected by debt? Second, what is the equilibrium level of debt, and is debt value-enhancing or value-reducing in the presence of endogenous product specification? Third, how does the decision sequence (or the type of venture that is financed) affect the cost of debt?

[5] See Showalter (1995).

We find that in both games leverage increases the quality of both firms' products. In game *Q* the high quality rises because marginal revenue is increasing with respect to quality. In game *S* this effect is augmented by the aforementioned agency distortion, leading to an even higher quality level for the same level of debt. In both games, the response of the low quality producer (hereafter, firm 2) is to increase its quality level. On the other hand, while leverage pushes up both qualities, it reduces product differentiation in almost all cases, whether measured as the ratio or as the difference between the high and the low qualities.[6] This happens because the anticipated less aggressive pricing behavior of the high quality firm (hereafter, firm 1) allows the low quality to increase more than proportionately. Besides, cost increments due to a given quality increment are higher for the high quality, due to the convexity of the cost function. Despite reduced differentiation, in both games debt increases both product prices. Hence, in the presence of leverage firms compete less aggressively in prices (as in Showalter (1995, 1999b)), but more aggressively in the quality stage of the game.

Turning to the desirability of debt, we find that when product qualities are endogenous, *leverage always reduces the levered firm's value*. This is in sharp contrast with the result in Showalter (1995, 1999b), that under demand uncertainty some debt is always desirable from the firm's point of view.[7] The crucial difference between Showalter's and our result lies in the fact that quality choices are exogenous in Showalter's analysis, while in this paper they are determined as part of the analyzed game. The aggressive competition in the quality stage induced by an amount debt, whether already contracted or anticipated, is clearly the dominant effect.

Game *Q* is closer to Showalter (1995, 1999b), since qualities are given at the second stage of the game, the choice of the financial structure. Despite the fact that firm 1's quality decision is not subject to any agency distortion (as, for instance, in game *S*), that firm's value is reduced because the anticipation of leverage in the financial structure of firm 1 induces firm 2 to raise its quality.[8] It can be shown that, had the low quality been fixed at any given level, debt would increase firm 1's value; when, however, the response of firm 2 is taken into account, the result is the opposite.

[6] To be precise, the ratio is always reduced, while the difference can only increase when the cost-of – quality function is very flat; such cases may result in entry-deterring behavior on the part of the high quality firm and are ignored in our analysis.
[7] We do not examine possible entry deterrence, in which debt may be avoided if a firm seeks to deter entry.
[8] It is assumed that quality levels, once chosen, are irreversible.

Since debt reduces firm value, it may come as a surprise to find that in the equilibrium of game *Q* firm 1 is levered. This paradox can be explained by noting that both agency and strategic effects depend on the cumulative probability of bankruptcy caused by a certain amount of debt, rather than on the debt amount itself.[9] In game *Q*, the optimal degree of bankruptcy risk turns out to be independent of quality choices—and in fact of any decision—depending only on parameters, such as the width of the distribution of the taste parameters and the variability of its upper end. Thus, all firm 1 does in stage 2 is to determine the amount of leverage that brings the probability of bankruptcy to the desired level, given the quality choices made at the first stage. Anticipating this behavior at the first stage, firm 2 raises its product's quality, knowing that its rival will accommodate such a move through leverage at the second stage. Hence, unless there is some mechanism allowing firm 1 to commit to taking no debt at the second stage, the latter is unavoidable.

In game *S*, where quality choices are made subsequent to the financial decision of firm 1, the debt-related agency distortion shifts the quality-reaction-function of firm 1 outwards at the second stage. This leads to an even higher rise of the lower quality, therefore, a further reduction of firm 1's value, compared to game *Q*.

While our results suggest that in a frictionless world the introduction of a higher quality under demand uncertainty is better financed by all-equity, in real world situations frictions such as limited availability of equity funding or favorable tax treatments may make debt necessary and/or desirable. We show that the cost of such debt in terms of agency-cum-strategic effects is lower when financing the market introduction of an already developed new product (game *Q*), compared to financing both stages of development and commercialization (game *S*). This happens because in Game *S* the owner of an already levered firm will chose a quality level well above the value-maximizing one.

In the remainder of this section we complete the review of the literature. Despite the importance financial structure has on investment, pricing and output decisions in oligopolistic industries, these effects have received relatively little attention, as noted in the 1991 survey by Harris and Raviv. Most of the studies that

[9] Note that an amount of leverage sufficiently small as to not induce bankruptcy even at the worst possible state of demand causes neither agency costs, nor strategic effects.

have been added since that survey are empirical ones,[10] and overall the topic seems to be relatively neglected in both the economics and financial literature. Existing theoretical studies have mostly dealt with homogenous product oligopolies, and have examined the effect of leverage on pricing and output, on barriers to entry, on the feasibility of entry deterrence and on R&D spending.[11] A few more recent works have also examined leverage effects in industries with differentiated products. In all these works product specification has been considered exogenous; hence, the demand functions have been taken as given in modeling the firms' interaction.[12] Wanzenried (2003) treats product differentiation parametrically and shows that the desirable amount of debt is decreasing in product differentiation. This concurs with our conclusion that debt and product quality are substitute mechanisms in relaxing price competition. This paper is, to our knowledge, the first study in the literature to examine the effect of leverage on product differentiation.

In the next section we present the general model. Section III presents the benchmark case of all-equity firms under uncertainty. Section IV examines the pricing stage in the presence of debt. Section V determines the equilibrium amount of leverage in game Q where quality choices precede the financial decision. Section VI examines how debt affects prices, qualities, product differentiation and firm value. Section VII analyzes the game S where the financial decision precedes quality choices. Section VIII compares the cost of debt in the two games. Section IX concludes.

II. The Model

We consider a market where two single product firms, firm 1 and firm 2, produce differentiated products. Each consumer j buys one unit of a certain type or nothing at all. The purchase of product i, $i=1,2$, yields utility

$$U_i^j = u_i t^j - p_i ,$$

[10] See, for instance, Phillips (1994), Chevalier (1995), Chevalier and Scharfstein (1996) and Showalter (1999a).

[11] See Brander and Lewis (1986), Maksimovic (1988), Poitevin (1989) and Jensen and Showalter (2004).

[12] See Maksimovic and Titman (1991), Showalter (1995), and Dasgupta and Titman (1998).

where u_i is a quality index, t^j is a consumer taste parameter and p_i the product's price. Utility from non purchase is zero. The utility function adopted implies that at equal prices consumers unanimously prefer the product with the higher u; without loss of generality, we assume $u_1 \geq u_2$. The consumer taste parameter t is uniformly distributed in $[a,b]$, $b > a > 0$, with density equal to one.

The consumer indifferent between the two qualities as well as the one indifferent between purchasing the lower quality or nothing are characterized by

$$t_B = (p_1 - p_2)/(u_1 - u_2)\text{, and } t_A = p_2 / u_2, \tag{1}$$

respectively. Hence, the market shares of firms 1 and 2 are $b - t_B$ and $t_B - a$, respectively. We assume that

$$2a < b < 4a \tag{2}$$

which implies that i) $t_B > a$, and ii) $t_A \leq a$, *i.e.*, the two firms have positive market shares and cover the entire market (natural duopoly).

On the supply side we assume variable production cost to be the same for both firms and, without loss of generality, to be equal to 0.[13] Production requires also a fixed cost $F(u)$, with $F' \geq 0$ and $F'' \geq 0$, which is sunk upon the choice of quality.

In the absence of uncertainty the financial choice of a firm is irrelevant. We introduce uncertainty over the consumer taste distribution (demand uncertainty) by assuming that b is distributed within a given interval $\left[\underline{b}, \overline{b}\right]$ according to some function $G(b)$ with expectation $\hat{b}$. Thus, firm 1 faces uncertainty directly in its turf, while firm 2 faces no uncertainty.[14] Firms are assumed to be risk-neutral throughout the paper.

In order to simplify the analysis we restrict the fluctuations of b so that the market will always remain duopoly and covered. This means that equation (2) always holds, which implies $\overline{b} < 2\underline{b}$. This assumption, while innocuous with respect to the interesting results, does simplify the analysis by avoiding functional form changes that would be necessary were the market to become a monopoly, or remain uncovered for some *ex post* realizations of the width of the taste distribution.

[13] In this case prices can be interpreted as the excess over unit cost.
[14] Of course, the distribution of *b* affects firm 2 indirectly, through the price reaction function of its rival.

We examine two three-stage games in which decisions on the same variable are taken simultaneously by the two firms and are observable at the moment the next decision in the sequence is made. Both games are named after the variable that is chosen at the first stage. In game *S* financial structure is chosen in stage 1, quality in stage 2, and prices in stage 3. In game *Q* qualities are chosen in stage 1, financial structure in stage 2 and prices in stage 3. In order for financial structure to play any role all three decisions (no matter their sequence) must be taken under uncertainty.[15] Figure 1 shows the timing of the games.

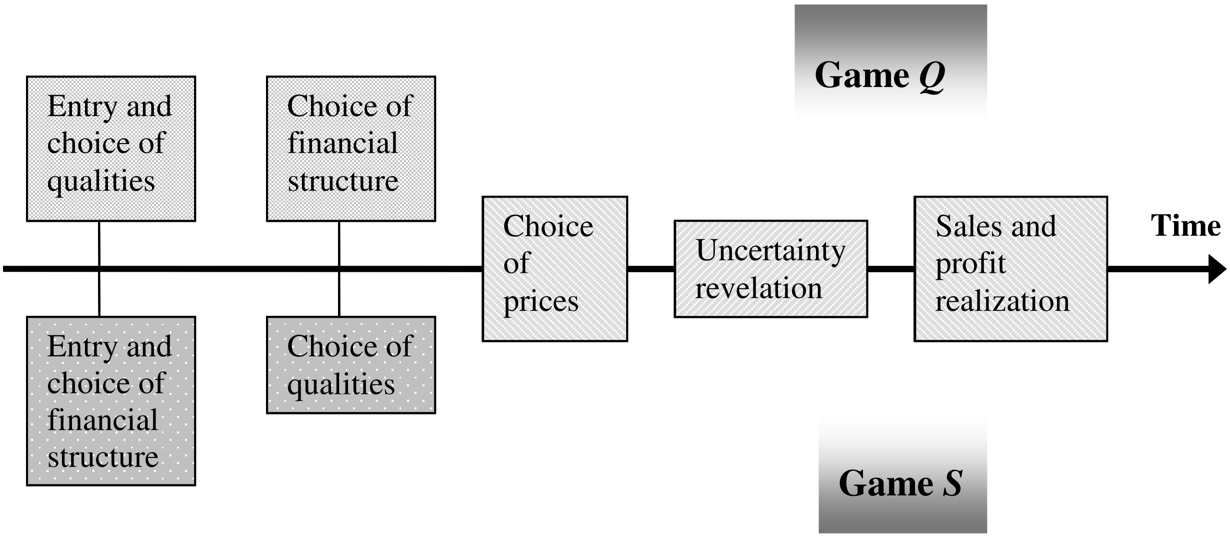

Figure 1: The game structure

III. Uncertainty and the all-equity firm

Before examining our main models, in this section we consider a benchmark case, game *E,* where equity is the only available financing source. Hereafter, a $\hat{}$ over a variable denotes its expected value.

In the last stage firm 1 faces a stochastic profit $\Pi_1 = p_1(b - t_B) - F_1$, while the profit of firm 2 is deterministic. Hence, prices are chosen by maximizing the following pair of profit functions

$$\hat{\Pi}_1 = p_1(\hat{b} - t_B) - F_1 \quad \text{and} \quad \Pi_2 = p_2(t_B - a) - F_2, \tag{3}$$

[15] Otherwise, the Modigliani-Miller theorem holds. When there is debt in the firm's capital structure the revelation of uncertainty before prices are chosen will also reveal whether default will take place, implying that the debt holders will write into the debt contract provisions for taking control of the firm in such cases. Hence, financial structure will have no effect on market equilibrium, unless *all three* decisions are taken before uncertainty is resolved. This assumption is also more realistic.

with respect to prices, which yields the following reaction functions

$$p_1 = (1/2)\left(\hat{b} \cdot \Delta u + p_2\right) \text{ and } p_2 = (1/2)(-a \cdot \Delta u + p_1), \tag{4a}$$

where $\Delta u \equiv (u_1 - u_2)$. Solving the system (4a) yields equilibrium prices as functions of qualities,

$$p_1^e\left(a, \hat{b}; u_1, u_2\right) = \hat{X}_1 \Delta u \text{, and } p_2^e(\cdot) = \hat{X}_2 \Delta u, \tag{4b}$$

where $\hat{X}_1\left(a, \hat{b}\right) \equiv \left(2\hat{b} - a\right)/3$, and $\hat{X}_2\left(a, \hat{b}\right) \equiv \left(\hat{b} - 2a\right)/3$. The superscripts *e* and *E*, indicate equilibrium values of the price subgame and the entire game *E*, respectively. Noting that

$$t_B^E = \left(\hat{b} + a\right)/3 \tag{4c}$$

it becomes obvious that $\hat{X}_i$, *i*=1,2, represent equilibrium market shares. Substituting (4b) into the profit functions yields the reduced form profit functions

$$\hat{\Pi}_1^E(\cdot) = \hat{X}_1^2 \Delta u - F(u_1) \tag{5a}$$

and

$$\Pi_2^E(\cdot) = \hat{X}_2^2 \Delta u - F(u_2). \tag{5b}$$

Since the revenue of firm 1 can be written as $\hat{X}_1^2 \Delta u = \hat{X}_1 p_1^e$, it is obvious that the term $\hat{X}_1$ can be interpreted as firm 1's marginal revenue with respect to a change in its *price sub-game equilibrium* price, p_1^e, an interpretation which will be useful later on.[16]

Maximizing (5a) with respect to u_1 yields

$$u_1^E\left(a, \hat{b}\right) = F'^{-1}\left(\hat{X}_1^2\right). \tag{6}$$

With respect to u_2, we note that $R_2' < 0$. Since the width of the consumer distribution always assures full market coverage, the profit maximizing u_2 is obtained at the minimum level consistent with $t_A \leq a$. Since t_A decreases in u_2, the optimal u_2 sets $t_A = a$, *i.e.*,

$$u_2^E\left(a, \hat{b}\right) = \left(\frac{\hat{b} - 2a}{\hat{b} + a}\right) u_1^e\left(a, \hat{b}\right) = \frac{\hat{X}_1 - a}{\hat{X}_1 + a} u_1^e\left(a, \hat{b}\right). \tag{7}$$

[16] This marginal revenue must be distinguished from marginal revenue due to a *unilateral* price change.

IV. THE PRICING STAGE UNDER DEBT

Let now debt be available as an alternative form of financing. Since at the moment where prices are chosen qualities and financial structure have already been decided, the analysis of the pricing stage is common to both games *Q* and *S*.

Assume that both firms have arrived at the last stage of the game partially financed by debt, the promised repayment of which amounts to D_k, $k=1,2$. Firm 2 is solvent if $D_2 \le p_2 \cdot (t_B - a) - F(u_2)$. From (1), the value of t_B depends only on prices and qualities which are chosen *prior* to uncertainty resolution, hence, it is not stochastic.[17] Since a is not stochastic either, the RHS of this expression contains no stochastic element, and the Modigliani-Miller theorem implies that the financial structure of firm 2 is irrelevant. Hereafter, we set $D_2 = 0$.

Turning to firm 1, it is clear from (3) that, for any (p_1, p_2), its revenue function varies monotonically with the realization of *b*. Let us define b_z as the value of *b* at which firm 1 is just able to repay its debt, *i.e.*,

$$p_1(b_z - t_B) - F(u_1) \equiv D_1, \tag{8}$$

where t_B is given by (1). Firm 1 is solvent for $b \in [b_z, \bar{b}]$ and in default for $b \in [\underline{b}, b_z)$. Let $\hat{b}_z \equiv E[b \mid b \ge b_z]$ denote the truncated expectation of *b* conditional on firm 1's solvency. Let also X_{iz} and $\hat{X}_{iz}$, *i*=1,2, denote the values of X_i and $\hat{X}_i$ when b and $\hat{b}$ are replaced by b_z and $\hat{b}_z$, respectively, and define $r \equiv \hat{X}_{1z} / \hat{X}_1$.

If $D_1 = 0$ firm 1 can never go bankrupt and $\hat{b}_z = \hat{b}$. From (8), an increase in D_1 increases *ceteris paribus* both b_z and $\hat{b}_z$: an increase in debt raises the cumulative probability of bankruptcy. Hence *r* is a monotonically increasing index of firm 1's bankruptcy risk.

Within each game *h=Q,S*, once debt has been contracted and qualities chosen, firm 1's owner will choose p_1 in order to maximize the firm's equity value,

[17] Since *b* is never part of firm 2's market share, uncertainty affects firm 2 only through its rival's reaction function. The latter is based on expected values and cannot be modified *ex-post*. Hence, the profit of firm 2 does not depend on the precise realization of *b*, and for the same reason its expression in equation (3) is non stochastic.

$$E_1 = \Pi_1 - D_1 = \int_{b_z}^{\bar{b}} \left[p_1 \left(b - b_z \right) \right] dG_b(b), \tag{9}$$

rather than total firm value. Setting the derivative of (9) equal to zero we get, after taking (8) into account,

$$p_1 = (1/2) \cdot \left(p_2 + \hat{b}_z \Delta u \right) \tag{10}$$

The expression in (10) is similar to the corresponding one in (4a) except that $\hat{b}$ has been replaced by $\hat{b}_z$. This is an important difference, however, since the former is a parameter while the latter a decision variable, determined at previous stages of the game. Positive amounts of debt imply $\hat{b}_z > \hat{b}$; debt shifts, therefore, the price reaction function of the leveraged firm, thus making its response softer. Firm 2, on the other hand, still maximizes total value and its reaction function is still given by the corresponding expression in (4a).

Solving (10) simultaneously with (4a) we find equilibrium prices:

$$p_i \left(\hat{b}_z; u_1, u_2 \right) = \hat{X}_{iz} \Delta u \,, \; i = 1, 2 \,. \tag{11}$$

It is clear that, *ceteris paribus*, an increase in debt increases *both* prices in both games. In terms of the Fundenberg and Tirole (1984) zoology, debt acquisition corresponds to a *puppy dog* strategy, similar to increasing product differentiation. Bankruptcy risk and product differentiation are "substitutes" in softening price competition in the sense that a given level of equilibrium price can be targeted by various $\hat{X}_{iz}$ and Δu levels.

Substituting last stage equilibrium prices from (11) into (9) and (8) we obtain, respectively, firm 1's price-maximized equity,

$$E_1 = (3/2) \hat{X}_{1z} (\hat{X}_{1z} - X_{1z}) \Delta u G(b_z) \equiv H_1(b_z) \Delta u \,, \tag{12}$$

the corresponding face value of debt,

$$D_1 = \hat{X}_{1z} \left(\frac{3X_{1z} - \hat{X}_{1z}}{2} \right) \Delta u - F(u_1) \equiv H_2(b_z) \Delta u - F(u_1) \,, \tag{13}$$

and total firm value,

$$V_1 = \int_{\underline{b}}^{\bar{b}} p_1 \left(b - \frac{p_1 - p_2}{\Delta u} \right) dG(b) - F(u_1) = (1/2) Y \Delta u - F(u_1) \,, \tag{14}$$

with $Y = \hat{X}_{1z}\left(3\hat{X}_1 - \hat{X}_{1z}\right)$, as functions of debt and qualities. Equations (12) and (13) will mainly be used in game *S*.

V. EQUILIBRIUM IN GAME *Q*

Firm 1 chooses its financial structure by maximizing (14) with respect to *D*, taking qualities as given. The following auxiliary result demonstrates a monotonic relation of debt to $\hat{X}_{1z}$, at the expense of a minor restriction on the distribution of the random variable.

Lemma 1: *If* $\partial\hat{X}_{1z}/\partial X_{1z} \le 1$, *then* $\left(d\hat{X}_{1z}/dD_1\right) \ge 0$.[18]

Proof: See the appendix.

The major implication of Lemma 1 is that, *for given qualities,* $\hat{X}_{1z}$ can replace debt as the decision variable. Hence, at the second stage of game *Q* firm 1 maximizes (14) with respect to $\hat{X}_{1z}$ instead of D_1. This maximization yields the following result:

Proposition 1: *In game Q there exists a unique optimal capital structure for the high quality firm that faces uncertainty in its market. This structure always contains a positive amount of debt, and is equal to the debt level that sets* $r^Q \equiv \hat{X}_{1z}^Q/\hat{X}_1 = 3/2$, *provided this relation yields* $b_z \in \left[\underline{b},\overline{b}\right]$; *otherwise the optimal capital structure consists entirely of debt financing.*[19] *A necessary condition for interior solution is* $\hat{X}_1 \in \left(1,(14/9)\right]$, *while for* $\hat{X}_1 \in \left[(14/9),(7/3)\right)$ *we always have an all-debt structure.*

Proof: For given qualities, $dV_1/dD_1 = \Delta u \cdot Y' \cdot \left(d\hat{X}_{1z}^Q/dD_1\right)$, $i = 1,2$, where $Y_1' = dY_1/d\hat{X}_{1z}$. From Lemma 1 the term in parenthesis is positive, hence $\left(dV_1/dD\right) \approx Y' = 3\hat{X}_1 - 2\hat{X}_{1z}$. If firm 1 uses only equity, $D = 0$ $\Leftrightarrow$

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[18] The assumption $\partial\hat{X}_{1z}/\partial X_{1z} \le 1$ (equivalently, $\left(\partial\hat{b}_z/\partial b_z\right) \le 3$), is innocuous with respect to all interesting results, while significantly simplifying the analysis by reducing the number of cases to be examined. The benchmark case of a uniform distribution satisfies this condition.

[19] The notion of an all-debt capital structure is a *limit case*, since it is not compatible with the subsequent price-setting decision of firm 2, where price is chosen by maximizing the value of equity.

$\hat{X}_1 = \hat{X}_{1z} \Leftrightarrow Y' > 0$, which means that some debt will always be used. Setting $Y' = 3\hat{X}_1 - 2\hat{X}_{1z} = 0$, we obtain the interior solution $\hat{X}_{1z}^{Q} = (3/2)\hat{X}_1$. The second order conditions can be easily shown to hold. If at $\hat{X}_{1z} = \bar{X}_1 \equiv (2\bar{b} - a)/3$, $Y' > 0$, we have a corner solution with all-debt financing, QED.

In order to see the intuition of Proposition 1, we re-write Y' as

$$Y_1' = dY_1 / d\hat{X}_{1z} = \hat{X}_1 - 2\left(\hat{X}_{1z} - \hat{X}_1\right). \tag{15}$$

Since $\hat{X}_1$ is the expected market share of firm 1, any strategic effect that succeeds in increasing the equilibrium price increases revenue by an amount $\hat{X}_1 dp_1$. Hence, the first term of the derivative in (15) represents the marginal benefit from an increase in debt. Debt, however, implies also an agency cost in the form of price distortion at the third stage. This agency cost is represented by the second term in (15) which is proportional to the difference $\hat{X}_{1z} - \hat{X}_1 = \hat{X}_1 (r-1)$, hence, proportional to the degree of bankruptcy risk. Since the second term is zero for the unlevered firm and $\hat{X}_{1z}$ is continuous in $\left[\hat{X}_1, \bar{X}_1\right]$, some positive amount of debt will always be contained in the optimal financial structure. An interior solution requires sufficient leverage so that the marginal cost of debt (2nd term) becomes equal to the (constant) marginal benefit of debt (1st term). In a corner solution the fluctuation in the taste parameter *b* is not sufficient to set marginal cost equal to the marginal benefit of debt. It can be shown that with the restrictions imposed on parameter values an interior solution requires a skewed distribution. For all symmetric distributions (including the uniform) the optimal financial structure turns out to be the corner solution of 100% debt. This result is due to the requirement that the market be a natural duopoly at any realization of the random variable *b*.[20]

Two interesting features of game *Q* derive from Proposition 1. First, the condition in (15) determines the risk of bankruptcy rather than debt itself.[21] Second, while quality choices affect equilibrium debt, they do not affect equilibrium

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[20] The condition for an internal solution requires $6\hat{b} - 4\bar{b} \le a$. A necessary condition for this for a symmetric distribution is that $3\underline{b} \le a + \bar{b}$, which cannot hold given the natural duopoly assumption for all values of the random parameter.

[21] Recall that $\hat{X}_1$ is a parameter, depending only on consumer tastes and the distribution of the random variable. Thus, changes in $\hat{X}_{1z}$ are equivalent to changes in *r*.

bankruptcy risk, r^Q; the latter depends only on consumer tastes and the distribution of the random variable. Hence, in game Q the optimal bankruptcy risk is like a parameter determined prior to the game. For any qualities chosen in the first stage, debt simply adjusts to the necessary level in order to set the ratio $\hat{X}_{1z} / \hat{X}_1$ equal to r^Q.

Since qualities are chosen before debt through total-value maximization, equilibrium qualities are given by (6) and (7) with $\hat{X}_1$, $\hat{X}_2$, replaced by $\hat{X}_{1z}^Q$, $\hat{X}_{2z}^Q$, as determined by Proposition 1.[22] Using $\hat{X}_{iz}^Q$, equilibrium qualities, and (11), one can straightforwardly determine equilibrium price levels.

VI. Prices, Differentiation and Firm Value

In this section we perform comparative statics analysis on endogenous variables, focusing mainly on the comparison between the levered and the unlevered equilibrium. We start by comparing equilibrium qualities in the two situations.

Proposition 2: *Even though qualities are chosen before financial structure, the presence of leverage in equilibrium increases both optimal quality levels,* i.e., $u_i^Q \geq u_i^e$, $i = 1, 2$.

Proof: See the appendix.

This result is due to the fact that *for any given qualities,* debt increases both prices. Thus, the anticipation of debt increases both firms' marginal revenue from quality improvements, shifting firm 1's reaction function in the qualities space to the right, and that of firm 2 upwards. Since the reaction function of the lower quality is upward sloping—from (7), we have $u_1^Q > u_1^e$ and $u_2^Q \geq u_2^e$.

Hereafter, we restrict the form of the cost-of-quality function to $F(u_i) = \lambda u_i^{\beta}$, $i = 1, 2,$ with $\beta > 1$ in order to satisfy the convexity requirement $F'' > 0$. Such a form has been commonly used in many studies.[23] While being sufficiently general in order to approximate most smooth convex functions, it allows us to obtain well defined

[22] $\hat{X}_{2z}$ is a linear transformation of $\hat{X}_{1z}$, exactly like $\hat{X}_2$ of $\hat{X}_1$.

[23] See, for instance, Motta *et al.* (1997).

results. The size of the convexity factor β turns out to be significant for some of our results, while λ, a scale factor, plays no role in the results and will be normalized to 1. Using this specific form of the cost function we are able to show that

Proposition 3: *In game* Q *the value of firm 1 is a decreasing function of bankruptcy risk.*

Proof: See the appendix.

Proposition 3 shows that it is optimal for the high quality firm to opt for an all-equity structure when quality choices are endogenous. Despite this, though, firm 1 cannot avoid leverage, unless there is some commitment mechanism allowing it to do so. The reason for this is that, since r^Q only depends upon parameter values, firm 1 behaves as if it were being committed from the outset to a given degree of bankruptcy risk. Firm 2 knows, therefore, that any quality choice it makes at the first stage will induce its rival to take the necessary amount of leverage in order to establish the bankruptcy risk at the r^Q level. Having assured a softer reaction from its rival, firm 2 increases its quality above u^e, leading at the same time firm 1 to a total-value reduction. This can be seen by writing:

$$\frac{dV_1^Q}{dr} = \frac{\partial V_1^Q}{\partial Y}\frac{\partial Y}{\partial r} + \frac{\partial V_1^Q}{\partial u_1}\frac{\partial u_1}{\partial r} + \frac{\partial V_1^Q}{\partial u_2}\frac{\partial u_2}{\partial u_1}\frac{\partial u_1}{\partial r} \qquad (16)$$

The envelope theorem implies that at the optimal choice of u_1 the second term of (16) is zero. Assume for the sake of the argument that the financial decision at the second stage yields an interior solution, implying that the first term is also zero. The third term is negative: its first component is negative, since an increase in u_2, with fixed u_1, implies a reduction in product differentiation; its second component is positive since the quality reaction function of firm 2 has positive slope; its third component has been shown to be positive in Proposition 2. Clearly, the negativity of dV_1^Q/dr is due to the fact that firm 1's leverage induces firm 2 to upgrade its quality.[24] Note also that fixing both qualities at their unlevered equilibrium levels would have resulted in $dV_1^Q/dr > 0$, since the second term of (16) would become positive ($u_1^e < u_1^Q$), and the third zero. This implies that, when products are differentiated, some amount of

[24] The proof contained in the appendix shows that this result is not limited to the neighborhood of the interior solution, holding for all admissible values of the parameters.

leverage increases the value of the levered firm (as in Showalter), but only as long as the rival's quality is exogenous. When the reaction of rival quality to changes in debt is taken into account, debt is no longer profitable.

Proposition 1 implies that firm 1 wishes to commit before stage 1 to undertaking the smallest possible bankruptcy risk. When the maximum realization of *b* does not deviate too much from the mean of the distribution, even 100% debt (corner solution) implies limited bankruptcy risk.[25] The latter increases with the width of the distribution of *b* until the interior solution ($r = 3/2$) is reached.[26] It follows that a reduction in the fluctuations of *b* acts like a market-imposed (but desirable) commitment. On the other hand, increases in demand uncertainty reduce the value of firm 1, even under risk neutrality.

Product differentiation can be expressed either as a quality ratio, u_1/u_2, or as the difference $\Delta u = u_1 - u_2$. The latter is a more important expression since it constitutes a component of equilibrium prices and firm values.

Proposition 4: *i) Leverage reduces the* u_1/u_2 *ratio, relative to the all-equity case; ii)* $\forall r^Q, \hat{X}_1$, $\exists \overline{\beta} \in (1,2)$ *such that* $\forall \beta \geq \overline{\beta}$ *leverage reduces* Δu*, relative to the all-equity case.*

Proof: See the appendix.

The above result shows that in most cases—including the commonly used function $F(u) = (1/2)u^2$—debt reduces product differentiation.[27]

Debt affects product differentiation in two different ways. The first one is related to the BLS effect, which tends to raise both qualities by increasing the marginal revenue from quality increments. This BLS effect, however, has an ambiguous overall impact on differentiation, since on the one hand the increase in marginal revenue is higher for the high quality, but on the other hand the convexity of the cost function implies that any given increase in marginal revenue translates into a

[25] When *b* does not fluctuate at all even all-debt financing bears no bankruptcy-risk.
[26] With a uniform distribution this never occurs within the limits we have imposed on the width in order to maintain covered market in all cases.
[27] When the optimal financial structure is all-debt (as is the case with the uniform distribution of *b*) we may find values of β sufficiently close to 1 for which differentiation is increased by debt. No such cases exist, however, when the optimal financial structure contains both equity and debt.

more important increase of the low quality. It turns out that, unless the cost function is very flat, the BLS effect tends to reduce product differentiation. In addition, debt also affects differentiation because, as shown in our analysis, the two are substitutes in achieving any given level of equilibrium prices.[28] For most degrees of convexity (at least $\forall\beta \geq 2$) both the BLS effect and the substitution effect work towards reducing differentiation.

We also examine the effect of debt on prices, which has three components. The first operates through distorting the objective of the decision maker at the last stage. This effect induces a softer reaction of the leveraged firm (BLS effect), which in turn results in, *ceteris paribus*, higher prices for both products.

The second component operates through the increase in qualities and also tends to increase prices, since *ceteris paribus* consumers are willing to pay higher prices for higher qualities (quality effect). The third effect of debt on prices operates through the impact of debt on product differentiation and has the opposite direction: a reduced Δu implies, *ceteris paribus,* stiffer price competition and lower prices (reduced-differentiation effect). The next result (proven in the appendix) shows that the BLS and quality effects together dominate the reduced-differentiation effect.

Proposition 5: *In game Q leverage increases both prices.*[29]

The following conclusions emerge from game Q. First, for *given* qualities it is always optimal for the high quality producer to include debt in its financial structure. Second, the availability of debt financing affects both quality choices upwards, despite the fact that quality levels are decided before the choice of financial structure. Third, the availability of debt financing reduces product differentiation. Fourth, despite the reduction in product differentiation, debt leads to higher prices for both products. Fifth, the availability of debt reduces the value of firm 1.

VII. Other Types of Uncertainty

In this section we examine the robustness of the above conclusions under alternative specifications of uncertainty. We consider two cases.

[28] Recall, from (11), that debt and product differentiation are "substitutes" in producing a given change in equilibrium prices.
[29] The proof with respect to the low quality price holds for any cost of quality function.

Generalization 1: A *constant number of consumers*.

The variability of the upper end of the taste distribution on firm 1's value and behavior has a dual effect, since as b increases both the size of the market and the dispersion in consumer taste increase. More dispersion in consumers' tastes implies *ceteris paribus* less competition, hence a higher revenue for both firms. This subsection eliminates the uncertainty over the market size and shows that uncertainty over the "stiffness" of competition alone is sufficient to generate the results described in Propositions 1-3.

Let now the total number of consumers be given and equal to K, and assume b to be uniformly distributed over $\left[\underline{b},\bar{b}\right]$, with density equal $\left(b-a\right)^{-1}$, instead of 1 (its value in the previous section). The sales of firm 1 are now $K\dfrac{b-t_B}{b-a}=K\left(1-\dfrac{t_B-a}{b-a}\right)$; without loss of generality we normalize K to 1. Now uncertainty in the upper range of the taste scale implies also uncertainty over the density of the distribution, thus affecting *both* firms' revenue. The revenue of firm 2 now becomes random and that firm's financial structure may include leverage, implying also the possibility of default as well. As the intuition suggests, while firm 1's revenue is increasing in the realization of b, the opposite holds for the revenue of firm 2.[30]

In the case where both firms are all-equity financed, price maximization of their expected profit functions yields the following reaction functions

$$p_1=\left(1/2\right)\left(\varphi\left(\underline{b}\right)\cdot\Delta u+p_2\right),\quad\text{and}\qquad p_2=\left(1/2\right)\left(-a\cdot\Delta u+p_1\right)\qquad\text{(4a')}$$

where $\varphi(\underline{b})\equiv\Phi(\underline{b})^{-1}+a$, with $\Phi(\underline{b})\equiv\dfrac{1}{\bar{b}-\underline{b}}\ln\dfrac{\bar{b}-a}{\underline{b}-a}$.[31] On the other hand, when both firms are levered, maximizing the expected *equity* results in the following reaction functions:

$$p_1=\left(1/2\right)\left(\varphi(b_z)\cdot\Delta u+p_2\right),\qquad\text{and}\qquad p_2=\left(1/2\right)\left(-a\cdot\Delta u+p_1\right)\qquad\text{(10')}$$

where $\varphi(b_z)=\Phi(b_z)^{-1}+a$, with $\Phi(b_z)\equiv E\left[\dfrac{1}{b-a}\middle|b\geq b_z\right]$, which are similar to those in (10) with $\varphi(b_z)$ replacing $\hat{b}$. Consequently, equilibrium prices of the levered firms are as in (11) with $\hat{X}_{iz}$, i=1,2 redefined for the purposes of this subsection, with $\varphi(b_z)$

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[30] This can be directly observed from equations (3') in the appendix.

[31] Expressions (4a') are the equivalent to (4a) in the previous analysis.

replacing $\hat{b}_z$. Comparing (10′) to (4′) and noting that $\varphi(b_z) > \varphi(\underline{b})$, shows that, *ceteris paribus,* leverage raises the equilibrium prices in this model as well. Observe also that firm 2's debt does not affect the pricing stage, since the firm 2 price reaction in (10') is the same as in (4a). As a consequence, the financial structure of the low quality producer is irrelevant for equilibrium values, exactly like in the previous section. In the appendix it is shown that the price-maximized total value of firm 1 is exactly like (14) with the aforementioned redefinition of $\hat{X}_{1z}$; hence, all the steps for the proofs of Propositions 1-3 remain unchanged. Since the results of Game *Q* carry through to the case where uncertainty does not affect total market size, they must be mainly attributed to uncertainty over the degree of competition and the resulting relative market shares.

Generalization 2: ***Double-sided uncertainty***

Assume both upper and lower taste limits are random, $b \in \left[\underline{b}, \overline{b}\right]$ and $a \in \left[\underline{a}, \overline{a}\right]$ with a common distribution. Since willingness-to-pay may reflect changes in income,[32] this case may reflect uncertainty over the general economic situation. In order to preserve the natural duopoly assumption, let the ratio (b/a) be fixed at some value $\kappa \in (2,4)$. The all-equity firm equations (3)-(7) remain virtually unchanged, with $\hat{a}$ replacing a. Since firm 2 is now facing uncertainty as well, its financial structure is no longer trivial. In the presence of debt in the structure of firm 2 we have, over and above (8), the additional equation:

$$D_2 = p_2 \cdot (t_B - a_z) - F(u_2), \tag{8″}$$

with firm 2 being solvent for $a \in \left[\underline{a}, a_z\right]$ and in default for $a \in \left(a_z, \overline{a}\right]$. Let $\hat{a}_z \equiv E\left[a \middle| \; a \le a_z\right]$ denote the truncated expectation of a conditional on firm 2's solvency. At the third stage firm 2 chooses its price by maximizing its equity value

$$E_2 = \Pi_2 - D_2 = \int_{\underline{a}}^{a_z} \left[p_2 \left(a_z - a\right)\right] dG(a), \tag{9″}$$

which yields the reaction function

$$p_2 = (1/2)(-\hat{a}_z \cdot \Delta u + p_1) \tag{10″}$$

[32] See Tirole (1988), p. 97, footnote 1.

Combining (10) with (10″) we see that the solution of the price stage is still given by (11), with $\hat{X}_{iz}$, i=1,2, now redefined, with $\hat{a}_z$ replacing a; this redefinition holds throughout the entire subsection. Obviously, for given qualities debt increases both prices. Replacing equilibrium prices into (4c) we note that the identity of the indifferent consumer $t_B = \left(\hat{b}_z + \hat{a}_z\right)/3$ is still deterministic, but the market shares, $b - t_B$, and $t_B - a$, are both stochastic, and so are firm revenues. Comparing (10″) to the price reaction function of firm 2 in (10′) reveals that, while firm 2's revenue becomes stochastic under both modifications of uncertainty, only under double-sided uncertainty its financial structure is relevant in determining the equilibrium.

Substituting optimal prices into (5a) we see that the total value of firm 1 is still given by (14) with the aforementioned redefinition of $\hat{X}_{1z}$; hence, Lemma 1 and Propositions 1-3 hold for firm 1 in this case, as well. Substituting optimal prices into (5b) we obtain the price-maximized value of firm 2:

$$V_2 = \int_{\underline{a}}^{\overline{a}} p_2\left(\frac{p_1 - p_2}{\Delta u} - a\right) dG(a) - F(u_2) = (1/2)\, Y_2 \Delta u - F(u_2), \tag{14″}$$

where $Y_2 = \hat{X}_{2z}(3\hat{X}_2 - \hat{X}_{2z})$, with $\hat{X}_2 = \left(\hat{b}_z - 2\hat{a}\right)/3$. Since (14″) is symmetric to (14) and $\left(d\hat{X}_{2z}/dD_2\right) \geq 0$,[33] Proposition 1 holds for firm 2 as well.[34, 35] Proposition 2 obviously holds for this case as well, while the only thing that changes with respect to the proof of Proposition 3 is that, if we now define $x \equiv \hat{X}_1/\hat{a}$, the lower limit of x is still 1 but its upper limit may be now higher than 7/3. This, however, does not affect the proof, therefore Proposition 3 holds in this case as well.

In conclusion, the effects of leverage in Game Q are robust as to the varying specifications of uncertainty. Leverage softens price competition and increases quality

[33] The equivalent of Lemma 1.
[34] The proof is omitted since it is absolutely symmetric to that of Proposition 1.
[35] The proof of Proposition 1 does not require a uniform distribution. However, if such a distribution is imposed on the fluctuations of b, one-sided uncertainty always results in a corner solution with all-debt financing for firm 1. With double-sided uncertainty this outcome is not necessary, as there may well be an interior solution in both a_z and b_z. Consider, for instance, the case $\overline{b} = 4\underline{b}$, implying also $\overline{a} = 4\underline{a}$. Then the interior solutions at both ends imply that we must have feasible solutions to the following system: $\hat{X}_{1z} = (3/2)\hat{X}_1$, $\hat{X}_{2z} = (3/2)\hat{X}_2$, which reduces to: $6\hat{b} - 4\hat{b}_z + \hat{a}_z = 0$, $\hat{b}_z - 6\hat{a} + 4\hat{a}_z = 0$. Set now $(b/a) = 2.5$ and the solution to the system becomes $\hat{b}_z = (66/17)\underline{b} < \overline{b}$, $\hat{a}_z = (90/68)\underline{a} > \underline{a}$.

levels while reducing the value of the high quality firm. These conclusions hold under either one- or double-sided uncertainty, as well as under fixed or varying market size.

VIII. Game *S*

In this section we perform a robustness check of Proposition 5 with respect to the order of moves, by asking whether debt can be value-enhancing for firm 1 if contracted *before* the actual determination of quality levels. Thus, in Game *S* firm 1 chooses its financial structure at the first stage, while at the second stage both firms simultaneously choose the quality level of their product. If firm 1 carries a positive amount of leverage from the first stage, its quality reaction-function is obtained by maximizing equity rather than total firm value, due to the agency problem, therefore both qualities are functions of firm 1's debt level.[36] At the third stage both firms decide their price. For given debt and qualities, the pricing stage of game *S* is similar to that of game *Q*.

Analytically, the high quality is chosen in game *S* by maximizing the value of equity as given in (12), subject to the constraint in (13). Besides being essential steps for the proof of Proposition 6, the following three lemmas 2 to 4 offer some intuition on the relation between debt, qualities and the degree of bankruptcy risk. Their proof is relegated to the appendix.

We deal first with the *ceteris paribus* impact of a change in the high quality on the degree of bankruptcy risk. An increase in the high quality a) raises firm 1's market share (for given prices), b) raises rival price, c) raises own price, and d) raises cost. The first two of these effects tend to reduce the number of bankruptcy states while c) and d) have the opposite effect. Assuming *b* is distributed uniformly, it can be shown that c) and d) dominate, hence:[37]

Lemma 2: *For any given debt level* D_1 *and any given lower quality level* u_2 *an increase in the optimally chosen quality level* u_1 *increases the bankruptcy risk* b_z.

Next, we deal with exogenous changes in debt and show that:

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[36] Recall that the capital structure of firm 2 is irrelevant

[37] The assumption that *b* is distributed uniformly, is maintained throughout this subsection. While none of the results holds uniquely under this type of distribution, this assumption simplifies all the proofs considerably.

Lemma 3: *For any given bankruptcy-inducing debt level* D_1 *both equilibrium qualities exceed their optimal levels in the absence of debt.*

Lemma 4: *The equilibrium quality levels are increasing functions of the debt level if the latter is bankruptcy-inducing.*

We have shown that if the equilibrium structure of game *S* contains debt, both qualities are higher than in the no-debt case (as in game *Q*), the difference depending directly on the amount of debt contracted. Thus, debt increases bankruptcy risk both directly and indirectly through quality increments. It now remains to find the financial structure that maximizes firm 1's value. In order to do this we start by checking whether this optimal structure should contain any debt at all, and as we show:

Proposition 6: *In game* S *the optimal financial structure of firm 1 is all equity.*[38]

Proof: Let $u_1^e\left(D_1\right)$ be the value-maximizing u_1 for levels of debt $D_1 \geq \tilde{D}_1$,[39] where $\tilde{D}_1$ is the minimum bankruptcy-inducing debt. Substituting u_1^e and u_2^e from (7) into (14), we re-write the price-maximized firm value as

$$V_1^S = a\hat{X}_{1z}^S\left(\frac{3\hat{X}_1 - \hat{X}_{1z}^S}{a + \hat{X}_{1z}^S}\right)u_1^e - F\left(u_1^e\right), \tag{17}$$

where the parameter $b_z\left(D_1\right)$, and therefore the value of $\hat{X}_{1z}^S\left(D_1\right)$, have been set equal to their corresponding equilibrium values of the quality stage. We show that the optimal structure contains no debt by showing that the best firm 1 can do with debt is inferior to the no debt solution. For this observe that

$$\mathrm{Max}_{D_1}\left\{V_{1z}^S\left(b_z\left(D_1\right), u_1^e\left(D_1\right)\right)\right\} \leq \mathrm{Max}_{D_1}\left\{\mathrm{Max}_{b_z}\left\{V_{1D}\left(b_z, u_1^e\left(D_1\right)\right)\right\}\right\}.$$

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[38] Or, equivalently, it may contain any amount of debt low enough as to exclude any *bankruptcy risk* and the ensuing *agency cost*: for $D_1 \leq \tilde{D}_1$ it makes no difference whether we include debt or not in the financial structure.
[39] *I.e.,* in the presence of bankruptcy possibilities, with $b_z > \underline{b}$.

It is easy to see that the coefficient of u_1^e in (17) is maximized when $b_z = \underline{b}$. In such a case, however, the optimal structure turns out to be all-equity, or, more generally, contains no bankruptcy-inducing debt, QED.

Bringing Propositions 3 and 6 together shows that, independently of the sequence of moves, *when quality choices are endogenous, debt always reduces* the value of the levered firm, implying that the optimal financial structure is all-equity. Unlike game *Q*, the structure of game *S does* allow firm 1 to avoid leverage. The equilibrium value of firm 1 is, therefore, higher in game *S*. This result is, however, an analytical artifact, since the structure of game *S* assumes a commitment to no leverage *after* the quality choices are made. In reality, while debt before the stage of product development can be credibly avoided by proceeding to product development using only equity financing, it is not obvious how firm 1 can still credibly commit at the quality stage that it will take no debt at a later stage. Thus, game *S* shows that contracting debt *before* the rival quality choice does not change the negative impact of the strategic effects of leverage on the value of firm 1. On the contrary, the next section shows that these effects are more pronounced when debt is used in order to finance product development.

IX. THE COST OF DEBT

The above discussion focuses exclusively on agency costs and their strategic implications, abstracting from "frictions" such as taxes, transactions costs, or limits to the availability of equity financing, which are important in most real world situations. If such factors are taken into account, some leverage may still be desirable in game *S*. For this reason, in what follows we compare the effects of debt on firm value and other variables of interest, between the two games.

We start by noting that the agency costs and strategic effects of debt that played a role in Propositions 3 and 6 were related to the size of the bankruptcy risk, which we measured by the quantity $\hat{b}_z - \hat{b}$. Suppose we keep this bankruptcy risk exogenously given and constant in both games *Q* and *S*, which corresponds to a given value of $b_z \in (\underline{b}, \overline{b})$. What would be the effects of the alternative game structures on quality levels and other variables of interest in the two games? Let $u_{iz}^h,\ h = Q, S$,

$i = 1, 2,$ denote the quality levels that results from the two games with the exogenously given b_z. We can now prove the following result, assuming always a uniform distribution for the taste parameter.

Proposition 7: *For any exogenously given value* $b_z \in (\underline{b}, \bar{b})$ *the quality levels in game S exceed those in game Q, or* $u_{iz}^Q < u_{iz}^S$, $i = 1, 2$.

Proof: See the appendix.

The above result is due to the fact that in game *S*, not only p_1, but also u_1 is chosen under equity maximization. The role of this double distortion is illustrated in Table 1, where we compare the values of some relevant variables under the two game structures, for exogenously given levels of b_z. At each value of b_z correspond two lines, the first (white) presenting equilibrium values for game *Q* and the second (light grey) for game *S*.[40]

We first note that for any entry of the table $u_1^S > u_1^Q$, while the corresponding face value of debt is greater in game *Q*.[41] Since u_1^Q represents the firm-value maximizing quality level, the $u_1^S - u_1^Q$ difference represents the magnitude of the agency distortion of the quality choice. From (7), $u_1^S > u_1^Q$ implies $u_2^S > u_2^Q$, as well. We already know from the discussion following Proposition 3 that $u_2^Q > u_2^e$ and that this is at the root of the firm-value reducing impact of debt. Obviously, expecting its rival to choose quality by maximizing equity instead of total value induces firm 2 to further increase its quality, thereby further reducing the value of firm 1: in all entries of Table 1, $V_1^Q > V_1^S$, for the same level of bankruptcy risk. Hence, the strategic cost

[40] While we are using for convenience the equilibrium superscripts to distinguish between the two decision sequences, it must be recalled that these are not equilibrium values of the entire game since b_z is exogenous.

[41] Alternatively, instead of comparing games *Q* and *S* for equal levels of bankruptcy risk b_z, one could perform the comparisons for equal level of debt, as follows: first, for a given value of b_z calculate the equilibrium values of Q and the corresponding value of D_1^Q that verifies (13); then start with that value as exogenously given and calculate the corresponding parameters of *S*. This type of comparison has been avoided because it is computationally heavy. Note, however, that, since $\partial b_z / \partial D_1 \geq 0$ and $\partial u_1^S / \partial b_z \geq 0$, the difference $u_1^S - u_1^Q$ would have been even higher had the comparison been performed using equal amounts of debt, instead of the same level of bankruptcy risk.

of debt is greater when quality choices are affected by the choice of financial structure. *Ceteris paribus,* product development-and-commercialization projects must use less debt than those aiming at only commercializing given qualities.

Similar conclusions can be reached by looking at the *equilibrium* level of the debt/value (leverage) ratio, $\mu_1^h \equiv \dfrac{V_1^h - E_1^h}{V_1^h}$, $h = Q, S$. Note that $D_1^h \neq V_1^h - E_1^h$, since the former is the promised repayment while the latter the value of debt in equilibrium. Clearly, μ_1^h is the financially meaningful degree of indebtedness. All the observations in the μ_1^h column show that in order to cause the same bankruptcy risk (same b_z), it takes a larger leverage ratio in game *Q* than in game *S*. This is again another consequence of the added quality distortion in game *S*.

b_z	D_1^h	u_1^h	E_1^h	V_1^h	μ_1^h	u_2^h	V_2^h
2.1	319358	713678	351390	669663	0.475	110570	31989
	295170	742166	365416	659458	0.445	114983	17746
2.2	393609	717829	282094	665786	0.576	119638	37945
	362579	757033	297501	649621	0.542	126172	20897
2.3	469971	721741	219280	661282	0.668	128529	44397
	433162	771986	234546	637791	0.632	137477	24265
2.4	548349	725420	163452	656182	0.751	137242	51342
	506930	787014	177330	623918	0.716	148895	27841
2.5	628650	728869	115085	650516	0.823	145774	58783
	583890	802111	126649	607953	0.792	160422	31618

Table 1[42]

In conclusion, the double distortion introduced by the decision sequence of game *S* makes the cost of debt heavier in terms of firm value than in game *Q*. This implies that relatively more debt will be used in financing only product commercialization, as compared to financing both product development and commercialization.

[42] For every value of b_z, the first and second lines correspond to equilibrium values of games *Q* and *S*, respectively. Calculations are based on the following parameter values: $a = 1$, $\overline{b} = 3$, $\underline{b} = 2.05$, $\lambda = 1.2$, and $\beta = 3$. All values except those of μ^h, $h = Q, S$, have been multiplied by 10^6.

X. CONCLUSIONS

In the context of an address model of vertical differentiation we introduce uncertainty over the realization of the upper end of the taste distribution. Our purpose is to examine the interaction between debt and product differentiation, two variables that turn out to be substitutes in relaxing price competition. While the low end of the consumer taste distribution is known with certainty, the position of the high end is uncertain and only its distribution is known. This situation corresponds to the introduction of a new and improved version of a product. While the new product is definitely of higher quality compared to the existing product, the exact quality level of both products is determined simultaneously. We perform our analysis with two alternative decision sequences, represented by games *Q,* and *S*, according to whether qualities are decided before or after financial structure. The former represents financing projects aiming at the commercialization of given qualities, while the latter to projects involving product development as well.

Independently of the decision sequence, debt increases both qualities. Unless the cost function is very flat with respect to quality, debt reduces product differentiation. This is due to the reaction of the lower quality which, knowing that competition is softened by the presence of debt will opt for a higher quality level. This confirms the substitutability between debt and differentiation in relaxing price competition. Equilibrium prices are higher, compared to situations where firm 1 uses only equity financing.

Despite higher prices and independently of the decision sequence, when quality choices are endogenous debt always reduces firm value. This happens because, knowing that rival leverage will soften competition anyways, the low-quality firm chooses a quality level that is much closer to that of its rival. This conclusion is in sharp contrast with previous literature, where in a differentiated-products environment with Bertrand-type competition debt is value enhancing. These results are robust to alternative specifications of uncertainty.

Turning now to the decision sequence, we observe that making the financial decision before the rival firm's quality decision increases the cost of debt. This is due to the fact that the prior choice of leverage induces a second agency distortion: not only the price but also the quality of firm 1 is now chosen under equity instead of total value maximization. Qualities are even higher with respect to their optimal level, compared to game *Q*, resulting in an even lower value. Thus, in the presence of frictions that make some debt desirable, or perhaps unavoidable, the cost of the latter is higher in the *S* game. Our model predicts that commercialization projects are more likely to carry a larger degree of leverage compared to product development projects.

We have not considered entry in our models, and we have also limited our firms to the introduction of a single quality. This is justified by our natural duopoly assumption for all levels of the random taste parameter. We adopted this assumption in order to focus our results on the effects of debt on the microeconomic equilibrium of the two-firm game. In more complex situations the high quality firm may choose to enter with more than one quality in order to displace the lower quality rival.[43] In such cases, however, the entry game may obscure the impact of the debt on the microeconomic equilibrium, which was the focus of our study.

[43] See, for instance, Constantatos and Perrakis (1997).

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Appendix:

Proof of Lemma 1: Substituting the subgame equilibrium prices into (8) we get

$$\hat{X}_{1z}^{h}\Delta u\left[X_{1z}^{h}+(1/2)\left(X_{1z}^{h}-\hat{X}_{1z}^{h}\right)\right]=D_{1}^{h}+F\left(u_{1}^{h}\right),\ h{=}Q{,}S. \tag{A.1}$$

Note that the above expression implies that X_{1z}^{h} is a function $X_{1z}^{h}\left(\frac{D_{1}^{h}+F\left(u_{1}^{h}\right)}{\Delta u};k\right)$, where k represents a vector of parameters characterizing the taste distribution. Differentiating both sides of (A.1) with respect to D_1 for fixed qualities, and taking into account the restriction $\partial\hat{X}_{1z}^{h}/\partial X_{1z}^{h}\leq 1$, we can easily show that $\partial X_{1z}^{h}/\partial D_{1}\geq 0$, which obviously implies $\partial\hat{X}_{1z}^{h}/\partial D_{1}\geq 0$, QED.

Proof of Proposition 1: In the text

Proof of Proposition 2: The choice of u_1 is always given by (6) with $\hat{X}_1$ or $\hat{X}_{1z}^{Q}$, according to the levered or unlevered case. In an analogous manner, the lower quality is given by (7) with $\hat{X}_2$ or $\hat{X}_{2z}^{Q}$. The expressions (6) and (7) are increasing in $\hat{X}_1$ and $\hat{X}_2$, respectively. Since $\hat{X}_{iz}^{Q}\geq\hat{X}_{i}$, $i=1,2$, the convexity of $F(u)$ implies $u_{i}^{Q}>u_{i}^{*}$. QED.

Proof of Proposition 3: Substituting optimal qualities into (14), setting $\hat{X}_{1z}/\hat{X}_{1}\equiv r$, $\hat{X}_{1}/a\equiv x$ (*i.e.*, normalizing by *a*) and simplifying, we get $V_{1}^{Q}\left(r;x\right)=A\frac{2\beta-rx-1}{rx+1}$, where $A\equiv\left[\frac{rx^{2}\left(3-r\right)}{2\beta}\right]^{\frac{\beta}{\beta-1}}$; note that $\frac{dA}{dr}\equiv A'=A\frac{\beta}{\beta-1}\cdot\frac{3-2r}{r\left(3-r\right)}$. Differentiating with respect to *r* yields

$$\begin{aligned}\frac{dV_{1}^{Q}\left(r;x\right)}{dr}&=A'\frac{2\beta-rx-1}{rx+1}-A\frac{2\beta x}{\left(rx+1\right)^{2}}\propto\\ &-A\frac{2\left(\beta-1\right)rx\left(3-r\right)-\left(3-2r\right)\left(rx+1\right)\left(2\beta-rx-1\right)}{\left(rx^{2}+1\right)\left(\beta-1\right)\left(3-r\right)}.\end{aligned} \tag{A.2}$$

The sign of the RHS of the above expression is negative if the numerator is positive; the latter can be written as $3-2r-7rx+6x+6rx^{2}-4r^{2}x^{2}+\left(-6+4r+2r^{2}x\right)\beta$, where $r\in\left(1,3/2\right]$ and $x\in\left(1,7/3\right]$. The coefficient of β is obviously positive, implying that the expression is increasing in β. Further, the additional term is positive for $r=1$, decreasing in r for all values of $x \in (1, 7/3]$, and equal to zero for $r = \frac{3}{2}$ and $\beta = 1$ for all values of x. Therefore, the numerator of (A.2) is positive and $dV_1^Q / dr < 0$, $\forall \beta > 1$, QED.

Proof of Proposition 4: For part i) note that from (7) we can also write

$$u_2^e = \hat{X}_2^Q \left(u_1^e - u_2^e\right) / a, \quad u_2^Q = \hat{X}_{2z}^Q \left(u_1^Q - u_2^Q\right) / a, \quad j = 1{,}2 \tag{A.3}$$

Again, since $\hat{X}_{2z}^Q > \hat{X}_2^Q$, $\left[\left(u_1^e - u_2^e\right) / u_2^e\right] > \left[\left(u_1^Q - u_2^Q\right) / u_2^Q\right]$ which leads to $\left(u_1^e / u_2^e\right) >$ $\left(u_1^Q / u_2^Q\right)$, thus proving part i).

For part ii) assume first less than all debt-financing (interior solution to equation (15)). From (7) we get $\Delta u^Q - \Delta u^e \geq 0 \quad \Leftrightarrow \frac{u_1^Q a}{a + \hat{X}_{2z}^Q} \geq \frac{u_1^e a}{a + \hat{X}_2}$, where $u_1^e = F'^{-1}\left(\hat{X}_1^{\,2}\right)$ from (6), and $u_1^Q = F'^{-1}\left(Y_1/2\right)$ since firm 1 chooses quality as to maximize the total firm value, given by (14). Replacing $\hat{X}_{2z}^Q = \left(\hat{X}_{1z}^Q - a\right)/2$, $\hat{X}_2 = \left(\hat{X}_1 - a\right)/2$ from the definitions of the corresponding variables, and re-arranging, we obtain that $\Delta u^Q - \Delta u^e \geq 0$, implies

$$\frac{F'^{-1}\left(Y_1/2\right)}{\left(a + \hat{X}_{1z}^Q\right)} \frac{\left(a + \hat{X}_1\right)}{F'^{-1}\left(\hat{X}_1^{\,2}\right)} \geq 1. \tag{17}$$

Note that a) the second fraction in (17) is invariant with respect to changes in $\hat{X}_{1z}^Q$, depending only on parameters, and b) for the all-equity firm $\hat{X}_{1z}^Q = \hat{X}_1$, therefore at zero debt the LHS of (17) equals 1. Hence, in game Q the necessary and sufficient condition for Δu to increase is

$$\frac{d}{d\hat{X}_{1z}^Q}\left(\frac{F'^{-1}\left(\frac{Y_1}{2}\right)}{a + \hat{X}_{1z}^Q}\right) \geq 0, \ \forall \hat{X}_{1z}^Q \in \left[\hat{X}_1, \frac{3\hat{X}_1}{2}\right). \tag{A.4}$$

This derivative turns out to be positive iff

$$\beta \leq \frac{\left(3\hat{X}_1 - 2\hat{X}_{1z}^Q\right)\left(\hat{X}_{1z}^Q + a\right)}{\hat{X}_{1z}^Q\left(3\hat{X}_1 - \hat{X}_{1z}^Q\right)} + 1. \tag{A.5}$$

The first term in the RHS of (A.5) is always less than 1 and its maximum value is at $\hat{X}_{1z}^{Q} = \hat{X}_1$. When optimal debt is less than 100% (interior solution), at the optimal capital structure of firm 1, $3\hat{X}_1 = 2\hat{X}_{1z}^{Q}$, which implies that the term in question is equal to zero. Hence, there is no convex quality cost function for which optimal leverage increases differentiation in game Q.

When, however, the optimal structure is all-debt, $\hat{X}_{1z}^{Q} = \frac{(2\overline{b} - a)}{3}$, and the RHS of (A.5) is then greater than 1, since the first term is positive. We can rewrite the RHS as $2 + \frac{a}{\hat{X}_{1z}^{Q}} - \frac{(\hat{X}_{1z}^{Q} + a)}{(3\hat{X}_1 - \hat{X}_{1z}^{Q})}$. The last two terms can be shown to be negative, implying that when the optimal structure is all-debt, the RHS of (A.5) lies between 1 and 2. Thus, there exist values of $\beta < \overline{\beta} \in (1,2)$ yielding convex cost-of-quality functions such that differentiation increases, but only when the distribution of the random factor is such that the optimal structure is all-debt.

Proof of Proposition 5: $p_2^Q \geq p_2^e$ by the market coverage condition and the fact that $u_2^Q > u_2^e$ from Proposition 2. Considering now the ratio of the high quality price with and without leverage, we note that, since $p_1^e = \hat{X}_1 \Delta u^e$, and $p_1^Q = r\hat{X}_1 \Delta u^Q$, it can be written as

$$\frac{p_1^e}{p_1^Q} = \frac{1}{r} \frac{\Delta u^e}{\Delta u^Q}. \tag{A.6}$$

Replacing equilibrium qualities, simplifying and setting $\frac{\hat{X}_1}{a} = \hat{x}_1$ and $\frac{\hat{X}_{1z}}{a} = \hat{x}_{1z}$, we get that

$$\frac{\Delta u^e}{\Delta u^Q} = \frac{u^e}{u^Q} \cdot \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} = \left[\frac{2\hat{x}_1^{\,2}}{\hat{x}_{1z}\left(3\hat{x}_1 - \hat{x}_{1z}\right)} \right]^{\frac{1}{\beta-1}} \cdot \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} \equiv L^{\frac{1}{\beta-1}} \cdot C_1, \tag{A.7}$$

where $L \equiv \frac{2\hat{x}_1^{\,2}}{\hat{x}_{1z}\left(3\hat{x}_1 - \hat{x}_{1z}\right)} < 1$, and $C_1 \equiv \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} \geq 1$. Substituting (A.7) into (A.6) we get

$$\frac{p_1^e}{p_1^Q} = L^{\frac{1}{\beta-1}} \cdot C_2 \tag{A.8}$$

where $C_2 \equiv C_1 \cdot \left(\hat{x}_1 / \hat{x}_{1z}\right) = \left(r\hat{x}_1 + 1\right) \big/ \left[r\left(\hat{x}_1 + 1\right) \right] < 1$. Since $L^{\frac{1}{\beta-1}} < 1$ as well, the RHS of (A.8) is also smaller than 1, QED.

GAME S

Proof of Lemma 2: For given D_1 and u_2 the optimal quality reaction of firm 1 is obtained by maximizing (12) subject to (13). Differentiating (13) with respect to u_1 we get $\frac{db_z}{du_1} = \frac{F' - H_2}{H_2' \cdot \Delta u}$. Differentiating (12) and replacing $\frac{db_z}{du_1}$ by the previous expression we get the first order condition as

$$H_1'(b_z)\frac{F'(u_1) - H_2(b_z)}{H_2'(b_z)} + H_1(b_z) = 0\,. \tag{A.9}$$

Observe that (A.9) defines a reaction function $u_1(b_z)$ that is independent of u_2 and D_1; this reaction function is valid as long as $\underline{b} \le b_z \le \bar{b}$. We rewrite (A.9) as $F' = H_2 - \frac{H_1 H_2'}{H_1'}$. Differentiating this last expression with respect to u_1 we find that the sign of $\frac{db_z}{du_1}$ is positive, QED.

Proof of Lemma 3: It suffices to show this result for the equilibrium quality level u_1^S, since the quality level u_2^S is an increasing function of u_1^S in all cases. From Lemma 1 the lowest value of u_1^S is when $b_z = \underline{b}$. Replacing this value into (A.9) we get that the lowest level of u_1^S, for any given bankruptcy-inducing debt level D_1, is given by the equation $u_1^S(\underline{b}) = F'^{-1}\left(H_2(\underline{b}) - \frac{H_1(\underline{b}) \cdot H_2'(\underline{b})}{H_1'(\underline{b})}\right)$. On the other hand the optimal quality u_1^* of the all equity (value-maximizing) firm is given by the equation $u_1^* = F'^{-1}\left(\hat{X}_1^{\,2}\right)$. It is easy to see that $H_2(\underline{b}) - \frac{H_1(\underline{b}) \cdot H_2'(\underline{b})}{H_1'(\underline{b})} > \left(\hat{X}_1\right)^2$, implying $u_1^S(\underline{b}) > u_1^*$, QED.

Proof of Lemma 4: For given u_2 and D_1, the equity maximizing choice of u_1 and b_z is given by the intersection of (13) and (A.9) in the (b_z, u_1) plane. By Lemma 2 we know that (A.9) defines an increasing function in that plane. We also observe that for given (D_1, u_2) there are two values of u_1 solving (13), of which the *larger* is the relevant one. Differentiating (13), we find $sign\left[\frac{\partial b_z}{\partial u_1}\right] = sign\left[F_1' - \frac{D_1 + F(u_1)}{u_1 - u_2}\right]$; this sign is positive for all relevant values of D_1. Hence, the optimal pair (b_z, u_1) is the intersection of two increasing functions. We examine these functions at the upper end of their range, where $b_z = \bar{b}$. Then (A.9) yields $u_1 = F'^{-1}\left[\left(\bar{X}_1\right)^2\right]$, $\bar{X}_1 \equiv \left(2\bar{b}-a\right)/3$, while (13) becomes $\left(\bar{X}_1\right)^2 \Delta u - F\left(u_1\right) = D_1$, with a corresponding value of u_1 greater than $u_1 = F'^{-1}\left[\left(\bar{X}_1\right)^2\right]$. Hence, (13) intersects (A.9) from below on a $\left(b_z, u_1\right)$ plane, and increasing D_1 shifts (13) right (down), implying that the equilibrium $\left(b_z,\ u_1\right)$ *increases* with D_1, QED.

Proof of Proposition 6: In the text

Proof of Proposition 7 : It suffices to show that $u_{1z}^Q < u_{1z}^S$. In game *Q* firm 1 chooses its quality by maximizing value, which is given by the RHS of (14). In game *S*, on the other hand, the firm maximizes equity, which corresponds to the RHS of (12), taking also (13) into account. This yields relation (A.9), which can be rewritten as

$$F'(u_{1z}^S) = H_2(b_z) - \frac{H_1(b_z)H_2{}'(b_z)}{H_1{}'(b_z)}, \tag{A.10}$$

where $H_1(b_z) = \left(3/2\right)\hat{X}_{1z}(\hat{X}_{1z} - X_{1z})[1-G(b_z)]$, $H_2(b_z) = \left(1/2\right)\hat{X}_{1z}(3X_{1z} - \hat{X}_{1z})$.

On the other hand, from (14) we get

$$F'\left(u_{1z}^Q\right) = \left(1/2\right)Y_1 = \frac{1}{2}\hat{X}_{1z}\left(3\hat{X}_1 - \hat{X}_{1z}\right). \tag{A.11}$$

Since $F\left(u\right)$ is convex, it suffices to show that the RHS of (A.10) exceeds the RHS of (A.11). By the definitions of X_{1z} and $\hat{X}_{1z}$ and the uniform distribution assumption we have $\frac{\partial \hat{X}_{1z}}{\partial b_z} = \frac{1}{3}$, $\frac{\partial X_{1z}}{\partial b_z} = \frac{2}{3}$, and replacing we get $H_1{}'(b_z) = -\left(1/3\right)\frac{\bar{b}-b_z}{\bar{b}-\underline{b}}\left[\frac{3b_z+\bar{b}}{2} - a\right]$, with $H_2{}'(b_z) = \frac{1}{6}(4\hat{X}_{1z} + 3X_{1z})$.

Replacing the above into (A.10) and simplifying, we find that the relation $F'\left(u_{1z}^Q\right) < F'\left(u_{1z}^S\right)$ is strictly equivalent to $3\hat{X}_1 < 3X_{1z} + \frac{(4\hat{X}_{1z}+3X_{1z})(\bar{b}-b_z)}{3b_z+\bar{b}-2a}$. This relation can be easily shown to hold if we reduce the RHS by replacing the coefficient 4 by 3 in the numerator of the fraction, QED.

Other Types of Uncertainty

Generalization 1: ***A constant number of consumers.***

Let $b \in \left[\underline{b}, \overline{b}\right]$ be distributed uniformly with density $(b-a)^{-1}$. The expected revenues of the all-equity firms 1 and 2 are now given—instead of (3)—by

$$\hat{\Pi}_1 = p_1 E\left[\frac{b-t_B}{b-a}\right] - F_1 = p_1\left(1 - \frac{t_B - a}{\overline{b} - \underline{b}} \ln \frac{\overline{b}-a}{\underline{b}-a}\right) F_1 = p_1\left[1 - (t_B - a)\Phi(\underline{b})\right] - F_1,$$

$$\hat{\Pi}_2 = p_2 \frac{t_B - a}{\overline{b} - \underline{b}} \ln \frac{\overline{b}-a}{\underline{b}-a} - F_2 = p_2 (t_B - a)\Phi(\underline{b}) - F_2, \tag{3'}$$

where $\Phi(\underline{b}) \equiv \frac{1}{\overline{b}-\underline{b}} \ln \frac{\overline{b}-a}{\underline{b}-a}$. The main change in this formulation is that uncertainty in the upper range of the taste scale affects *both* firms' revenue. Thus, the revenue of firm 2 now becomes random and that firm's financial structure may include leverage, implying also the possibility of default as well. Observe also that firm 1's (2's) revenue is an increasing (decreasing) function of the parameter b. Maximizing and solving the system yields

$$p_1 = (1/2)\left([\Phi(\underline{b})^{-1} + a]\cdot \Delta u + p_2\right), \qquad p_2 = (1/2)\left(-a\cdot\Delta u + p_1\right) \tag{4a'}$$

Setting $\varphi(\underline{b}) \equiv \Phi(\underline{b})^{-1} + a$, we get the equilibrium prices given by (4b) , with $\varphi(\underline{b})$ replacing $\hat{b}$.

In the presence of debt for both firms we have the two additional equations

$$D_1 \equiv p_1\left(\frac{b_z - t_B}{b_z - a}\right) - F_1 = p_1\left(1 - \frac{t_B - a}{b_z - a}\right) - F_1,$$

$$D_2 = p_2 \cdot \frac{t_B - a}{b_x - a} - F(u_2), \tag{8'}$$

with firm 1 being solvent for $b \in \left[b_z, \overline{b}\right]$ and in default for $b \in \left[\underline{b}, b_z\right)$, and firm 2 being solvent for $b \in \left[\underline{b}, b_x\right)$ and in default for $b \in \left[b_x, \overline{b}\right]$. Set $\Phi(b_z) = E[\frac{1}{b-a} | b \geq b_z]$ and $\varphi(b_z) = \Phi(b_z)^{-1} + a$, and let us redefine for the purposes of this subsection, $\hat{X}_{iz}$, i=1,2, with $\varphi(b_z)$ replacing $\hat{b}_z$. Hence, in the price subgame the two firms choose their prices by maximizing their equity values

$$E_1 = \Pi_1 - D_1 = \frac{p_1(t_B - a)}{\overline{b} - \underline{b}} \cdot \int_{b_z}^{\overline{b}} \left(\frac{1}{b_z - a} - \frac{1}{b-a}\right) db$$

$$E_2 = \Pi_2 - D_2 = \frac{p_2(t_B - a)}{\overline{b} - \underline{b}} \int_{\underline{b}}^{b_x} \left(\frac{1}{b-a} - \frac{1}{b_x - a}\right) db \tag{9'}$$

taking also (8') into account. The maximization yields the reaction functions

$$p_1 = (1/2)\left[\left(\Phi(b_z)^{-1} + a\right)\cdot \Delta u + p_2\right] = (1/2)\left(\varphi(b_z)\cdot\Delta u + p_2\right),$$

$$p_2 = (1/2)\left(-a\cdot\Delta u + p_1\right), \tag{10'}$$

which are similar to those in (10) with $\varphi(b_z)$ replacing $\hat{b}$. Consequently, (11) holds as well, with the aforementioned redefinition of $\hat{X}_{iz}$, *i*=1,2. Observe that $\varphi(b_z) > \varphi(\underline{b})$, implying that, *ceteris paribus,* leverage raises the equilibrium prices in this model as well. Observe also that firm 2's debt does not affect the pricing stage, since the firm 2 price reaction in (10') is the same as in (4a).

Instead of (13) and (14), the value of debt and the total value of firm 1 are now given by[44]

$$D_1 = \hat{X}_{1z}\left(\frac{3X_{1z} - \hat{X}_{1z}}{2(b_z - a)}\right)\Delta u - F(u_2), \tag{13'}$$

$$V_1 = p_1\int_{\underline{b}}^{\bar{b}}\left[1-\left(\frac{p_1 - p_2}{\Delta u} - a\right)\frac{1}{b-a}\right]db - F(u_1) = (1/2)Y_1\Delta u - F(u_1) \tag{14'}$$

where $Y_1 = \hat{X}_{1z}\left(1-\frac{\varphi(b_z)-2a}{3}\Phi(\underline{b})\right) = \hat{X}_{1z}\left(3\hat{X}_1 - \hat{X}_{1z}\right)$, a mere redefinition of Y (see equation (14)). Differentiating (13b) and noting that $\Phi(b_z) = E\left[(b-a)^{-1}\middle|b \geq b_z\right] \in \left[\left(\bar{b}-a\right)^{-1}, (b_z - a)^{-1}\right]$, we find that $\left(db_z/dD_1\right) \geq 0$, implying that b_z can replace D_1 as the decision variable, as in Lemma 1. Noting further that $\frac{\partial\varphi}{\partial b_z} > 0$, we can maximize (14') with respect to $\hat{X}_{1z}$. Since (14') is similar to (14), all the steps that prove Propositions 1-5 for the case of constant number of consumers are identical to those proving these propositions in Game *Q*.

[44] With $X_{1z} = \frac{2b_z - a}{3}$, $\hat{X}_{1z} = \frac{2\varphi(b_z) - a}{3}$.