Endogenous Product Qualities, Financial Structure, and Firm Value

by[1]

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Abstract: We examine the interaction between financial and microeconomic decisions in a differentiated duopoly under uncertainty as to the heterogeneity of consumer taste for quality. Financing is by equity and/or debt, with limited liability in case of bankruptcy. Product specification is *endogenous.* The upper-end of the taste distribution is uncertain, and so is the density which varies as to keep the size of the market constant. We consider a game where firms choose qualities (first stage), financial structure (second stage), and prices (third stage), and then uncertainty is resolved and the state of demand revealed. Once debt is contracted, the manager maximizes equity instead of total value. We find that leverage in the high-quality firm's financial structure a) increases both prices and qualities, b) reduces product differentiation for most parameter values, and c) reduces the value of the levered high quality firm because it induces an upgrade of the low quality product. Despite the negative profitability of strategic leverage in a differentiated products environment, the high quality firm will carry leverage in its structure, unless it has a means to commit *ex-ante* to using only equity.

Keywords: Vertical differentiation; uncertainty; financial structure; leverage; endogenous quality choice.

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I. INTRODUCTION

In this paper we examine the interactions between financial and microeconomic decisions in a differentiated oligopoly with *endogenous* product specification. More specifically, we analyze how leverage may affect the market outcome in a duopoly in which consumer demand for the basic product is already known, but the additional willingness-to-pay for a higher quality is uncertain. Such is the case, for instance, in a firm providing dial-up connection (low quality) to the internet and another one ready to introduce wireless connection (high quality). Both dial-up and wireless are technologically available at many quality levels, but each firm is to offer only a single product; similar situations arise in several high-tech applications. Other such examples are the building of a luxury hotel in a tourist area already served by family-type establishments, and the introduction of high speed train service in competition with regular train or bus service. While the willingness-to-pay for quality improvements of the existing service is known with certainty, since the product or service may have been available for quite a long time, the corresponding willingness-to-pay for quality improvements in the service is uncertain.

Although most studies in both economics and finance adopt the principle of separation of financial structure choice from decisions on investment, pricing and output, it is a well-known fact that in the presence of uncertainty these dual sets of decisions interact with each other. Jensen and Meckling (1976) were the first to point out that the presence of debt in the financial structure of a firm may induce the equity owners, who are assumed to control the operations of the firm, to undertake investments with negative contributions to the total value of the firm, provided those investments are associated with sufficiently higher risk. This loss in firm value is known as the *agency cost* of debt (JM effect) and is due to the fact that, under limited liability, a risk-neutral owner-manager undervalues the losses debt holders incur in states of bankruptcy, thus preferring riskier projects with even lower value.

In an oligopoly with demand uncertainty, however, besides the agency cost of debt this behavior may also create strategic effects that enhance the value of the levered firm, as Brander and Lewis (1986) were the first to point out. Limited liability induces the owner-manager of a levered firm to overvalue good demand states. In a Cournot oligopoly this corresponds to an outward shift of the levered firm's reaction function, resulting in higher profits of the levered firm at the expense of its all-equity rival.[4] In a differentiated-products Bertrand oligopoly with exogenous product specification, Showalter (1995) shows that limited liability induces the levered firm to behave less aggressively: debt increases prices and profits of *both* rivals. Thus, whether competition is in prices or quantities, in oligopolies some amount of debt raises firm value despite the agency cost (BL effect).

In our duopoly setting a firm introducing a new, higher-quality version always prefers sufficient consumer heterogeneity, for otherwise a price war with the seller of the basic product is unavoidable. When introducing an improved version, therefore, part of the seller's concern is how "wide" the market is, or, alternatively, how far the willingness-to-pay for high quality goes.[5] Assuming for simplicity a uniform taste distribution, this corresponds to uncertainty over the exact position of the high end of that distribution.[6] The presence of such uncertainty may affect the decision on how to finance the project, which in turn may have effects on prices and product specification as well as on firm value.

Using a vertically differentiated duopoly as in Shaked and Sutton (1983) we analyze a three-stage game where qualities are chosen at the first stage, financial structure of the high quality at the second, and prices at the third. We ask two sets of questions. First, how are prices, qualities and product differentiation affected by debt? Second, what is the equilibrium level of debt, and is debt value-enhancing or value-reducing in the presence of endogenous product specification?

We first show that firm 2's leverage has no relevance for the equilibrium outcome. This is due to the fact that the low end of the taste distribution is known with certainty; hence, firm 2 only faces uncertainty over the density of its market.[7]

[4] Of course, if both firms use debt as commitment to a more aggressive strategy they get trapped into a prisoner's dilemma situation. This conclusion is challenged in Hughes et al. (1998) where firms may acquire and share information prior to debt. The intuition of Brander and Lewis is also challenged in Faure-Grimaud (2000) and Povel and Raith (2004), where it is shown that debt does not make a firm more aggressive, and may even make its behavior softer. This difference is mainly due to the assumptions that a) liquidation has a cost for the owner of the firm, and b) liquidation does not occur with certainty, but rather with a probability that is increasing in the amount of default. Hence, even under limited liability, the lender does not become sole residual claimant.

[5] The willingness-to-pay for quality increments of the consumers with low taste for quality is easier to estimate since a) the price of the existing basic product is close to their willingness-to-pay for that quality, and b) their willingness-to-pay for higher qualities represents small increments over that for the basic product.

[6] The density of the distribution is also uncertain, since it is the inverse of the width. This guarantees that the market size remains constant, and all the results of the paper are solely due to uncertainty over consumer heterogeneity. In an earlier version of the paper we show that the results also hold in the case where the density remains constant and the market size varies with the width.

[7] In other words, even if firm 2 is uncertain about the size of its market, it knows with certainty the heterogeneity of its clientele.

Further, we find that leverage increases the quality of both firms' products, since debt relaxes competition, thus raising both products' marginal revenue with respect to quality. While leverage pushes up both qualities, it reduces product differentiation for most sets of relevant parameter values, whether measured as the ratio or as the difference between the high and the low qualities.[8] This happens because the anticipated less aggressive pricing behavior of the high quality firm (hereafter, firm 1) induces the low quality firm (firm 2) to increase the quality level of its product more than proportionately. Despite reduced differentiation, debt increases both product prices. Hence, in the presence of leverage firms compete less aggressively in prices (as in Showalter (1995, 1999b)), but more aggressively in the quality stage of the game.

Our most surprising result is that when product qualities are endogenous, *leverage always reduces the levered firm's value*. This is in sharp contrast with short run analysis (fixed qualities), where some debt increases the levered firm's profit.[9] The difference lies in the fact that, since the low quality firm rationally expects its levered rival to relax price competition, it has less incentive to do it itself by downgrading the quality of its product. The resulting reduced differentiation, in conjunction with higher quality costs, hurts the profits of the high quality firm. In the face of this result one would expect the high quality firm to avoid leverage, opting for all-equity financing. Note, however, that once qualities are chosen the issuance of debt becomes optimal. Thus, although the high quality firm would like to commit not to issue any debt, such a commitment is not possible: sequential rationality forces the high quality firm to include debt in its financial structure.

In the remainder of this section we complete the review of the literature. Despite the importance financial structure has on investment, pricing and output decisions in oligopolistic industries, the link between financial structure and these decisions had received relatively little attention in the early literature, as noted in the

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[8] To be precise, the ratio is always reduced, while the difference can only increase when the cost-of – quality function is sufficiently flat *and* the fluctuation of the upper-end of the taste distribution quite narrow. Even then, a subset of these cases is ruled out since it compromises the viability of the low quality firm..

[9] Our short-run result is in line with Showalter (1995, 1999b). Note that these papers treat differentiation in the "Bowley-Dixit-Spence" tradition, simply emphasizing imperfect substitutability due to brand differentiation. While Garella and Petrakis (2008) shows that a *vertical* dimension can be introduced to this type of models, in this work we follow the "pure vertical differentiation tradition" of Shaked and Sutton (1982) and Gabszewicz and Thisse (1980). Similarly in Lyandres (2006, 2007) debt is always desirable from the point of view of the firm. In that study, unlike ours, the nature of firms' interaction is exogenous.

1991 survey by Harris and Raviv.[10] Several studies have been added since that survey, both theoretical and empirical ones.[11] The theoretical research focused on the effect of leverage on pricing and output, on barriers to entry, on the feasibility of entry deterrence and on R&D spending. In all these works, either products are assumed homogeneous, or product specification is considered as exogenous. [12]

Using a model close to Showalter's, Wanzenried (2003) treats product differentiation parametrically and shows that the desirable amount of debt is decreasing in product differentiation. Unfortunately, the analysis of that paper has been simultaneously criticized by both Frank and Le Pape (2008), and Haan and Toolsema (2008), for using bankruptcy risk instead of debt as firms' choice variable. Interchanging strategic variables alters the nature of competition in a way analogous to that of using prices instead of quantities as strategic variables. Frank and Le Pape (2008) show that while the result in Wanzenried (2003) may still hold for some parameter values, when using debt as choice variable the relation between differentiation and debt is no longer monotonic. Note that, since in our paper only one firm's exposure to bankruptcy is strategically relevant, the results are not altered if one or the other variable is used as choice variable for the high quality producer.[13]

In the next section we present the general model. Section III presents the benchmark case of all-equity firms under uncertainty. Section IV examines the pricing stage in the presence of debt. Section V determines the equilibrium amount of leverage. Section VI examines how debt affects prices, qualities, product differentiation and firm value. Section VII concludes.

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[10] Maksimovic (1988), Poitevin (1989), Maksimovic and Titman (1991).

[11] For the empirical ones see, for instance, Phillips (1994), Chevalier (1995), Chevalier and Scharfstein (1996), Showalter (1999a) and Lyandres (2006).

[12] See Showalter (1995, 1999b), Dasgupta and Titman (1998), Jensen and Showalter (2004), and Lyandres (2007). Calveras *et al.* (2004) shows that limited liability induces firms in bad financial situation to bid more aggressively in a procurement auction. Firms with little left to loose are ready to accept such low rewards that only exceptionally favorable realizations of the random variable can make the project profitable.

[13] This is analogous to the fact that, while the oligopoly equilibrium crucially depends on whether we consider prices or quantities as choice variables, the monopoly outcome is independent of the choice variable used. See also Franck and Le Pape (2008), footnote 7.

II. THE MODEL

We consider a market where two single product firms, firm 1 and firm 2, produce differentiated products. Each consumer j buys one unit of a certain type or nothing at all. The purchase of product i, $i = 1,2$, yields utility

$$U_i^j = u_i t^j - p_i ,$$

where $u_i \in [0,\infty)$ is a quality index, t^j is a consumer taste parameter and p_i the product's price. Utility from non-purchase is zero. The utility function adopted implies that at equal prices consumers unanimously prefer the product with the higher level of u; without loss of generality, we assume $u_1 \geq u_2$. The consumer taste parameter t is uniformly distributed in $[a,b]$, $b > a > 0$, with density equal to $(b-a)^{-1}$.

The consumer indifferent between the two qualities as well as the one indifferent between purchasing the lower quality or nothing are characterized by

$$t_B = (p_1 - p_2)/(u_1 - u_2) \text{, and } t_A = p_2 / u_2 , \tag{1}$$

respectively. Hence, sales of firms 1 and 2 are $b - t_B$ and $t_B - \max\{t_A, a\}$, respectively. We assume that

$$2a < b < 4a \tag{2}$$

which implies that i) $t_B > a$, and ii) $t_A \leq a$, *i.e.*, the two firms have positive market shares and cover the entire market (natural duopoly); hence, the market share of firm 2 is now $t_B - a$.

On the supply side we assume variable production cost to be the same for both firms and, without loss of generality, to be equal to 0.[14] Production requires also a fixed cost $F(u)$, with $F' \geq 0$ and $F'' \geq 0$, which is sunk upon the choice of quality.[15]

In the absence of uncertainty the financial choice of a firm is irrelevant. We introduce uncertainty over the consumer taste distribution (demand uncertainty) by assuming that b is distributed within a given interval $\left[\underline{b},\bar{b}\right]$ according to some function $G(b)$ with expectation $\tilde{b}$. Hereafter, a ˜ over a variable denotes its expected value. In order to simplify the analysis we restrict the fluctuations of b so that the market will always remain duopoly and covered. This means that equation (2) always

[14] In this case prices can be interpreted as the excess over unit cost.

[15] These restrictions on $F(u)$ are sufficient for the proofs of Propositions 1 and 2, and the first part of Proposition 3; for the remaining results some further restrctions are necessary, and will be introduced in Section VI.

holds, which implies $\bar{b} < 2\underline{b}$.[16] Firms are assumed to be risk-neutral throughout the paper. In order to concentrate on the strategic effects of debt we keep the total market size constant while allowing uncertainty with respect to consumer heterogeneity. This means that as b varies randomly the height of the distribution is also random. Our results also hold when the height of the density stays constant, so that both the size of the market and the consumer heterogeneity are allowed to vary with b.[17]

We examine a three-stage game where both firms simultaneously choose, at the first stage their quality, at the second stage their financial structure, and at the third stage their price. In order for financial structure to play any role, all three decisions must be taken under uncertainty.[18] Figure 1 shows the timing of the game:

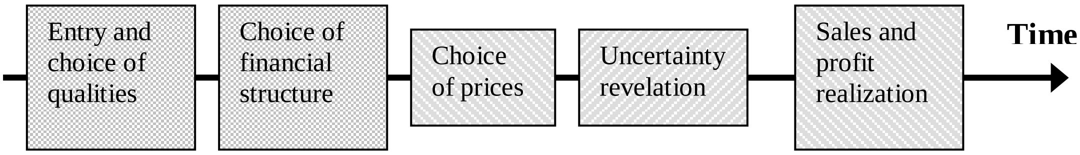

Figure 1: The game structure

For comparison purposes we also examine a game where firms are all-equity financed, or only carry riskless debt (see below). Our benchmark game is, thus, a two-stage game, similar to the above but with the second stage omitted.

III. PRICING STAGE

Let debt be available as an alternative form of financing, and assume that both firms have arrived at the last stage of the game partially financed by debt, the promised repayment of which amounts to D_k, $k = 1, 2$. For any quality-and-price choice of the two firms, let

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[16] This assumption, while innocuous with respect to the interesting results, does simplify the analysis by avoiding functional form changes that would be necessary were the market to become a monopoly, or remain uncovered for some *ex post* realizations of the width of the taste distribution, see Shaked and Sutton (1982).

[17] When the density does not vary, profit is linear in the random taste parameter, and several of the proofs and expressions become much simpler.

[18] Otherwise, the Modigliani-Miller theorem holds. When there is debt in the firm's capital structure the revelation of uncertainty before prices are chosen will also reveal whether default will take place, implying that the debt holders will write into the debt contract provisions for taking control of the firm in such cases. Hence, financial structure will have no effect on market equilibrium, unless *all three* decisions are taken before uncertainty is resolved. This assumption is also more realistic.

$$\Pi_1(b) \equiv p_1\left(\frac{b-t_B}{b-a}\right) - F(u_1) = p_1\left(1-\frac{t_B-a}{b-a}\right) - F(u_1), \tag{3a}$$

$$\Pi_2(b) \equiv p_2\left(\frac{t_B-a}{b-a}\right) - F(u_2), \tag{3b}$$

represent the operating profits of firms 1 and 2, respectively, at any realization of b.[19] Then, $\forall D_1$ define $b_z(D_1)$, and $\forall D_2$ define $b_w(D_2)$ such that

$$\Pi_1(b_z) \equiv D_1, \text{ and } \Pi_2(b_w) \equiv D_2. \tag{4}$$

For any given quality-price choice the profit of firm 1 is increasing in b, therefore b_z represents the *lowest* realization of the random variable that allows firm 1 to break-even:[20] firm 1 is solvent for $b \in \left[b_z, \bar{b}\right]$ and in default for $b \in [\underline{b}, b_z)$; we denote b_z the *default market size*. From (4), an increase in D_1 increases *ceteris paribus* b_z, thus increasing the bankruptcy interval $[\underline{b}, b_z)$ and the cumulative probability of bankruptcy. Let $\tilde{b}_z \equiv E(b \mid b \geq b_z)$ denote the truncated expectation of b conditional on firm 1's solvency. Our constant market size assumption requires that a larger width corresponds to a thinner density, hence, the profit of firm 2 depends negatively on the realization of b.[21] The value of b_w represents, therefore, the *highest* realization of the random variable that allows firm 2 to break-even, that firm being solvent for $b \in [\underline{b}, b_w)$ and in default for $b \in \left[b_w, \bar{b}\right]$.

Let $\Pi_i^*(b; u_1, u_2)$, $i = 1,2$, indicate the maximized profit of firm i for any given realization of b known *ex-ante* with certainty.[22] If $D_1 \geq \Pi_1^*(\bar{b}) \equiv \bar{D}_1$, then $b_z \geq \bar{b}$, so that at any possible demand state firm 1 goes bankrupt. It is natural to assume that $\bar{D}_1$ is an upper bound to the borrowing available to firm 1. On the other hand, when $D_1 \leq \Pi_1^*(\underline{b}) \equiv \underline{D}_1$,[23] then $b_z \leq \underline{b}$, implying that leverage induces no bankruptcy at any

[19] Strictly speaking, we need to multiply the revenue part of (3a) and (3b) with a constant in order to convert the shares of the constant market size of the two firms into sales; we normalize the constant to 1 without loss of generality.

[20] From (1), the value of t_B depends only on prices and qualities which are chosen *prior* to uncertainty resolution; hence, it is not stochastic

[21] If the size of the market is allowed to vary with b the profit of firm 2 non-stochastic. This change does not alter our results.

[22] The value of $\Pi_i^*(b; \cdot)$ can be easily found in the literature, see for instance Tirole (1988), p.297.

[23] It can be shown that $\Pi_1^*(\underline{b}) \geq 0$.

demand state (*riskless debt*); hence, $b_z = \underline{b}$. Since riskless debt produces neither agency cost nor strategic effects, the equilibrium with riskless debt is similar to the benchmark case of firms being all-equity financed, so any reference to the former includes also the latter and *vice versa*. Analogously, one can define an interval $\underline{D}_2, \bar{D}_2$ such that any debt level within that interval is feasible and exposes firm 2 to bankruptcy risk. As it turns out, firm 2's leverage—whether riskless or bankruptcy-inducing—has no relevance for the equilibrium outcome, implying that this interval is not important for our analysis. Hereafter, the presence of z or w in a subscript indicates the presence of bankruptcy-inducing debt in the financial structure of firm 1 or firm 2, respectively; when z (w) appears as subscript in a mathematical expectation, the expectation is conditional on $b \geq b_z$ ($b \leq b_w$). The presence of v in a subscript indicates absence of bankruptcy-inducing leverage.

In the last stage, the manager-owner of a levered firm maximizes the firm's expected equity value $\tilde{E}_{iz}$, $i = 1,2$, rather than its expected total value $\hat{V}_{iz} = \hat{E}_{iz} + \hat{D}_{iz}$. The problem of firm 1 is

$$\max_{p_1} \tilde{E}_{1z} \; b_z \; = \hat{\Pi}_{1z} \; b_z \; - \hat{D}_{1z} = p_1 \; t_B - a \; \cdot \int_{b_z}^{\bar{b}} \left(\frac{1}{b_z - a} - \frac{1}{b - a} \right) dG(b) \, , \tag{5}$$

while that of firm 2 is

$$\max_{p_2} \tilde{E}_{2w} \; b_w \; = \hat{\Pi}_{2w} \; b_w \; - \hat{D}_{2w} = p_2 \; t_B - a \; \int_{\underline{b}}^{b_w} \left(\frac{1}{b - a} - \frac{1}{b_w - a} \right) dG(b) \, . \tag{6}$$

Setting $\Phi(b_z) \equiv E\left[\frac{1}{b-a} \middle| b \geq b_z \right]$, and defining $\varphi(b_z) \equiv a + \Phi(b_z)^{-1}$, the resulting reaction functions are

$$p_1 = \; 1/2 \quad \varphi(b_z) \cdot \Delta u + p_2 \tag{7}$$

and

$$p_2 = \; 1/2 \quad -a \cdot \Delta u + p_1 \tag{8}$$

Note immediately that the best reply of firm 1 increases monotonically in D_1, while that of firm 2 is unaffected by the presence of D_2. This asymmetry stems from the fact that the two firms face different types of uncertainty: from (3ab) it is clear that firm 1 faces uncertainty over both the width and the density of its market, whereas firm 2 only faces uncertainty with respect to the latter.[24] Since the presence and magnitude of D_2 have no effect on the equilibrium outcome, we assume, hereafter, that firm 2 is all-equity financed, *i.e.*, $b_w = \underline{b}$. Define $X_{1z}(b_z) \equiv (2b_z - a)/3$, $X_{2z}(b_z) \equiv (b_z - 2a)/3$, $\hat{X}_{1z}(b_z) \equiv (2\varphi(b_z) - a)/3$, and $\hat{X}_{2z}(b_z) \equiv (\varphi(b_z) - 2a)/3$. Let also $X_i \equiv X_{iz}(\underline{b})$, and $\hat{X}_i \equiv \hat{X}_{iz}(\underline{b})$, $i = 1,2$. Solving (7) and (8) simultaneously we find equilibrium prices:

$$p_{iz}(b_z; u_1, u_2) = \hat{X}_{iz}\Delta u,\ i = 1, 2, \tag{9}$$

Holding qualities fixed, an increase in debt increases *both* prices. The consumer who is indifferent between the two products at equilibrium prices is characterized by

$$t_{Bz}(b_z) = \left[\varphi(b_z) + a\right]/3 \tag{10}$$

In the benchmark game either both firms are all-equity financed, or only riskless debt is allowed, therefore each firm decides its price by maximizing its total value. Since the amount of debt that is paid back is constant across all realizations of b and $b_z = \underline{b}$, $b_w = \overline{b}$, maximizing total value at the last stage corresponds to maximizing operating profit:

$$\max_{p_1} \tilde{\Pi}_1 = p_1 E\left[\frac{b - t_B}{b - a}\right] = p_1\left(1 - (t_B - a)E\left[\frac{1}{b - a}\right]\right) = p_1\left(1 - (t_B - a)\Phi(\underline{b})\right) \tag{11a}$$

$$\max_{p_2} \tilde{\Pi}_2 = p_2 E\left[\frac{t_B - a}{b - a}\right] = p_2\left(t_B - a\right)\Phi(\underline{b}) \tag{11b}$$

The reaction function of firm 2 remains unchanged from (8) while that of firm 1 becomes

$$p_1 = (1/2)\left(\varphi(\underline{b}) \cdot \Delta u + p_2\right) \tag{12}$$

The expression in (12) is similar to the corresponding one in (7) except that $\varphi(b_z)$ has been replaced by $\varphi(\underline{b})$. However, this is an important difference since the latter is a parameter, while the former a decision variable determined at previous stages of the game. Since positive amounts of debt imply $\varphi(b_z) \geq \varphi(\underline{b})$, debt shifts the price reaction function of the leveraged firm and makes its response softer.[25] Solving (12) and (8), last-stage equilibrium prices p_{1v} in the benchmark game are also given

[24] Firm 1 faces uncertainty over both the size and the heterogeneity (width of taste variability among its clientele) of its market, while firm 2 faces uncertainty only over the former.

[25] Recall that for $D_1 \leq \underline{D}_1$, $\hat{b}_z = \hat{b}$, and changes in debt do not affect the reaction function.

by (8-11) with $\hat{X}_{iz} = \hat{X}_i$, implying that *for given qualities* leverage results in higher prices. Under the same condition it can also be easily verified that $p_{1z} - p_{1v} \geq p_{2z} - p_{2v}$, *i.e.*, the levered firm 1 raises its price more than its rival does. Similarly, with respect to the indifferent consumer we have $t_{Bv} = t_B \;\; \varphi \;\; \underline{b} \;\;\; \leq t_B \;\; \varphi \;\; b_z \;\;\; = t_{Bz}$, as expected, since the higher increase in firm 1's price reduces its relative market share at any realization of *b*. In terms of the Fudenberg and Tirole (1984) zoology, debt acquisition corresponds to a *puppy dog* strategy, similar to increasing product differentiation. Bankruptcy risk and product differentiation are "substitutes" in softening price competition in the sense that a given level of equilibrium price can be targeted by various $\hat{X}_{iz}$ and Δu levels.

Substituting last stage equilibrium prices from (8) into (4) and (3) we obtain, as functions of debt and qualities, firm 1's price-maximized equity,

$$E_{1z} = \frac{\hat{X}_{1z}}{2}\Delta u[1 - G(b_z)]\left[2\Phi(b_z)\hat{X}_{1z} - \frac{3X_{1z} - \hat{X}_{1z}}{b_z - a}\right],$$

the face value of its debt as a function of the default market size,

$$D_1 = \hat{X}_{1z}\left(\frac{3X_{1z} - \hat{X}_{1z}}{2(b_z - a)}\right)\Delta u - F(u_1), \tag{13}$$

and its total firm-value,

$$V_{1z} = p_1 \int_{\underline{b}}^{\bar{b}} \left[1 - \left(\frac{p_1 - p_2}{\Delta u} - a\right)\frac{1}{b - a}\right] dG(b) - F \;\; u_1 \;\; = \;\; 1/2 \;\; Y \Delta u - F \;\; u_1 \;\; , \tag{14}$$

with $Y \;\; b_z \;\; = 2\hat{X}_{1z}\left(1 - \frac{\varphi(b_z) - 2a}{3}\Phi(\underline{b})\right) = \Phi(\underline{b})\hat{X}_{1z} \;\; 3\hat{X}_1 - \hat{X}_{1z}$. In the riskless debt case, total value is also given by (14) with $Y \;\; b_z = \underline{b} \;\; = 2\Phi(\underline{b})\hat{X}_1^{\,2}$:

$$V_{1v} \equiv V_1 \;\; D \leq \underline{D} \;\; = \hat{\Pi}_1 \;\; \cdot \;\; = \Phi(\underline{b})\hat{X}_1^{\,2}\Delta u - F(u_1) \tag{15}$$

Finally, the last-stage equilibrium value of debt and total-value of firm 2 is

$$V_{2z} = p_2 \cdot E\left[\frac{t_B - a}{b - a}\right] - F \;\; u_2 \;\; = \hat{X}_{2z}^{\,2}\Delta u \Phi(\underline{b}) - F \;\; u_2 \;\; . \tag{16}$$

It is clear from (14) and (16) that, while both firms' total value is affected by the financial structure of firm 1, firm 2's financial structure is irrelevant. Nonetheless, (16) is needed for the verification of the viability of that firm.

IV. EQUILIBRIUM

In stage 2 we concentrate on the determination of financial structure for firm 1. From equation (14) it is clear that the optimal amount of debt is chosen implicitly, by choosing the default market size b_z. For given qualities equation (4) defines an implicit function $b_z(D_1)$, thus making b_z the decision variable in the choice of structure. Since only one firm's potential bankruptcy has strategic effects—hence, only that firm's financial structure matters for the equilibrium outcome—the results are not altered if either debt or b_z is used as a choice variable for the high quality producer, given the equilibrium relation $b_z(D_1)$ between the two.[26] Thus, the criticism of Frank and Le Pape (2008), and Haan and Toolsema (2008) against such substitution of strategic variables, does not apply in our case.

Before proceeding with the determination of firm 1's financial structure in our duopoly context it is useful to examine that firm's optimal structure if it were the sole producer in the market, protected by some legal entry barrier. Under our natural duopoly assumption over the width of the distribution, if a single quality is supplied then the market remains uncovered. The last consumer purchasing the available product is now $t_m = p_m / u_m$, where the subscript m indicates the monopoly case. Replacing t_B by t_m in (5) and maximizing yields $p_{mz} = 1/2\ \varphi(b_z)u_1$. The price-maximized value of the monopolist is $V_{mz} = \varphi(b_z)u_1\left[1-\left(\frac{\varphi(b_z)}{2}-a\right)\Phi(\underline{b})\right]-F(u_1)$.

Letting $\varphi' \equiv \partial\varphi(b_z)/\partial b_z > 0$, we find that

$$\frac{\partial V_{mz}}{\partial b_z} \propto \varphi'\left[1-(\varphi(b_z)-a)\Phi(\underline{b})\right] = \varphi'\left[1-\frac{\Phi(\underline{b})}{\Phi(b_z)}\right] < 0,$$

since $\Phi(b_z) \leq \Phi(\underline{b})$. Thus, the value of the monopolist is reduced as b_z increases, implying that the optimal financial structure for the protected monopolist is the one that does not result in default-inducing debt. In the absence of strategic interaction with a rival, the advantage of leverage (BL effect) disappears, leaving only the agency cost (JM effect).

[26] We discuss this relation further on in this section.

Firm 1 chooses its financial structure by maximizing (14) with respect to b_z, taking qualities as given. Maximizing (14) with respect to $\hat{X}_{1z}$ yields the following result:

Proposition 1: *There exists a unique optimal default market size* b_z^* *for the high quality firm 1 that corresponds to a default-inducing amount of debt. If* $\hat{X}_{1z}\ \bar{b}\ \geq\ 3/2\ \hat{X}_1$, *then the optimal structure contains both debt and equity financing and* b_z^* *is given by*

$$\hat{X}_{1z}^* =\ 3/2\ \hat{X}_1 \Leftrightarrow 6\varphi\ \underline{b}\ - 4\varphi\ b_z^*\ - a = 0; \tag{17}$$

if $\hat{X}_{1z}\ \bar{b}\ \leq 3/2\,\hat{X}_1$, *then* $b_z^* = \bar{b}$ *(corner solution) and the optimal structure consists of all-debt financing.*[27]

Proof: Differentiating (14) for given qualities, dV_1/db_z is proportional to $dY_{1z}/d\hat{X}_{1z}\ = 3\hat{X}_1 - 2\hat{X}_{1z}$; setting the latter equal to zero reduces to $6\varphi(\underline{b}) - 4\varphi(b_z) - a = 0$. Since $\frac{\partial\varphi}{\partial b_z} > 0$, $\hat{X}_{1z}^*$ is increasing in b_z, and equal to $\hat{X}_1$ at $b_z = \underline{b}$, therefore there is a unique value of b_z satisfying (17); if this value is in the admissible interval $\left[\underline{b}, \bar{b}\right]$ then it defines from (14) the optimal default market size and corresponding amount of debt. Otherwise, at $b_z = \bar{b}$ we have $\hat{X}_{1z}\ \bar{b}\ \leq\ 3/2\ \hat{X}_1$ and the optimal structure is all-debt, QED.

In order to see the intuition of Proposition 1, we re-write

$$dY_{1z}/d\hat{X}_{1z} = \hat{X}_1 - 2\ \hat{X}_{1z} - \hat{X}_1\ . \tag{18}$$

Combining (9) and (15) we see that, in the absence of leverage, the expected sales of firm 1 is proportional to $\hat{X}_1$. Any strategic effect that succeeds in increasing firm 1's equilibrium price by 1 unit, increases, therefore, its revenue by an amount proportional to $\hat{X}_1 dp_1$. Hence, the first term of the derivative in (18) represents the marginal benefit from an increase in bankruptcy-inducing debt. Such debt, however,

[27] The notion of an all-debt capital structure is a *limit case*, since it is not compatible with the subsequent price-setting decision of firm 2, where price is chosen by maximizing the value of equity.

implies also an agency cost in the form of price distortion at the third stage. This agency cost is proportional to the second term in (18) which is proportional to the difference $\hat{X}_{1z} - \hat{X}_1 = \hat{X}_1 (r - 1)$, where $r(b_z) \equiv \hat{X}_{1z} / \hat{X}_1$, with $r'(b_z) > 0$, and $r(\cdot) \geq 1$. Since r is monotonically increasing in the $\Phi(\cdot)/\Phi(\cdot_z)$ ratio, it can be interpreted as an index of relative bankruptcy risk. Hence, the agency cost is proportional to the degree of bankruptcy risk. Since the second term is zero for the unlevered firm and $\hat{X}_{1z}$ is continuous on $\left[\hat{X}_1, \bar{X}_1\right]$, some positive amount of debt will always be contained in the optimal financial structure. An interior solution requires sufficient leverage so that the marginal cost of debt (2nd term) becomes equal to the (constant) marginal benefit of debt (1st term). If the range of the taste parameter b is not sufficient to set the marginal agency cost equal to the marginal benefit of debt, there is a corner solution.

An interesting feature deriving from Proposition 1 is that, while quality choices affect equilibrium debt, they do not affect equilibrium bankruptcy risk. From (17), the optimal value of r, $r^* = \min\left(3/2, \hat{X}_{1z}(\bar{b})/\hat{X}_1\right)$, implying that r^* depends only on consumer tastes and the distribution of the random variable.[28] Hence, the optimal bankruptcy risk is like a parameter determined prior to the game. For any qualities chosen at the first stage, debt simply adjusts to the necessary level in order to set the ratio $\hat{X}_{1z}/\hat{X}_1$ equal to r^*. Note also that the optimal bankruptcy risk and corresponding structure depend crucially on the shape of the distribution of the random parameter b. At the end of the next section we provide a numerical example with the same parameter values but two different distributions of b, one of which yields a corner and the other an interior solution.

Since condition (17) determines the optimal default market size and risk of bankruptcy (rather than the optimal amount of debt), we still need to determine whether there exists a unique feasible amount of debt $D_1 \in \left(\underline{D}_1, \bar{D}_1\right)$, corresponding to the value of b_z^* determined from Proposition 1. In the appendix it is shown that:

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[28] Recall that $\hat{X}_1$ is a parameter, depending only on consumer tastes and the distribution of the random variable. Thus, changes in $\hat{X}_{1z}$ are equivalent to changes in *r*.

Lemma 1: *When* $\hat{X}_{1z}(\bar{b}) \geq 3/2\,\hat{X}_1$, *a sufficient condition for* b_z^* *to correspond to a unique value* $D_1^* \in [\underline{D}_1, \bar{D}_1]$ *is* $6b_z - 4\varphi(b_z) - a \geq 0$, $\forall b_z \in [b_z^*, \bar{b}]$.[29] *When* $\hat{X}_{1z}(\bar{b}) \leq 3/2\,\hat{X}_1$, sign $\partial D_1/\partial b_z$ *is positive in an open neighborhood to the left of* $\bar{b}$, *implying that* $b_z^* = \bar{b}$ *corresponds to* $D_1^* \leq \bar{D}_1$.

We now turn to stage 1. Since qualities are chosen before financial structure, they are decided by maximizing total firm value. However, since the optimal value of b_z is independent of qualities and depends only on parameters, optimal qualities can be expressed as functions of b_z. The high quality is decided by maximizing equation (14)—or (15) for the riskless debt case—with respect to u_1, which yields

$$u_{1z}^* \equiv u_1^*(b_z^*) = F'^{-1}\left(\frac{Y(b_z^*)}{2}\right), \tag{19}$$

with $u_{1v}^* \equiv u_{1z}^*(\underline{b}) = F^{-1}(\Phi(\underline{b})\hat{X}_1^2)$ for the riskless case. With respect to u_2, we note that the derivative of the RHS of (16) is proportional to $dV_2/du_2 = -\hat{X}_{2z}^2\Phi(\underline{b}) - F'(u_2) < 0$. Since the width of the consumer distribution always assures full market coverage, the profit maximizing u_2 is obtained at the minimum level consistent with $t_A \leq a$. Since t_A decreases in u_2, the optimal u_2 sets $t_A = a$, *i.e.*, $u_{2j}^* = p_2/a$, $j = z, v$, with p_2 replaced by (9), yielding

$$u_{2z}^* \equiv u_2^*(b_z^*) = \left(\frac{\varphi(b_z) - 2a}{\varphi(b_z) + a}\right) u_{1z}^*(b_z^*) = \frac{\hat{X}_{1z} - a}{\hat{X}_{1z} + a} F'^{-1}\left(\frac{Y}{2}\right),$$

$$u_{2v}^* \equiv u_2^*(\underline{b}) = \left(\frac{\varphi(\underline{b}) - 2a}{\varphi(\underline{b}) + a}\right) u_1^*(\underline{b}) = \frac{\hat{X}_1 - a}{\hat{X}_1 + a} F'^{-1}\left(\Phi(\underline{b})\hat{X}_1^2\right). \tag{20}$$

The equilibrium of this game is summarized as follows. Optimal qualities are given by (19), (20) with $\hat{X}_{1z}^* = \min\{3/2\,\hat{X}_1, \hat{X}_{1z}(\bar{b})\}$; the optimal financial structure of firm 1 contains leverage $D_{1z}^* \equiv D_1(\hat{X}_{1z}^{*-1}(b_z))$, *i.e.*, the necessary amount of debt for (4) to hold at the equilibrium value of $b_z^* \equiv \hat{X}_{1z}^{*-1}(b_z)$, while the financial structure of

[29] Note, however, that even if the condition in the Lemma is not satisfied the only problem that may arise is that there may be from (13) more than one debt level corresponding to the optimal default market size.

firm 2 is irrelevant, hence undetermined; optimal prices are given by (9) with the appropriate substitutions. When debt-financing is unavailable, or available at very limited amounts $D_1 \leq \underline{D}_1$, firm 1's financial structure becomes irrelevant, as well, and equilibrium qualities and prices are given by (20), (19) and (9) with $\hat{X}_{1z}$ replaced by $\hat{X}_1$. Equilibrium firm values $\hat{V}_{1j}^*, \hat{V}_{2j}^*,\ j = z, v,$ are obtained by substituting optimal qualities and $\hat{X}_{1z}^*$ or $\hat{X}_{2z}^* \equiv \hat{X}_{2z}\ \ \hat{X}_{1z}^{*\,-1}\ \ b_z$ into (14)-(16). The above described equilibrium requires that both firms are active in the market. Since $\hat{V}_{1j}^* \geq \hat{V}_{2j}^*,\ j = z, v,$ we assume at this point that $\hat{V}_{2j}^* \geq 0,\ j = z, v.$[30]

V. Comparative Statics: Equilibrium Prices, Differentiation and Firm Value

In this section we perform comparative statics focusing mainly on the comparison between the levered and the unlevered equilibrium. We start by comparing equilibrium qualities in the two situations.

Proposition 2: *When bankruptcy-inducing leverage is allowed at the second stage, both equilibrium optimal quality levels are higher than they would have been had firms been limited to default-free debt, or all-equity financing,* i.e., $u_{iz}^* \geq u_{iv}^*,\ i = 1, 2$.

Proof: It is easy to check that both (18) and (19) are increasing in $\hat{X}_{1z}$. Since $\hat{X}_{1z} \geq \hat{X}_1$, the convexity of $F(u)$ implies $u_{iz}^* \geq u_{iv}^*$. QED.

This result is due to the fact that *for any given qualities,* debt increases both prices. Thus, the anticipation of debt increases both firms' marginal revenue from quality improvements, shifting firm 1's reaction function in the qualities space to the right, and that of firm 2 upwards. From (19) the reaction function of the lower quality is upward sloping, therefore, $u_{1z}^* > u_{1v}^*$, and $u_{2z}^* \geq u_{2v}^*$.

Hereafter, we restrict the form of the cost-of-quality function to $F\ \ u_i\ \ = \lambda u_i^{\beta}$, $i = 1, 2,$ with $\beta > 1$ in order to satisfy the convexity requirement $F'' > 0$. While being

[30] In standard vertical differentiation models, such as this one, once the viability of firm 2 is assured, that of firm 1 follows *a fortiori*.

sufficiently general in order to approximate most smooth convex functions, it allows us to obtain well defined results.[31] The size of the convexity factor β turns out to be significant for some of our results.

Before deriving the remaining results a technical point needs to be addressed. The specification of the cost function requires that firm 2's viability be investigated, instead of simply assumed.[32]

Lemma 2: Under our demand-cost assumptions $\forall \beta \geq 2.3565$ the low-quality firm is viable in equilibrium for all admissible parameter values and all probability distributions. For $\beta \in (1, 2.3565)$ the viability of firm 2 is parameter- and distribution-dependent, holding if $\beta \left(\dfrac{\hat{X}_1 + a}{\hat{X}_{1z} - a} \right)^{\beta - 1} \geq \dfrac{\hat{X}_{1z}(3\hat{X}_1 - \hat{X}_{1z})}{a(\hat{X}_{1z} - a)}$.

Proof: See the appendix.

From Lemma 2 and its proof it becomes obvious that the scale factor λ does not affect the viability of the low quality firm. Since it can also be shown not to affect any of the results that follow, it is hereafter normalized to 1.

Equilibrium product differentiation can be expressed either as a quality ratio u_{1j}^*/u_{2j}^*, or as the difference $\Delta u_j^*(r^*, \hat{X}_1; \beta) = u_{1j}^*(r^*, \hat{X}_1; \beta) - u_{2j}^*(r^*, \hat{X}_1; \beta)$, $j = z, v$, with $r^* \equiv 1$ in the riskless-debt case. The difference $\Delta u_j^*(r^*, \hat{X}_1; \beta)$ is very important since it constitutes a component of equilibrium prices and firm values.

Proposition 3: *When leverage financing is available a) the optimal quality ratio* u_1^*/u_2^* *is lower compared to the case where equity (riskless leverage) is the only source of financing for firm 1. b) Concerning* Δu^*, $\forall \beta \geq 2$, $\Delta u_z^* \leq \Delta u_v^*$, *while for* $\beta \in (1, 2]$, *i) when* $r^* = 3/2$ *(interior solution to (17)),* $\Delta u_z^* \leq \Delta u_v^*$, *always, while ii) for*

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[31] The frequently used function $F(u_i) = u_i^2/2$ is a special case of the above, with $\lambda = 1/2$, $\beta = 2$.

[32] We are indebted to a referee for pointing this out. By "*viability*" we mean *ex ante* positive expected value in equilibrium. This should not be confused with "solvency", which describes whether in equilibrium the *ex post* pay-offs are able to repay each firm's debt. Recall that the low-quality firm's expected total value is independent of that firm's leverage, while this is obviously untrue for its solvency.

$r^* < 3/2$ *(100% leverage),* $\exists \underline{\beta}(\hat{X}_1) \in (1,2)$ *such that* $\forall (\hat{X}_1, \beta)$ *with* $\beta \geq (<) \underline{\beta}(\hat{X}_1)$,

$\Delta u_z^* \leq (>) \Delta u_v^*$.

Proof: See the appendix.

The above result shows that product differentiation may increase *only* when there is a corner solution of 100% debt *and* the value of β is sufficiently close to 1. For all but the lowest admissible values of the parameter β in our cost function—including $\beta = 2$, as in the commonly used function $F(u) = (1/2)u^2$—debt reduces product differentiation for all market size parameters. Debt affects product differentiation in two different ways. The first one is related to the BL effect, which tends to raise both qualities by increasing the marginal revenue from quality increments. This BL effect, however, has an ambiguous overall impact on differentiation, since on the one hand the increase in marginal revenue is higher for the high quality, but on the other hand the convexity of the cost function implies that any given increase in marginal revenue translates into a more important increase of the low quality. It turns out that, unless the cost function is very flat (β has values below 2), the BL effect tends to reduce product differentiation. In addition, debt also affects differentiation because, from (12), the two are substitutes in achieving any given level of equilibrium prices. For sufficiently high degrees of convexity (at least $\forall \beta \geq 2$) both the BL effect and the substitution effect work towards reducing differentiation.

Next, we examine the effect of debt on prices, which has three components. The first operates through distorting the objective of the decision maker at the last stage. This effect induces a softer reaction of the leveraged firm (BL effect), which in turn results in, *ceteris paribus,* higher prices for both products.

The second component operates through the increase in qualities and also tends to increase prices, since, *ceteris paribus,* consumers are willing to pay higher prices for higher qualities (quality effect). The third effect of debt on prices operates through the impact of debt on product differentiation and has the opposite direction: a reduced Δu implies, *ceteris paribus,* stiffer price competition and lower prices (reduced-differentiation effect). The next result (proven in the appendix) shows that the BLS and quality effects together dominate the reduced-differentiation effect.

Proposition 4: *In the presence of bankruptcy-inducing debt in the financial structure of firm 1, both equilibrium prices are higher than in the case where firm 1 is unlevered.*[33]

Proof: See the appendix.

What was shown by equation (9) and the discussion surrounding equation (12) for *given* qualities (short run) turns out to hold even when qualities adjust to their equilibrium level (long run). The fact that prices go up is good news for the two firms, but does not account for the entire story, since in equilibrium qualities are higher, and therefore their production requires higher fixed cost. Thus, while leverage enhances short-run profitability, exactly as in Showalter (1995), its long-run impact on firm value is ambiguous. We analyze next the impact of debt on firm 1's value and obtain a rather surprising result, given the fact that debt makes a firm less aggressive and competition is in strategic complements.

Proposition 5: $\forall D_1 \geq \underline{D}_1$ *the value of firm 1 is a decreasing function of debt, with* $V_{1v}^* > V_{1z}^*$*, the equilibrium value of the all-equity (riskless) firm.*

Proof: See the appendix.

Proposition 5, which is a central result of this paper, has a striking feature: the strategic use of *two* instruments, each of them known to relax price competition and increase firm value, has the exact opposite result. This happens because by relaxing price competition debt reduces firm 2's need for differentiation and allows that firm to bring the quality level of its product closer to that of firm 1's. By doing so, firm 2 mitigates the effect of debt on price competition. Firm 1 sells now at a price which is higher relative to the unlevered scenario, but not sufficiently so as to compensate for the increased fixed cost of its product. Hence, when quality choices are endogenous, it is optimal for the high quality firm to opt for an all-equity structure, or to only carry riskless debt. This argument is illustrated by writing:

$$\frac{dV_{1z}^*}{dr} = \frac{\partial V_{1z}^*}{\partial Y}\frac{dY}{dr} + \frac{\partial V_{1z}^*}{\partial u_1}\frac{du_{1z}^*}{dr} + \frac{\partial V_{1z}^*}{\partial u_2}\frac{du_{2z}^*}{du_1}\frac{du_{1z}^*}{dr} \tag{21}$$

[33] The proof with respect to the low quality price holds for any cost of quality function.

The envelope theorem implies that at the optimal choice of u_1 the second term of (21) is zero. Assume for the sake of the argument that the financial decision at the second stage yields an interior solution, implying that the first term is also zero.[34] The third term is negative: its first component is negative, since an increase in u_2, with fixed u_1, implies a reduction in product differentiation; its second component is positive since the quality reaction function of firm 2 has positive slope; its third component has been shown to be positive in Proposition 2. Clearly, the negativity of dV_{1z}^{*}/dr is due to the fact that firm 1's leverage induces firm 2 to upgrade its quality.[35] Note also that fixing both qualities at their unlevered equilibrium levels would have resulted in $dV_{1z}^{*}/dr > 0$, since the second term of (21) would become positive ($u_{1v}^{*} < u_{1z}^{*}$), and the third, zero. This implies that when products are differentiated, some amount of leverage increases the value of the levered firm (as in Showalter (1995)), but only as long as the rival's quality is exogenous. When the reaction of rival quality to changes in debt is taken into account debt on the one hand is no longer profitable, but on the other hand cannot be avoided in the absence of a firm commitment on the part of firm 1 not to choose a levered financial structure once qualities have been irreversibly fixed.

Despite the optimality of an all-equity structure, we know from Proposition 1 that *in equilibrium* firm 1 *is* levered. The reason is that, since $r^{*} > 1$ and depends only upon parameter values, qualities during the first stage are decided upon the expectation that later on in the game (2nd stage) firm 1 will take up some debt. Hence, firm 2 chooses its quality expecting (correctly) that, no matter its choice, at the next stage its rival will take up the necessary amount of leverage in order to reach r^{*}. Having assured a softer price reaction from its rival due to debt, firm 2 increases its quality to a level u_{2z}^{*} above u_{2v}^{*}, causing at the same time a reduction in the total-value of firm 1.

The following conclusions emerge from the entire game. First, for *given* qualities it is always optimal for the high quality producer to include debt in its financial structure. Second, the availability of debt financing affects both quality

[34] Proposition 5 holds even if the equilibrium financial structure is an all-debt one. The interior solution assumption is used here just to present the intuition in a clearer and more concise manner.

[35] The proof contained in the appendix shows that this result is not limited to the neighborhood of the interior solution, but holds for all admissible values of the parameters.

choices upwards, despite the fact that qualities are decided before the financial structure. Third, the availability of debt financing reduces product differentiation. Fourth, despite the reduction in product differentiation, debt leads to higher prices for both products. Fifth, the availability of debt reduces the value of firm 1.

Table 1 illustrates most of the above results. We use the following parameter values: $a=1,\ \underline{b}=2.1,\ \overline{b}=3.9,$ and consider two cases.[36] The first one is a uniform distribution, with the average value of b at the midpoint of 3, while the second one is a stepwise uniform, with 2/3 of the probability mass in the region $2.1, 3$.[37] The results in columns 3-10 have been multiplied by 10^4. Note that the uniform (asymmetric) distribution of b yields corner (interior) solution for b_z^* and recall that b_z^* is independent of the cost parameter β. Note also that the value of the lower-quality producer increases with its rival's leverage, a result not proven in the paper.

β	b_z^*	u_{1v}^*	u_{1z}^*	u_{2v}^*	u_{2z}^*	Δu_v^*	Δu_z^*	V_{1v}^*	V_{1z}^*	V_{2v}^*	V_{2z}^*
3, asym.	3.769	6518	6914	1266	2623	5252	4291	3926	2848	157	750
3, unif.	3.9	6657	7056	1479	2736	5178	4320	3934	2939	195	728
2, asym.	3.769	6373	7170	1237	2720	5136	4550	2485	1240	020	225
2, unif.	3.9	6648	7468	1477	2896	5171	4572	2456	1252	009	149

Table 1: Numerical Example

VI. CONCLUSIONS

In the context of an address model of vertical differentiation we introduce uncertainty over the realization of the upper end of the taste distribution. Our purpose is to examine the interaction between debt and vertical product differentiation, two variables that turn out to be substitutes in relaxing price competition. We consider two firms, one selling a product of standard technology and the other introducing a product of new technology. Both products—basic and new technology—can be offered at a single quality level chosen out of a range of technologically feasible qualities. No matter the quality choices of the two firms, the high-tech product is

[36] Recall that $\lambda=1$. Changing the value of λ will affect the quality levels and market values. However, all the propositions remain intact.

[37] Note that for the chosen parameter values the value of firm 2 turns out to be negative $\forall\beta<2$.

always viewed as superior quality by consumers (no leapfrogging). Both firms choose their quality level simultaneously (no incumbent-entrant relations). We assume that, while the low end of the consumer taste distribution is known with certainty, the position of the high end is uncertain and only its distribution is known. This implies that the two firms face an asymmetric situation: while the willingness-to-pay for quality improvements of the standard product is known, the corresponding one for a new-technology product is uncertain. After specifying their product and before announcing their price, both firms can underwrite debt.

Since the lower quality faces uncertainty only with respect to the density of its market, its debt is of no importance for the equilibrium outcome. On the contrary, the high quality is always levered in equilibrium and its leverage has the following important consequences for the equilibrium outcome. First, in anticipating leverage, both firms increase their quality. Second, except for a small set of admissible parameters determined in the text, debt reduces equilibrium product differentiation. Knowing that its rival will be levered, the low quality firm expects softer price competition, for any product specification. Hence, it chooses a product quality that, in the absence of debt, would have been unprofitable due to triggering very stiff price competition. Third, *despite reduced differentiation*, long-run equilibrium prices are higher, compared to situations where firm 1 uses only equity financing. Fourth, despite the higher prices, the value of the high quality firm is reduced, compared to the case of all-equity financing. The fall in the value of the high quality firm is due to the upgrading of the lower quality. This conclusion is in sharp contrast with short-run analysis where qualities are given and debt enhances the total value of the high quality firm. Even if leverage is value-reducing, the high quality firm cannot commit to all-equity financing because after qualities have been chosen, it becomes rational to include debt in its financial structure.

The above conclusions are robust if one allows the market size to vary together with consumer heterogeneity. In such a case the height of the density remains constant even though its width changes with the parameter *b*. The results of this case are virtually identical to the ones in this paper. Some of the conclusions are also robust to altering the decision sequence so that the financial decision precedes the choice of quality, as shown in a companion paper (see Constantatos and Perrakis, 2010). In fact, the prior choice of leverage induces a second agency distortion that further increases the cost of debt, since, not only the price but also the quality of firm 1 is chosen under distorted incentives (equity instead of total value maximization).

The assumption of market coverage helps to keep matters tractable by imposing a corner solution to the choice of the lower quality. It is by no means essential for the intuition developed in the paper, since the presence of debt always reduces the low quality's need for differentiation.

We have not considered entry in our model. While our natural duopoly assumption rules out—for all levels of the random taste parameter—the possibility of three firms surviving in the market, entry may take the form of a new firm displacing some of the incumbents by introducing a superior quality. Assume that a number of potential entrants threaten to introduce a quality between the two incumbent ones. The lower incumbent firm being threatened by such entry can only ensure its survival by producing a quality level as close to the high quality, as it is allowed by the non-negative profit requirement (see Constantatos and Perrakis (1999)). Our results must now hold stronger since the urge of the low quality firm to avoid displacement will transform any gains from relaxing competition through debt to (unprofitable) quality improvements.

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Appendix:

Proof of Lemma 1: To assess the feasibility of this amount of debt we differentiate (13) with respect to b_z. We have

$$\text{sign}\left(\frac{\partial D_1}{\partial b_z}\right) = \text{sign}\left[\; b_z - a \quad 3X_{1z} - 2\hat{X}_{1z} \;\; \frac{\partial \hat{X}_{1z}}{\partial b_z} + \hat{X}_{1z} \;\; \hat{X}_{1z} - a \;\right]. \tag{A.1}$$

The positivity of this sign, which would determine a monotone increasing relation between the amount of debt and the default market size, is difficult to ascertain for the entire range $b_z \in \left[\underline{b}, \bar{b}\right]$ for all admissible parameter values and all probability distributions; it can only be verified numerically for specific cases. Fortunately the clear results of Proposition 1 allow us to simplify this assessment, since they allow us to reduce the search to a subset of the range of b_z. From (A.1) it is easy to see that the following is a *sufficient* condition for $dD_1/db_z \geq 0$:

$$6b_z - 4\varphi(b_z) - a \geq 0 \tag{A.2}$$

At $b_z = \bar{b}$ we have $\Phi(b_z) = \Phi(\bar{b}) = (\bar{b} - a)^{-1}$, implying that $\varphi(\bar{b}) = \bar{b}$, and the above clearly holds strictly. To show that the corresponding optimal debt D_1^* is feasible, we note that Proposition 5, proven in the next section, shows that the optimal value of the levered firm 1 is lower than the corresponding value in the absence of leverage, implying that $D_1^* < \bar{D}_1$ and is, therefore, feasible, QED. Now suppose that $\exists b_z < \bar{b}$ solving the equation $6\varphi(\underline{b}) - 4\varphi(b_z) - a = 0$. At $b_z = \bar{b}$ the corresponding debt D_1, which is no longer optimal, is still feasible since the levered firm 1 value is again less than the unlevered one. A sufficient condition for expression (A.1) to be positive and the function $D_1(b_z)$ to be monotone over the relevant range is that (A.2) must hold for $b_z \in [b_z^*, \bar{b}]$, where $b_z^* = \varphi^{-1}\left(\frac{6\varphi(\underline{b}) - a}{4}\right)$, QED.

Proof of Proposition 1: In the text.

Proof of Proposition 2: In the text.

Proof of Lemma 2: Replacing u_{2z}^* from (20) into (16) we get that firm 2 is viable if:

$$\hat{V}_{2z} = \hat{X}^2_{2z}\Delta u^*_z \Phi(\underline{b}) - F\ \ u^*_{2z}\ \ = \left[\frac{a\hat{X}^2_{2z}}{\hat{X}_{2z}+a}\Phi(\underline{b}) - \lambda\ \ u^*_{1z}\ {}^{\beta-1}\left(\frac{\hat{X}_{2z}}{\hat{X}_{2z}+a}\right)^{\beta}\right]u^*_{1z} \geq 0$$

Replacing $u^*_{1z} = \left[\frac{1}{\lambda\beta}\frac{Y}{2}\right]^{\frac{1}{\beta-1}}$, we find that the above boils down to

$$\frac{a\hat{X}^2_{2z}}{\hat{X}_{2z}+a} = \frac{a\ \ \hat{X}_{1z}-a\ \ ^2}{2\ \ \hat{X}_{1z}+a} \geq \frac{\hat{X}_{1z}\ \ 3\hat{X}_1 - \hat{X}_{1z}}{2\beta}\left(\frac{\hat{X}_{1z}-a}{\hat{X}_{1z}+a}\right)^{\beta}$$

A simple rewriting of the above yields the expression in the text. Let now $\hat{x}_{1z} \equiv \frac{\hat{X}_{1z}}{a}$, $\hat{x}_1 \equiv \frac{\hat{X}_1}{a}$, in which case the expression in the text reduces to

$$\frac{(\hat{x}_{1z}+1)^{\beta-1}}{(\hat{x}_{1z}-1)^{\beta-2}} \geq \frac{\hat{x}_{1z}(3\hat{x}_1 - \hat{x}_{1z})}{\beta}. \tag{A.3}$$

The admissible parameter values from the natural duopoly condition are $\hat{x}_{1z} \in\ 1, 7/3$ and $\hat{x}_1 \leq \hat{x}_{1z} \in\ 1, 7/3$. Since the LHS of (A.3) increases and the RHS decreases as β increases, there exists a unique minimum value $\beta = 2.3565$ such that (A.3) holds for all higher β's and all admissible parameter values, QED. For $\beta \in\ 1, 2.3565$ there exist values and distributions such that $\hat{x}_1$ and $\hat{x}_{1z}$ violate (A.3), QED.

Proof of Proposition 3: For part i) note that from (18) and (19) we get

$$\frac{u^*_1}{u^*_2}\ \ b_z\ \ = \left(\frac{\hat{X}_{1z}-a}{\hat{X}_{1z}+a}\right)^{-1} \tag{A.4}$$

The RHS of (A.4) is decreasing in $\hat{X}_{1z}$ and $\hat{X}^*_{1z} > \hat{X}_1$, hence, $u^*_{1v}/u^*_{2v}\ >\ u^*_{1z}/u^*_{2z}$, QED.

For part ii) assume first that the financial structure decision (17) admits an interior solution. From (19)-(20) we get that for $\Delta u^*_z - \Delta u^*_v \geq 0$ it is necessary and sufficient that $\frac{u^*_{1z}a}{a+\hat{X}^*_{2z}} \geq \frac{u^*_{1v}a}{a+\hat{X}_2}$, where $u^*_{1z}, u^*_{1v},$ are given by (19). Manipulating the definitions of $\hat{X}_{2z}$, $\hat{X}_2$, we get $\hat{X}_{2z} =\ \ \hat{X}_{1z} - a\ \big/2$, $\hat{X}_2 =\ \hat{X}_1 - a\ \big/2$. Using these expressions and re-arranging, we obtain that $\Delta u^*_z - \Delta u^*_v \geq 0$ is equivalent to

$$\frac{F'^{-1}\ Y\ b_z^*\ \big/ 2}{a+\hat{X}_{1z}^*}\frac{a+\hat{X}_1}{F'^{-1}\ \hat{X}_1{}^2}\ \geq 1. \tag{A.5}$$

Note that if the firm carries no bankruptcy-inducing debt, the LHS of (A.5) equals 1. Since the second fraction of the LHS of (A.5) is independent of $\hat{X}_{1z}$, the necessary and sufficient condition for $\Delta u_z^* - \Delta u_v^* \geq 0$ reduces to

$$\frac{d}{d\hat{X}_{1z}}\left(\frac{F'^{-1}\ Y\!\big/_2}{a+\hat{X}_{1z}}\right) \geq 0,\ \forall \hat{X}_{1z} \in \left[\hat{X}_1, \frac{3\hat{X}_1}{2}\right).$$

This derivative turns out to be positive iff

$$\hat{X}_{1z}+a\ \frac{Y'}{\beta-1} \geq Y \Leftrightarrow \beta \leq \frac{3\hat{X}_1 - 2\hat{X}_{1z}\quad \hat{X}_{1z}+a}{\hat{X}_{1z}\ \ 3\hat{X}_1\ -\hat{X}_{1z}} + 1. \tag{A.6}$$

When optimal debt is less than 100% (interior solution), at the optimal capital structure of firm 1, $3\hat{X}_1 = 2\hat{X}_{1z}^*$, which implies that the first term in the RHS of (A.6) is equal to zero. Hence, there is no convex quality cost function for which optimal leverage increases differentiation.

When, however, the optimal structure is all-debt, $\hat{X}_{1z}^* = \dfrac{(2\bar{b}-a)}{3}$, and the RHS of (A.6) is then greater than 1, since the first term is positive. We can rewrite the RHS as $2+\dfrac{a}{\hat{X}_{1z}^*}-\dfrac{(\hat{X}_{1z}^*+a)}{(3\hat{X}_1-\hat{X}_{1z}^*)}$; the sum of the last two terms can be shown to be negative and lie within the interval $(-1,0)$, implying that $\exists \underline{\beta} \in\ 1,2$ such that (A.6) is satisfied for any value of $\beta < \underline{\beta} \in\ 1,2$. Hence, when the distribution of the random factor is such that the optimal structure is all-debt there may exist values of $\beta < \underline{\beta} \in\ 1,2$ yielding convex cost-of-quality functions such that differentiation increases relative to the riskless-debt case.

Proof of Proposition 4: Concerning the price of the low quality, by the market coverage condition and the fact that $u_{2z}^* > u_{2v}^*$ (Proposition 2) we have immediately that $p_{2z}^* \geq p_{2v}^*$. Considering the high quality price with and without leverage, we note that, since $p_{1v}^* = \hat{X}_1 \Delta u_v^*$ and $p_{1z}^* = r\hat{X}_1 \Delta u_{1z}^*$, we can write

$$\frac{p_{1v}^*}{p_{1z}^*} = \frac{1}{r}\frac{\Delta u_v^*}{\Delta u_z^*}. \tag{A.7}$$

Replacing equilibrium qualities, simplifying and setting $\frac{\hat{X}_1}{a} = \hat{x}_1$ and $\frac{\hat{X}_{1z}}{a} = \hat{x}_{1z}$, we get that

$$\frac{\Delta u_v^*}{\Delta u_z^*} = \frac{u_{1v}^*}{u_{1z}^*} \cdot \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} = \left[\frac{2\hat{x}_1^{\,2}}{\hat{x}_{1z} \;\; 3\hat{x}_1 - \hat{x}_{1z}} \right]^{\frac{1}{\beta-1}} \cdot \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} \equiv L^{\frac{1}{\beta-1}} \cdot C_1, \tag{A.8}$$

where $L \equiv \frac{2\hat{x}_1^{\,2}}{\hat{x}_{1z} \;\; 3\hat{x}_1 - \hat{x}_{1z}} < 1$, and $C_1 \equiv \frac{r\hat{x}_1 + 1}{\hat{x}_1 + 1} \geq 1$. Substituting (A.7) into (A.6) we get

$$\frac{p_{1v}^*}{p_{1z}^*} = L^{\frac{1}{\beta-1}} \cdot C_2 \tag{A.9}$$

where $C_2 \equiv C_1 \cdot \;\; \hat{x}_1 / \hat{x}_{1z} \;\; = \;\; r\hat{x}_1 + 1 \; \big/ \big[r \;\; \hat{x}_1 + 1 \;\; \big] < 1$. Since $L^{\frac{1}{\beta-1}} < 1$ as well, the RHS of (A.9) is also smaller than 1, QED.

Proof of Proposition 5: Using (20) we obtain $\Delta u = 2u_1 / \;\; 1 + rx$. Replacing Δu into (14), we obtain $V_{1z} = u_1 \left[\frac{\Phi \;\; \underline{b} \;\; \hat{x}^2 r \;\; 3 - r}{1 + rx} - u_1^{\beta-1} \right]$. Replacing u_1 from (19) into the latter and simplifying, we get $V_{1z}^* \;\; r;\hat{x}_1 \;\; = A\frac{2\beta - r\hat{x}_1 - 1}{r\hat{x}_1 + 1}$, where $A \equiv \left[\Phi \;\; \underline{b} \;\; \frac{r\hat{x}_1^{\,2} \;\; 3 - r}{2\beta} \right]^{\frac{\beta}{\beta-1}} > 0$, with $A' \equiv \frac{dA}{dr} = A\frac{\beta}{\beta - 1} \cdot \frac{3 - 2r}{r \;\; 3 - r} \geq 0$. Differentiating $V_{1z}^* \;\; r;x$ with respect to *r* yields

$$\frac{dV_1^* \;\; r;\hat{x}_1}{dr} = A'\frac{2\beta - r\hat{x}_1 - 1}{r\hat{x}_1 + 1} - A\frac{2\beta\hat{x}_1}{r\hat{x}_1 + 1 \;\; ^2} \propto$$

$$- A\frac{-2r^3\hat{x}_1^{\,2} + 3r^2\hat{x}_1^{\,2} - 2r^2\hat{x}_1 \;\; - 2r + 3 + \;\; -3 + 2r + r^2\hat{x}_1 \;\; 2\beta}{r\hat{x}_1 + 1 \;\; ^2 \;\; \beta - 1 \;\; 3 - r} \tag{A.10}$$

The sign of the RHS of the above expression is negative if the numerator is positive. The coefficient of β in the numerator is obviously positive, implying that the numerator is increasing in β. Hence, if the numerator is positive for $\beta = 1$, it is positive for all β. Setting $\beta = 1$ the numerator of (A.10) simplifies to $3 - 2r \;\; r^2 x_1^{\,2} - 1 \;\; > 0$, therefore $dV_1^Q / dr < 0$, $\forall \beta > 1$, QED.

Numerical example

We use the following parameter values: $a=1,\ \underline{b}=2.1,\ \bar{b}=3.9$ and consider two cases. The first one is a uniform distribution, with the average value of b at the midpoint of 3, while the second one is a stepwise uniform, with 2/3 of the probability mass in the region $\left[2.1,3\right]$. For the uniform case we have $\Phi(\underline{b})=\dfrac{1}{\bar{b}-\underline{b}}\ln\dfrac{\bar{b}-a}{\underline{b}-a}$, from which $\varphi(\underline{b})=2.857$. Given that $\varphi(\bar{b})=\bar{b}=3.9$, the LHS of the second part of relation (17) yields 0.541, a positive number implying an all-debt financial structure. For the asymmetric case, though, setting $\hat{b}=(\underline{b}+\bar{b})/2=3$, we have $\Phi(\underline{b})=\dfrac{2}{3(\underline{b}-\hat{b})}\ln\dfrac{\hat{b}-a}{\underline{b}-a}+\dfrac{1}{3(\underline{b}-\hat{b})}\ln\dfrac{\bar{b}-a}{\hat{b}-a}$, yielding $\varphi(\underline{b})=2.723$; for $b_z=\bar{b}$ the LHS of (17) equals $-0.263<0$. Hence, the optimal structure contains equity, and for $b_z>\hat{b}$ the conditional distribution becomes uniform, implying that $\Phi(b_z)=\dfrac{1}{\bar{b}-b_z}\ln\dfrac{\bar{b}-a}{b_z-a}$. Solving (17) we find $b_z=3.769$, implying a positive equity value at the optimal market structure.