

Full length article

Structural design of stiffened plates of industrial duct walls with relatively long panels undergoing large deformations



Ali Rezaiefar, Khaled Galal*

Department of Building, Civil and Environmental Engineering, Concordia University, Montréal, Québec, Canada H3G 1M8

ARTICLE INFO

Article history:

Received 18 December 2015

Received in revised form

4 July 2016

Accepted 29 August 2016

Available online 16 September 2016

Keywords:

Stiffened plate

Large deformations

FE analysis

Industrial ducts

ABSTRACT

Structural design of stiffened plates of the walls of rectangular-sectioned industrial ducts is currently based on the strip method that follows the linear flexural beam theory. The current design method assumes that the displacements generated in the plate are small and the membrane forces that generate in the plate are negligible. This paper addresses the current design method and discrepancies associated with it to present a new design approach that takes into account the effects of large displacements on the load-deformation and load-stress behavior of stiffened plates. Two different sets of formulas are presented based on Finite Element analysis of rectangular plates with relatively long aspect ratios that estimate the maximum stress and deformation in the typical and edge panels of stiffened plates. These formulas are then used to establish design equations for stiffened plates. Based on the findings of this paper, a reduction of 30–40% in the plate thickness could be achieved if the new design approach is selected over the conventional design method.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

Many industries require transferring large amounts of air and flu gas between different points as part of their production or treatment procedures. Depending on the nature of flu gas and the volume of transfer, ducts with different shapes and sizes are used in various parts of a given industry. Most duct line assemblies consist of various segments connected to each other to form a conduit line that transfers large amounts of air or flu gas in industrial facilities. A segment of a duct line assembly forms a straight line or an elbow with circular, rectangular or transfer cross-sections. A typical rectangular duct segment consists of four plates sealed together to form the walls of a rectangular section resting on supporting frames at two ends. The wall plates are stiffened by rib elements at certain intervals throughout the length of the duct segment as shown in Fig. 1 to form stiffened plates with relatively long rectangular panels. This paper studies the structural design of the stiffened plates of the walls of large rectangular sectioned industrial ducts.

ASCE [4] defines various loads that apply to duct line assemblies considering the duct segment geometry, the method of application, environmental conditions, the indoor or outdoor location, the nature of flue gases transferred, the flow temperature and position of duct segment in the flow system. Although there are various external loads (dead, live, wind and earthquake loads)

applicable to the duct segment, the main load acting on duct walls is the internal hydrostatic pressure that transfers the air or flu gas inside the ductline. The hydrostatic pressure inside the duct is positive if the fans that generate the flow are installed upstream but in most cases the pressure is negative (suction) since the fans are installed at the outlet of the flow [4].

Structural design of duct walls consists of two stages namely, local and global design [4]. In the local stage, the plate thickness and stiffener arrangements are designed under the internal pressure whereas in the global stage the whole duct segment is designed for the external loads.

One of the major aspects in the analysis and design of industrial ducts and similar structures is the load-displacement behavior of the stiffened plates. Although the effects of large deformations (i.e. geometrical nonlinearities) on the load-displacement behavior of rectangular plates are well known, the design of plates in duct-work industries is mainly based on linear analysis without consideration of large deformations effects. ASCE [4] briefly mentions the membrane forces induced in the plate as a result of large deformations without giving any further technical guidance. Since then, the effects of large deformations on the failure criteria of long rectangular plates have been studied by several researchers. Young and Budynas [25] presented nonlinear load-deformation curves of rectangular plates for various plate boundary conditions and aspect ratios considering large deformation effects. More recently, Thanga et al. [20] studied the case of stiffened plates of duct walls while considering large displacements and allowing for partial yielding of the plate within an acceptable limit. They

* Corresponding author.

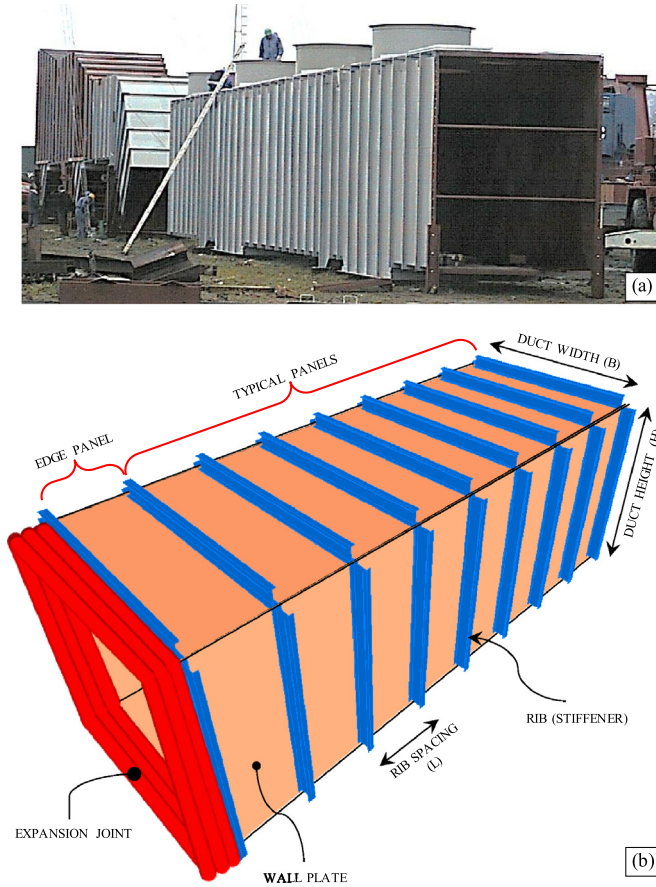


Fig. 1. (a) Large industrial rectangular duct segments under construction (courtesy of fives-solios Inc.). (b) Schematic view of a typical rectangular duct segment.

concluded that a design approach that is based on large deformation analysis would be more economic than the linear method stated in ASCE [4] guideline. Following these findings, the load-displacement and load-stress relations for long rectangular plates were then formulated by El-Aghoury and Galal [8] using Bezier equations. Similar findings were reported by Liu [15] for ducts with wall plate thicknesses ranging between 3/16" and 3/8". From the above, it can be noted that there is a need for developing design equations that takes into account the effect of large deformations using boundary conditions that represent stiffened plates of duct walls.

The duct line segments are supported against the external loads by means of structural frames and footings or through connections with other segments. The term "structural design of duct walls" in this paper refers to the local design of duct wall plates. After the design of duct walls and segments, the duct line assembly shall be checked for thermal expansion. Thermal expansion might cause destructive axial force if restrained and should be released along the duct line by using expansion joints and elbows. It is practical in the design of duct lines to use expansion joints at one or both ends of straight segments [4]. An expansion joint may transfer shear, moment, torsion or a combination of all of them between two adjacent segments of the duct line but it generally provides full release of axial forces. Thus the boundary conditions at the edge panel (see Fig. 1-b) at the location of the expansion joint are different from those of the typical panels of the wall plate. Therefore, this paper focuses on evaluating the linear and nonlinear behavior of stiffened plates of duct walls considering the large deformation effects as well as the effects of expansion joints at the ends of the duct segment on the boundary

conditions.

2. Analysis and design of long-spanned rectangular plates

The current procedure of the design of long-spanned plates with aspect ratio greater than 2.0 is based on the application of flexural beam analogy by using a "unit-width strip" at the middle of the plate as shown in Fig. 2. The analysis of this unit-width beam element (design strip) could be linear or nonlinear depending on the nature of design.

2.1. Linear analysis of long rectangular plates under uniform lateral pressure

The principal assumption in the linear analysis of plates under uniform lateral pressure is that the deformations are small and strains are infinitesimal compared to the plate thickness so the classic beam bending equations apply to the design strip of the plate as a beam element.

Using the unit-width beam analogy, the cross-sectional properties of the plate are calculated based on the plate thickness, t , as shown in Eq. (1).

$$\begin{cases} I = t^3/12 \\ S = t^2/6 \end{cases} \quad (1)$$

where S is the section modulus and I is the moment of inertia of the unit-width strip. Applying the beam bending analogy to the unit-width strip of the plate for the three boundary condition combinations shown in Fig. 2, results in the elastic stresses and deflections as shown in Table 1. In this table, q is the lateral uniformly distributed load, σ is bending stress, δ is the plate deflection and λ is the plate length-to-thickness ratio (L/t). The boundary conditions indicators "S" and "C" represent the simply-supported (hinged) and clamped (fixed) ends respectively. The design stresses and deflections in Table 1 are calculated by applying the section modulus and moment of inertia in the classical bending moment equations, respectively.

The formulas presented in Table 1 are useful for the linear analysis of long (i.e. $S > 2.0$) rectangular plates based on the unit-width strip method. These equations could be interpreted in conventional design formulas in forms of Eq. (2) and Eq. (3) for the three applied boundary conditions combinations.

$$(q/F_y)\lambda^2 = \begin{cases} 1.33(\sigma_{\max}/F_y); (S - S) \\ 2(\sigma_{\max}/F_y); (C - C) \\ 1.33(\sigma_{\max}/F_y); (C - S) \end{cases} \quad (2)$$

$$(q/E)\lambda^4 = \begin{cases} 6.4(\delta/t); (S - S) \\ 32(\delta/t); (C - C) \\ 15.42(\delta/t); (C - S) \end{cases} \quad (3)$$

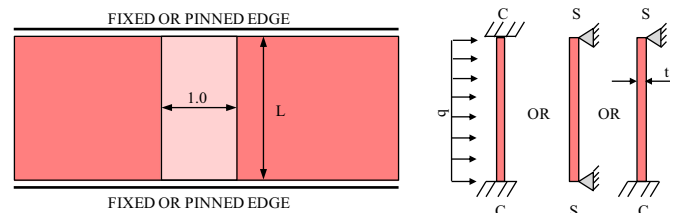


Fig. 2. Unit-width strip and beam analogy.

Table 1
Stresses and deflections of long plates when using unit-width strip method and beam analogy.

Boundary conditions	S-S	C-C	C-S
σ	$3qL^2x(L-x)$	$q(6xL-L^2-6x^2)/(2t^2)$	$0.75q(3xL-4x^2)/t^2$
σ^- (max)	0	$0.5qL^2$; [at $x=0, x=L$]	$0.75qL^2$; [at $x=0$]
σ^+ (max)	$0.75qL^2$; [at $x=0.5L$]	$0.25qL^2$; [at $x=0.5L$]	$0.422qL^2$; [at $x=0.625L$]
δ/t	$q(xL^3-2Lx^3+x^4)/(2Et^4)$	$qx^2(L-x)^2/(2Et^4)$	$q(xL^3-3Lx^3+2x^4)/(4Et^4)$
δ/t (max)	$5qL^4/(32E)$ [at $x=0.5L$]	$qL^4/(32E)$ [at $x=0.5L$]	$12qL^4/(185E)$ [at $x=0.5785L$]

In Eqs. (2) and (3), the length-to-thickness ratio (i.e. λ) of plate is calculated for the given load (i.e. uniform pressure, q) by replacing the (σ_{max}/F_y) and (δ/t) ratios with the maximum stress and deflection limits set by the design criteria (i.e. design codes adopted by the engineer). The final design results are the plate length and thickness that fit into the length-to-thickness ratio calculated from these design equations. Since the plate gauges are limited, it is practical to select the thickness of the plate from the available gauges then design the ribs spacing using Eqs. (2) and (3). In cases where the presence of expansion joints dictates the boundary conditions of the edge panel, a practical design calls for keeping the same plate thickness while reducing the rib spacing for the edge panel. In cases where the stress criterion is used, the rib spacing ratio for the edge panel would be calculated using Eq. (2) as:

$$\lambda_{edge}/\lambda_{typical} = (1.33/2.0)^{0.5} = 0.815$$

and similarly for the deflection criterion using Eq. (3):

$$\lambda_{edge}/\lambda_{typical} = (15.42/32)^{0.25} = 0.833$$

Thus, as a general rule it is practical to design the rib spacing at the edge panels of duct segments within the range of 80% of the typical rib spacing when expansion joints are used.

2.2. Nonlinear analysis of plates under uniform lateral pressure with large deformations

Applying uniform lateral pressure to a rectangular plate results in deformations throughout the plate with the peak deflection at the centroid. A relationship between applied lateral pressure and the resulting deflections could be established if the types of stresses generated in the plate due to the applied pressure are known.

In general, plate stresses are categorized into three main types by the classic plates and shells theory namely the in-plane (tangential) stresses, the out-of-plane (shear) stresses and the bending (flexural) stresses [16]. One of the governing parameters in the classic plate theory is the thickness of the plate. In extremely thin plates, the effects of bending and shear stresses on the overall load-deformation behavior are minimal and any out-of-plane deflection is significantly greater than the thickness of the plate. Plates with such behavior are classified as membrane elements in most of the available references including Timoshenko and Woinowsky-Kreiger [21]. On the other hand in thicker plates, the effects of bending stresses on the overall plate behavior increase with the thickness of plate while the tangential stresses and lateral displacements reduce significantly. In cases where the deflections are small compared to the thickness of the plate, the tangential stresses are negligible and the plate is considered as “pure bending plate” [21].

In general, Kirschhoff-Love plate theory is applied to thin plates where the plate thickness is at least one degree smaller than its width [21] as shown in Eq. (4).

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = q/D \quad (4)$$

where w is the deformation at any point, x and y stand for the coordinates of the point where the deflection is desired, q is the applied uniform pressure and D is the flexural rigidity of the plate calculated as:

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (5)$$

in which E and ν are the modulus of elasticity and Poisson's ratio of the plate material respectively. There are several available solutions for Eq. (4) when applying various boundary conditions in terms of closed-form and numerical approaches in the literature, including the solutions given by Timoshenko and Woinowsky-Kreiger [21].

The contribution of shear stresses in the overall response of thick plates (e.g. thick concrete slabs) to the applied pressure must be considered in the plate governing equations. Such thick plates are often called the “Mindlin-Reissner plates” since their case was first presented and solved separately by Mindlin [17] and Reissner [19] as an extension to the Kirschhoff-Love plate theory. The case of thick plates with shear behavior is out of scope of this paper.

If the displacements of thin plates are relatively large, Eq. (4) does not result in accurate answers because it ignores the contribution of tangential stresses in the plate behavior. The load-deformation behavior of thin plates is governed by von-Kármán equations in the case of rectangular plates with large deformations. Different references defined large deformations based on portions of the thickness of the plate e.g. $\delta > 0.5t$ [11], $\delta > 1.0t$ [21,22] and $\delta > 0.2t$ [5] where δ is the maximum deflection normally at the center of plate as shown in Fig. 3. The von-Kármán equations are presented in Eq. (6) and Eq. (7) where F is a stress function of x and y [21].

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \quad (6)$$

$$\begin{aligned} \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \\ = \frac{t}{D} \left(\frac{q}{t} + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) \end{aligned} \quad (7)$$

A solution to von-Kármán equations for specific boundary conditions results in *stress* and *deformation* functions. The stress function (F) determines the tangential stresses in the middle surface of the plate and the deformation function, (w) defines the deflected shape of the plate as:

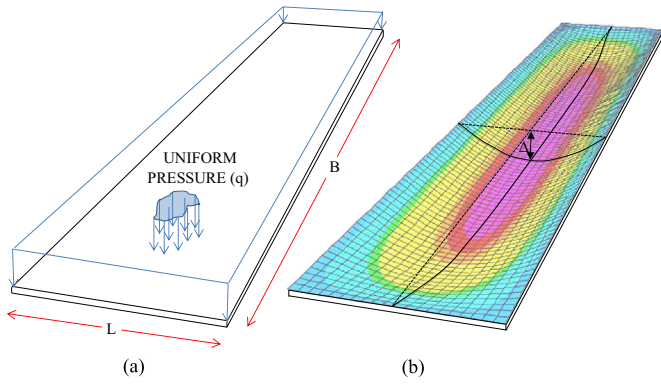


Fig. 3. Rectangular simply supported plate under uniform lateral pressure. (a) Plate dimensions and loading (b) Contour display of plate deflection.

$$\begin{cases} N_x = t \frac{\partial^2 F}{\partial y^2} \\ N_y = t \frac{\partial^2 F}{\partial x^2} \\ N_{xy} = -t \frac{\partial^2 F}{\partial x \partial y} \end{cases} \quad (8)$$

The bending and shear stresses could be then extracted from the classic plate theory considering small deformations. In other words, the distributed uniform load q could be resolved into two components q_1 and q_2 where q_1 is balanced in total by bending and shear stresses calculated using the equations of plates with small deformations and q_2 is balanced by the membrane (tangential) stresses calculated using Eq. (8) [21].

Although general closed form solutions to von-Kármán equations do not exist yet, several cases of plates with different geometry and boundary conditions are solved in the literature. First attempts were made by Levy in three separate reports for the National Committee for Aeronautics [11–13] to present a closed form solution to von-Kármán equations for specific boundary conditions and geometries of rectangular and square plates. In order to support and verify the previously derived solutions by Levy, Ramberg et al. [18] performed normal pressure experimental tests on 56 rectangular plates with clamped edges and 5 plates with freely supported edges. The results of those tests only qualitatively verified the results obtained from the theoretical solutions to the plate deformation equations, thus more improved solutions were required. Some improved solutions to von-Kármán equations for rectangular plates were obtained for the special cases of plates with aspect ratios of 1.5 and 2.0 under normal uniform pressure by Wang [23]. In similar attempts, other researchers [10,14,24] stated various closed form solutions for the same problem using different methods. Latest attempt to derive a closed form solution for the case of rectangular plate with large deformations was by Ibearugbulem et al. [9] where the maximum deflection at the center of rectangular plates simply-supported at four edges (SSSS) was calculated. Rectangular plates with aspect ratios varying from 1.0 to 2.0 were solved by this method and the solutions were verified with the previous methods by Timoshenko and Woinowsky-Krieger [9].

Aside from the closed form types of solutions that lead to lengthy mathematical formulations which are not simple to follow by the design engineers, approximate and numerical methods were also suggested in most of the previously mentioned references. Bakker et al. [5] used a shape function corresponding to the first buckling mode shape of the plate under in-plane compressive stresses in order to approximately determine the large deflection

behavior of rectangular simply-supported thin plates. In their approach, the large-deflection behavior of the plate under transverse loading was estimated as a function of the ratio of the pre-buckling to post-buckling stiffness of the plate. Both pre- and post-buckling stiffnesses are available in the literature for rectangular plates.

As in most of the available references on plate analysis, in this paper long rectangular plates are those with aspect ratios of 2.0 and greater, which mainly refer to panels of stiffened plates used as the walls of thin-walled structures such as industrial ducts, bunkers and similar structures. There is very limited research work available in the literature addressing the behavior of such plates under transverse pressure considering large deformations effects; this is in part due to the general definition of long plates as elements whose behavior could be resembled as one-way plate action using beam theory as mentioned in the previous section. Thanga et al. [20] used FE analysis in order to study the effects of yielding of steel material on the behavior of duct wall plates under lateral pressure considering the membrane forces induced by large deformation. They modeled a strip of plate with unit width and various lengths using shell elements. They established design equations for three different cases of through-thickness yielding namely 0%, 16.5% and 33% of the thickness of steel plate in order to increase the load carrying capacity. Same modeling approach (unit width strip geometry) was used by El-Aghoury and Galal [8] in a parametric study on the specific case of plates between two adjacent ribs of the walls of industrial ducts. The main result of their work was a very close approximation of the load-deformation of plates with large deformations under uniform pressure by applying the Bezier curves [6] based on numerical modeling. Both of last two references assumed that using a unit width strip model based on beam analogy give very accurate approximations of the case of long rectangular plates under lateral uniform pressure. Contrary to the traditional method of treating rectangular plates with aspect ratios higher than 2.0 as one-way plates, Wang and El-Sheikh [24] showed that two-way plate action still applies to plates with aspect ratios from 1.5 to 10.0 and the application of one-way action will result in over-estimated deflections calculation for relatively long plates.

3. Nonlinear Finite Element analysis of duct walls

The Finite Element modeling and analysis in this paper are done using standard SHELL181 element from the element library of the general purpose commercial software package ANSYS [1]. SHELL181 is a 4-node shell element with six degrees of freedom (translation in and rotation about three dimensional axes) which is designed for the analysis of thin to moderately-thick shell structures [7]. SHELL181 element covers three types of behavior namely “plate-type” which covers only the bending-type using the classical Mindlin/Reissner theory, “membrane-type” which neglects any bending and considers pure membrane behavior and “shell-type” that covers both bending and membrane behaviors by implementing the geometrical nonlinearities effects. The element is tested by ANSYS for the linear and nonlinear applications concerning large deformations, large rotations and large strains through benchmark verification examples [2]. SHELL181 stress output covers the normal and shear stresses namely S_x , S_y and S_{xy} at top, bottom and middle of each shell layer. It also provides the equivalent von-Mises stresses as well as other types of common failure criteria. More importantly, it provides stress resultants corresponding to membrane forces (N_{11} , N_{22}), In plane shear force (N_{12}), bending moments (M_{11} , M_{22}), in plane moment (M_{12}) and transverse shear forces (Q_{13} , Q_{23}) as shown in Fig. 4.

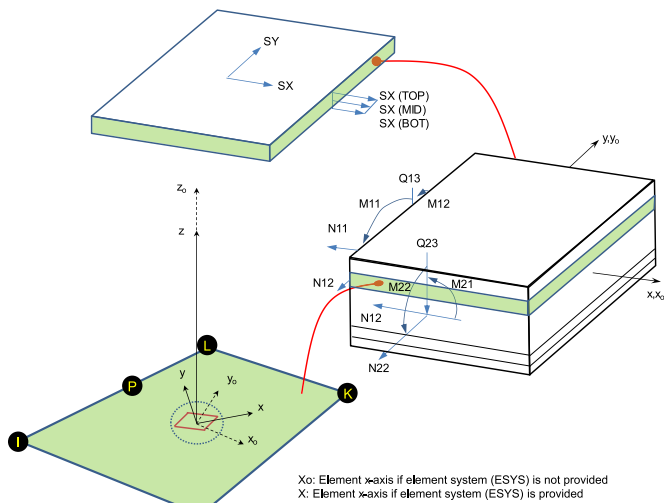


Fig. 4. SHELL181 geometry and stress output (ANSYS, 2010-b).

3.1. Validation of modeling and boundary conditions

Most of the FE models and experimental tests available in the literature for rectangular plates under lateral pressure consist of a unit-width strip or a single panel plate model with clamped, hinged or free boundary conditions assigned to each of the four edges. This approach results in geometry and boundary conditions for the edge and typical panels of duct walls as shown in Fig. 5.

The wall plate panel with boundary conditions shown in Fig. 5 (a) is modeled here to verify the element and analysis method. The unitless load displacement curve is shown in Fig. 6 in comparison to the results of [8] and [25]. Similarly in Fig. 7, the load-stress curves resulted from the FE analysis are presented against the available results in the literature.

Comparing the results of current study for the single panel plate with those available in the literature, it is seen that for the analysis of typical panel plates of duct walls independently (without considering the effects of expansion joints), the available equations in the literature are adequate in predicting the behavior of duct wall plates. In other words the plate model has no significant advantage over the existing unit-width strip method that applies the large displacement beam theory to plates.

Expansion joints at one or both ends of a duct segment release

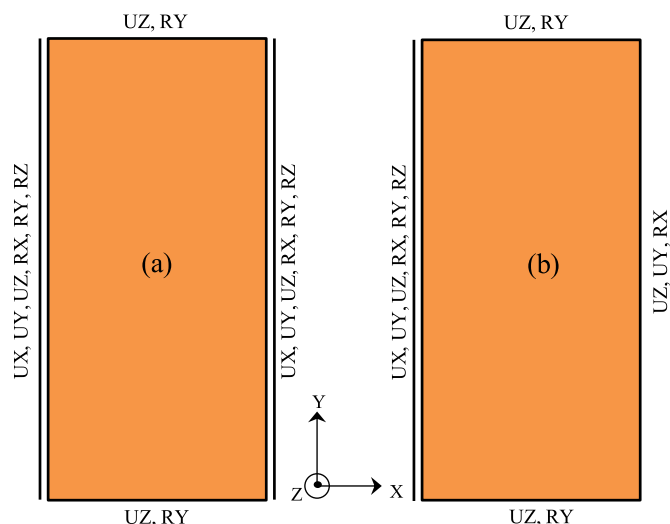


Fig. 5. Boundary conditions for the single-span plate model (a): Typical panel (b): Edge panel.

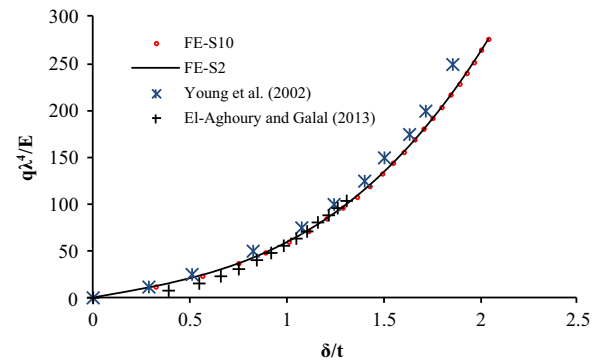


Fig. 6. Load-deformation curve of rectangular plate (typical panel).

the axial force and bending moment by providing discontinuity along the length of the stiffened plate. The equilibrium of forces and deformations changes with the release of axial force as shown in Fig. 8 for a sample three span beam (or plate strip) that is subjected to uniform load with consideration of large deformations. In case (a) the large deformations resulting from the distributed load produce the axial force N (membrane action) in each span and the reactions at the two ends of the beam as shown in the free body diagram in Fig. 8(a). If the axial restraint is removed at one end (or both ends), the axial force N could not generate due to the equilibrium of forces, hence the deflections will be larger and the large deflections result in cumulative in-plane displacements of the beam as shown in Fig. 8(b).

Thus, neither the strip method used in the previous works nor the single span plate modeling apply to the case of duct walls (typical and edge panels) if expansion joints are present in the duct assembly. In other words, a realistic model that represents the stiffened plates of duct walls must cover the geometry of the wall plates in a way that the assumed boundary conditions correspond to the existence of expansion joint at the edge of the duct segment. Such a plate model obviously cannot consist of only a single panel of the stiffened plate, thus in this paper multi-span plates are used. Fig. 9 shows the geometry, FE mesh and the boundary conditions of the 3D model of a duct segment that is designed based on the linear design method. In this model, the DOFs of plate elements at the corners of the duct segment are fully integrated to resemble the fully welded situation in reality. The rib stiffeners are also fully connected at the duct corners based on the assumption of moment connection at the ends of the ribs. Geometric nonlinearities (large deformation effects) are included in the analysis to verify the behavior of the duct segment.

Fig. 10 shows the absolute deformed shape of the duct segment model under internal uniform pressure in terms of displacements and rotations. The overall deformation of the wall plates is the sum of panel plates and rib stiffeners deformations. Since the rib elements are out of the scope of this paper, they are removed from the view in the figure.

It is seen in Fig. 10 that the deformed shapes of the duct wall plates are similar for all the panels except for the edge panel where the expansion joint exists. It is observed from the absolute displacement contours that the edge panel displacements are smaller than those of the typical panels considering the smaller length used for the edge panels. It is also observed that the free edge of the edge panel shows the maximum rotation at the middle due to the connectivity of wall plates at the duct corners that prevent the rotations. Considering the symmetry of the deformations and rotations, it is concluded that a multi-span wall plate model will well represent the full duct segment model for the analysis of the wall plate.

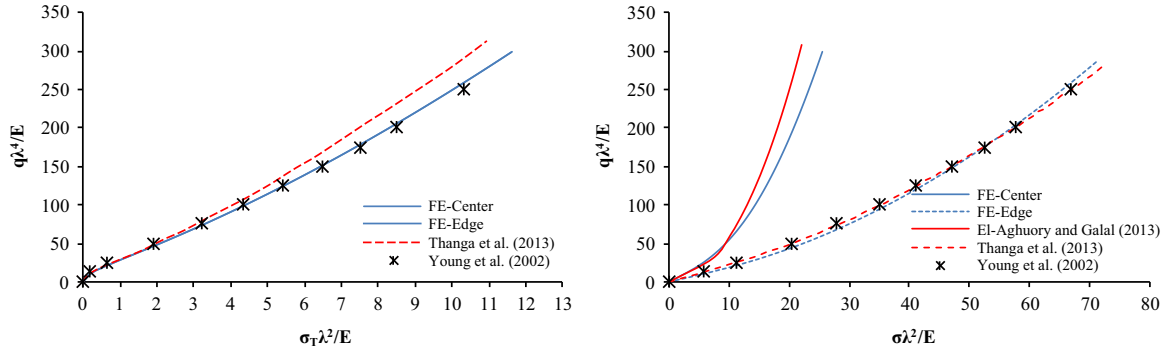


Fig. 7. Normalized tangential (membrane) and total stresses for single span rectangular plate with large deformations.

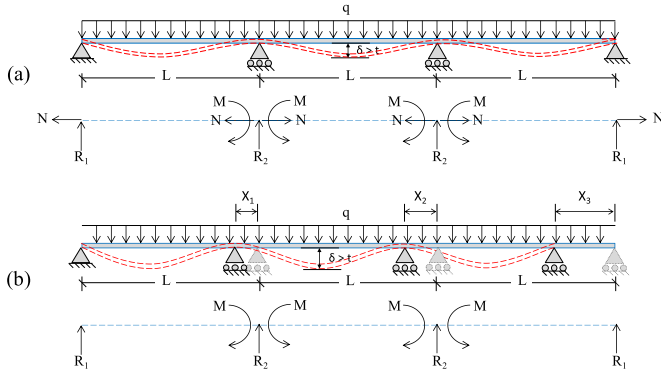


Fig. 8. Loading and free body diagram for the equilibrium of continuous beams with large deformation.

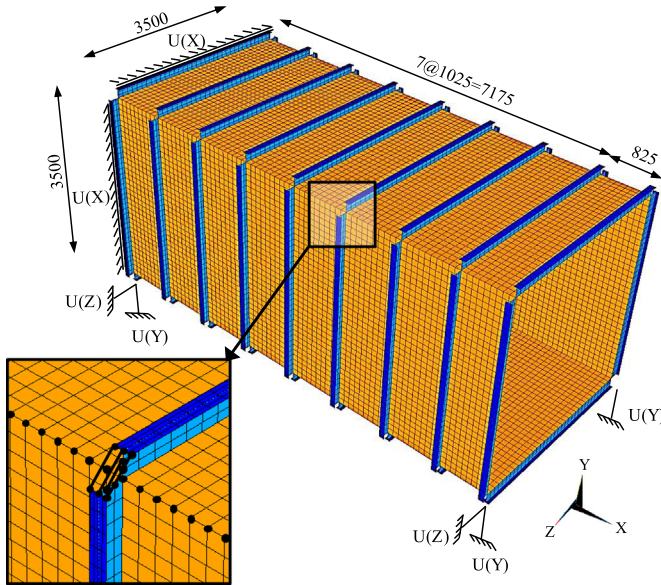


Fig. 9. Geometry and FE mesh of a sample 3D duct segment model.

3.2. Multi-span plates with internal line supports

The total displacement of a duct wall plate consists of the rib displacements and the plate displacement (see Fig. 10). In order to study the wall plate behavior, the plate load-displacement needs to be distinguished from that of the ribs. Thus, in the numerical models of this paper, multi-span plates with internal line supports

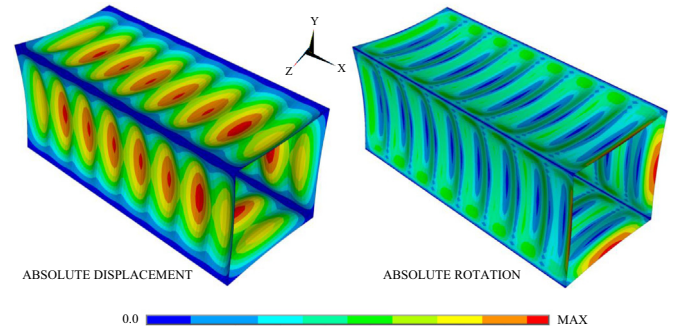


Fig. 10. Absolute deformation and rotation of the duct segment model under the applied internal pressure (rib elements removed from the view for clarity).

are used. Plates with internal line supports are an imaginary type of stiffened plates where the rib elements are assumed to be infinitely rigid so they could be replaced with appropriate boundary conditions. Fig. 11 shows the sketch of a 10 span segment of a duct wall plate with panel aspect ratio $S=5.0$ and internal line supports. The boundary conditions used in this model represent expansion joints at both ends of the segment. Equal rib spacing was used for all the panels in order to distinguish between the load-displacement behaviors of different panels.

As seen in Fig. 12, the plate undergoes in-plane as well as out-of-plane deformations under the applied uniform pressure when the large displacements (geometrical nonlinearities) are included in the analysis. It is also seen in the $D(Z)$ contours in Fig. 12 that various panels of the stiffened plate tend to follow different load-displacement patterns. Fig. 13 shows the load-displacement curves of different panels of stiffened plates with three aspect ratios $S=2$, $S=5$ and $S=10$.

As seen in Fig. 13 for plates with equal spans, the edge panel has the weakest load-displacement behavior i.e. under the same pressure it undergoes highest deformations. Similar observation

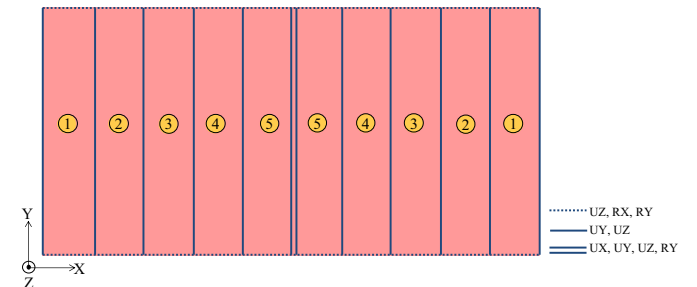


Fig. 11. General arrangement of duct wall plate models with internal and edge line supports resembling the restraints provided by the end panels, ribs and other walls of the duct when expansion joints exist at both ends of the segment.

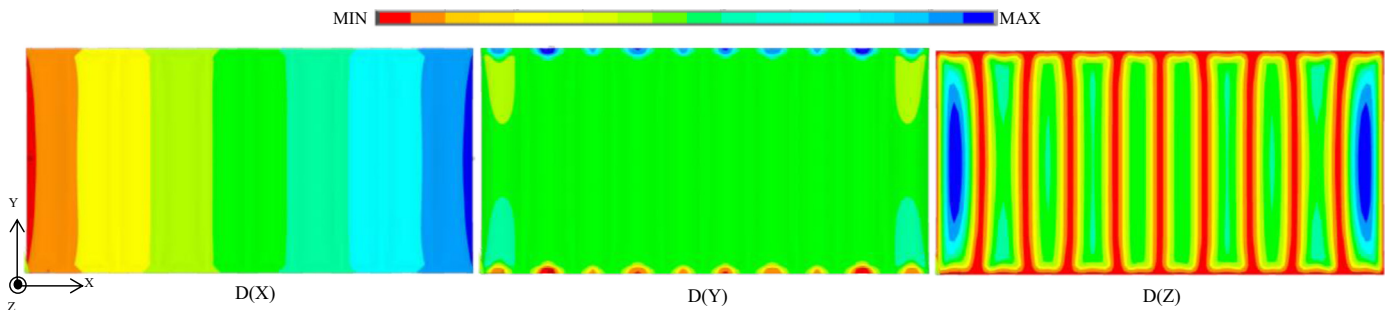


Fig. 12. Displacement contours in three global directions for 10-span plate model with internal line supports and span aspect ratio $S=5$.

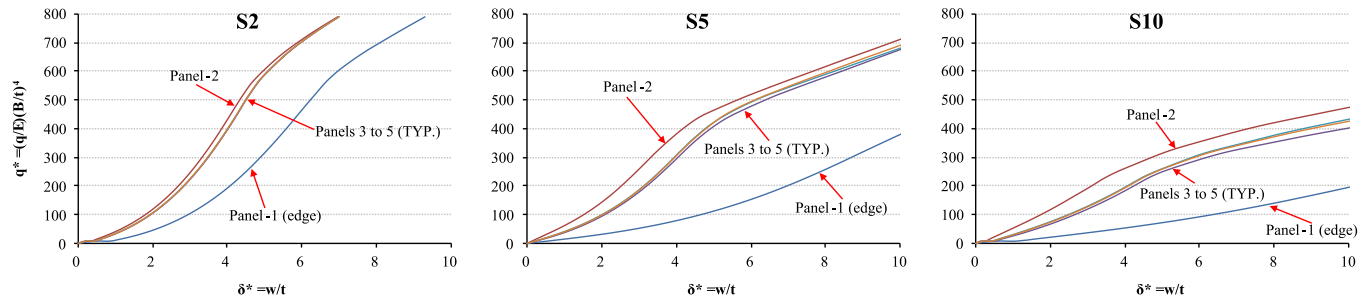


Fig. 13. Out of plane load-displacement charts for different panels of the plate with internal line supports in three aspect ratios $S=2$, $S=5$ and $S=10$.

exists for the case of linear analysis due to the boundary conditions provided by the expansion joint. On the other hand, the 2nd panel tends to have the lowest displacements under the same pressure in general comparing to the typical panels. The reason for this behavior is that due to the free rotational degree-of-freedom at the free edge of the edge panel and the continuity of the plate between the edge and 2nd panel, the rotation at one edge of the 2nd panel has a non-zero value. Thus the load-displacement curve of the 2nd panel shows slightly stiffer behavior (i.e. lower displacement under the same pressure) compared to the typical panels where the rotation at the edges are zero.

4. Design of duct wall plates based on nonlinear analysis

Based on the load-displacement charts in Fig. 13, two main categories for the panels of stiffened plates in this paper are the “edge” and “typical” while the design of typical panels could be conservatively applied to the 2nd panel accordingly. Similar to the conventional design, there are two criteria to be used in the nonlinear design of plates namely *stress* and *deflection*. In this section, nonlinear equations will be driven from the FE analysis of 10 span plates with panel aspect ratios from 2 to 10 for stress and

deflections criteria.

4.1. Stress criterion

The load-stress charts for the edge and typical panels of multi-span plates with various aspect ratios (S) from 2 to 10 are shown in Fig. 14. The stress in these charts is the maximum absolute von-Mises stress normalized with plate's yield stress.

The charts in Fig. 14 are estimated using a trend line with second order equation as shown in Eq. (9).

$$q\lambda^2/F_y = a(\sigma/F_y)^2 + b(\sigma/F_y) \quad (9)$$

The values of “ a ” and “ b ” constants depend on the plate aspect ratio for the edge and typical panels and could be found in Table 2.

Eq. (9) is applicable in both ultimate limit states and allowable stress design methods to calculate the length-to-thickness ratio for the given load when appropriate stress limits are used for σ/F_y .

4.2. Deflection criterion

The load-deformation charts of the typical and edge panels of the multi-span plates under uniform lateral pressure are shown in

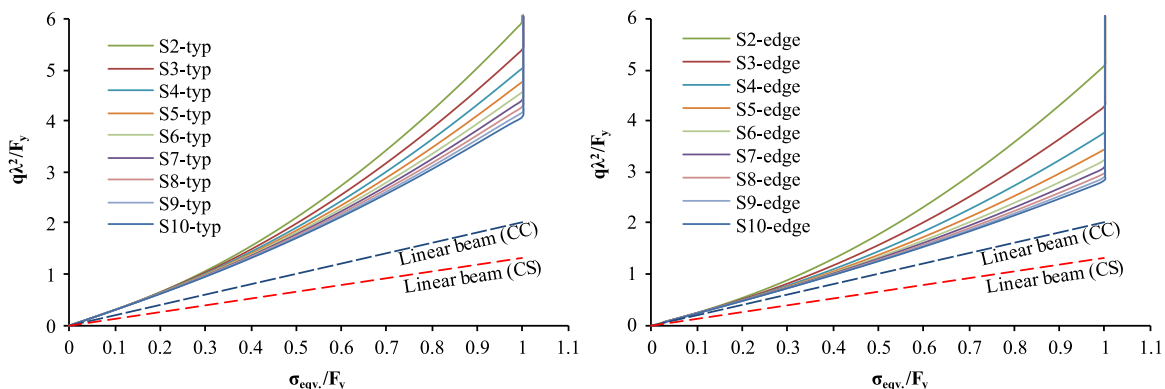


Fig. 14. Maximum von-Mises stress in typical and edge panels of multi-span plates with aspect ratios from 2 to 10.

Table 2
Aspect ratio numerators to be used in Eq. (9).

S	Typical panel		Edge panel	
	a	b	a	b
2	3.42	2.52	3.07	2.05
3	2.82	2.61	2.27	2.03
4	2.41	2.65	1.72	2.06
5	2.1	2.69	1.35	2.1
6	1.87	2.72	1.09	2.14
7	1.69	2.75	0.91	2.18
8	1.55	2.76	0.76	2.21
9	1.43	2.77	0.66	2.23
10	1.33	2.78	0.57	2.25

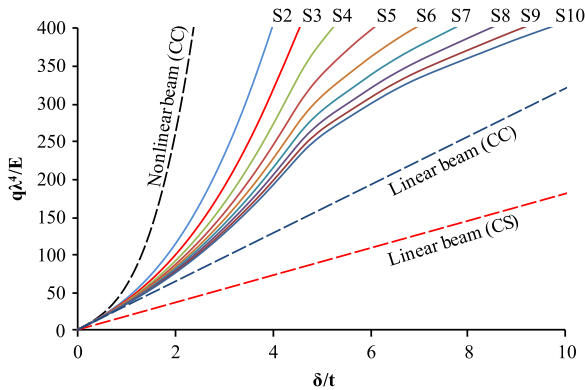


Fig. 15. Load-deformation charts for typical panel with aspect ratios S2 to S10.

Figs. 15 and 16. These charts reflect the nonlinear load-deformation behavior when the geometric nonlinearities are considered in the analysis.

Fig. 15 shows that for typical plates the nonlinear behavior has two trends. The first trend starts from the beginning of loading and follows a cubic function up to the point where the material yielding occurs. The second trend results from the addition of material nonlinearities after yielding in combination with stress redistributions to the original nonlinear load-displacement. The plastic behavior of plate after yielding is out of scope of design for the plates being used for the walls of industrial ducts. Thus, if yielding of material is set as the limit state i.e. the first trend of the charts in Figs. 15 and 16 are used for the design, the load-displacement behavior for the typical and edge panels could be estimated by a trend line with cubic equation as shown in Eq. (10).

$$q\lambda^4/E = a'(\delta/t)^3 + b'(\delta/t)^2 + c'(\delta/t) \quad (10)$$

where “a”, “b” and “c” account for the plate aspect ratio (S) effects

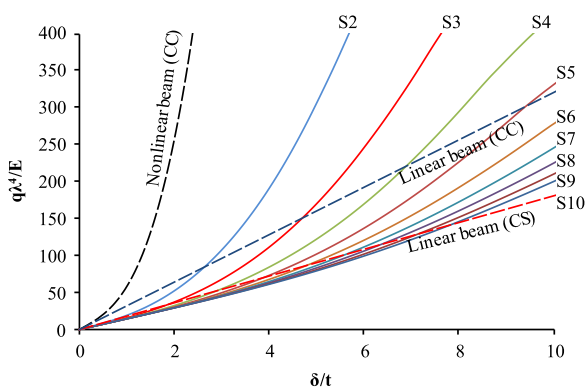


Fig. 16. Load-deformation charts for edge panel with aspect ratios S2 to S10.

Table 3
Aspect ratio numerators to be used in Eq. (10).

S	Typical panels			Edge panel		
	a'	b'	c'	a'	b'	c'
2	2.51	7.82	31.34	1.28	3.97	14.1
3	1.52	5.93	31.51	0.58	1.49	13.72
4	1.01	5.15	31.18	0.31	0.8	13.62
5	0.67	4.95	30.55	0.17	0.69	13.47
6	0.47	4.69	30.35	0.11	0.5	13.5
7	0.35	4.46	30.09	0.08	0.42	13.55
8	0.27	4.27	29.79	0.06	0.36	13.64
9	0.26	3.82	29.87	0.05	0.31	13.73
10	0.23	3.58	29.74	0.04	0.26	13.83

listed in Table 3.

4.3. Application of design equations

Design of duct wall plates consists of determination of plate thickness and length based on the stress limits defined by the adopted design code and verification of the deflections of the designed wall plate with the deflection limits set by the authorities. The calculation of plate maximum stress and deflection was covered in the previous section by means of Eqs. (9) and (10) for the typical and edge panels respectively. In this section, those equations are used to provide a more meaningful design approach by adopting the ultimate limit states for stress set by the two main North American steel design codes for plates. The ultimate stress for plate elements under bending and tension is $\sigma_u = 0.9F_y$ (CISC-S16.09), [3]. Thus for $\sigma_u/F_y = 0.9$ in Eq. (9):

$$q\lambda^2/F_y = a(0.9)^2 + b(0.9)$$

Replacing a and b numerators in this equation with their values from Table 2 for various panel aspect ratios will result in a power function that calculates the normalized plate length-to-thickness ratio ($\lambda/\sqrt{F_y}$) for the given lateral pressure:

$$\lambda_u/\sqrt{F_y} = \eta q^{-1/2} \quad (11)$$

where λ_u is the ultimate plate length-to-thickness ratio based on the ultimate stress limit ($0.9F_y$), and η is the plate aspect ratio factor ($\eta = 0.81a + 0.9b$). The values of η for various panel aspect ratios are presented in Table 4 for the edge and typical panels. The last row in Table 4 shows the value of η for linear design by replacing the ultimate limit state stress in Eq. (2) for the edge (C-S) and typical (C-C) panels.

The normalized plate length-to-thickness ratio calculated from Eq. (11) is plotted against a regular range of lateral pressures set by ASCE [4] in Fig. 17 for edge and typical panel plates.

The value of η in Table 4 indicates the effect of plate aspect ratio

Table 4
Aspect ratio numerators to be used in Eq. (11).

S	Typical panels η	Edge panel η
2	2.24	2.08
3	2.15	1.91
4	2.08	1.80
5	2.03	1.73
6	1.99	1.68
7	1.96	1.64
8	1.93	1.61
9	1.91	1.59
10	1.89	1.58
Linear	1.34	1.09

in the nonlinear design. In order to evaluate the effectiveness of nonlinear design in comparison with the conventional linear design, the length-to-thickness ratios resulted from Eq. (11) for typical and edge panels from nonlinear and linear designs are compared in Table 5.

As seen in Table 5, the plate length for edge panel is 84–93% of the typical panel length if nonlinear design is used showing that for shorter aspect ratios the edge panel design gets closer to that of the typical panel. It is also seen in the table that for the typical panels the nonlinear design results in 41–67% higher length-to-thickness ratios accordingly depending on the aspect ratio. In other words for the same plate length, the plate thickness resulted from the nonlinear design is 30–40% lower than what would result from the conventional linear design. It is worth to mention that although separate equations are presented for the design of edge and typical panels, it is practical not to use different plate thicknesses for the edge and typical panels. In other words, the design could be done by only calculating the length-to-thickness ratio for the typical panels using Eq. (9) and multiplying the length of the typical panel by the $\lambda_u(\text{edge})/\lambda_u(\text{typical})$ factor from Table 5 to get the length of the edge panel.

4.4. Displacement limits for the design of duct walls

Contrary to the ultimate limit states design or other approaches where the stress limitations are set by the codes, in the field of industrial structures design, the serviceability limit states (i.e. the displacement limits) are often set by the local authorities, mechanical engineers or the owner of the industrial facilities where the duct system is built in a way to keep the duct functional. A general rule of thumb followed by most of the design engineers for the displacement limits is a portion of the plate length (e.g. $L/50$, $L/75$, $L/100$ etc.). However this type of displacement limits tends to meet the serviceability demands of building structures, it might result in unnecessary plate thicknesses without addressing the true nature of wall behavior in thin-walled structures such as industrial duct assemblies. On the other hand the authors were unable to find any tabulated displacement limits set for the duct wall plates (or similar structures) or any explanation on the basis of such limits as $L/50$ or $L/100$ in the literature.

In order to be able to compare the nonlinear design approach to the conventional linear design method, this paper uses a different way to set the displacement limits that tends to be more practical for the plate elements such as duct walls. The displacements in this method are compared to the plate thickness. By using this method, replacing the δ/t with the desired displacement limit in Eq. (10) will result in plate length to thickness ratio for the applied pressure. Since the displacement limits are different from case to case, it is not possible to formulate the design as in the ultimate limit states in which a certain stress limit is set by the code. The ratio of maximum lateral pressure calculated from Eq. (10) to the

Table 5

Comparison between the length-to-thickness ratios resulted from the nonlinear and linear designs for edge and typical panels.

S	$\lambda_u(\text{edge})/\lambda_u(\text{typical})$	$\lambda_u(\text{typical})/\lambda_u(\text{linear})$	$\lambda_u(\text{edge})/\lambda_u(\text{linear})$
2	0.929	1.67	1.91
3	0.888	1.60	1.75
4	0.865	1.55	1.65
5	0.852	1.51	1.59
6	0.844	1.49	1.54
7	0.837	1.46	1.50
8	0.834	1.44	1.48
9	0.832	1.43	1.46
10	0.83	1.41	1.41
Linear	0.815	1.00	1.00

Table 6

Ratio of maximum lateral pressure resulted from the nonlinear approach to linear method, $q(\text{nonlinear})/q(\text{linear})$.

$(\delta/t)_{\max}$	S2	S3	S4	S5	S6	S7	S8	S9	S10
1.0	1.30	1.22	1.17	1.13	1.11	1.09	1.07	1.06	1.05
2.0	1.78	1.55	1.42	1.35	1.30	1.26	1.23	1.20	1.18
3.0	2.42	1.97	1.74	1.61	1.52	1.46	1.41	1.36	1.33
4.0	3.21	2.49	2.12	1.91	1.77	1.67	1.60	1.54	1.49
5.0	4.16	3.10	2.57	2.25	2.05	1.91	1.81	1.73	1.67
6.0	5.27	3.81	3.08	2.64	2.36	2.17	2.04	1.94	1.86
7.0	6.53	4.61	3.65	3.06	2.69	2.45	2.28	2.17	2.06
8.0	7.95	5.51	4.28	3.53	3.06	2.76	2.54	2.41	2.28
9.0	9.53	6.50	4.98	4.04	3.46	3.08	2.82	2.67	2.52
10.0	11.27	7.59	5.74	4.60	3.88	3.43	3.11	2.94	2.77

pressure resulted from Eq. (3) are shown in Table 6 for various plate panel aspect ratios and displacement limitations $(\delta/t)_{\max}$ from 1.0 to 10.0 respectively. As seen in Table 6, the effectiveness of the presented nonlinear design approach in displacement criterion depends highly on the applied displacement limit and plate panel aspect ratio.

5. Conclusions

Two sets of design equations have been set using verified numerical analysis of multi-span plates with internal line supports under uniform lateral pressure that take into account the effects of large deformations nonlinearities. These equations stand for the design of plate length and thickness by applying stress and displacement criteria to the plate structure. If used appropriately in conjunction with well-defined stress and deflection limits, a duct wall plate designed by these equations will be up to 40% more economical compared to the conventionally designed plates that use the linear beam theory.

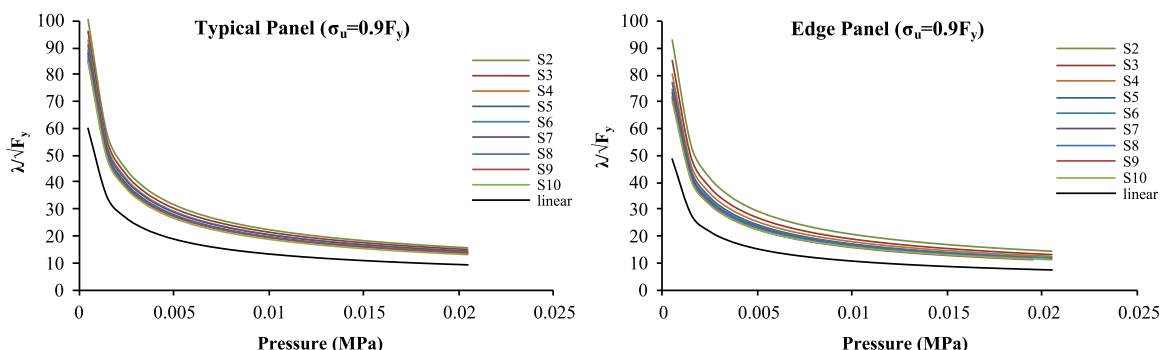


Fig. 17. Normalized plate length-to-thickness ratio for various lateral pressures on duct walls typical and edge panels.

Acknowledgement

The authors would like to express their appreciation for the financial support provided for this research by Solios Environment Inc., Strudes Inc., NSERC (Natural Sciences and Engineering Research Council of Canada) and FRQNT (le Fonds de Recherche du Québec - Nature et Technologies) under the "BMP Innovation" award program number 180371.

References

- [1] I. ANSYS, in: P. Kohnke (Ed.), ANSYS Mechanical APDL Theory reference, SAS IP, Inc., Canonsburg, PA, 2013, pp. 586–587.
- [2] I. ANSYS, ANSYS Mechanical APDL Element reference, SAS IP, Inc., Canonsburg, PA 2013, pp. 805–820.
- [3] AISC, Manual of steel construction, load and resistance factor design, Third edition, American Institute of Steel Construction, Inc., 2003.
- [4] ASCE. (1995). The structural design of air and gas ducts for power stations and industrial boiler applications. Air and gas duct structural committee of the energy division of the American Society of Civil Engineers (ASCE) Reston, VI.
- [5] M.C. Bakker, M. Rosmanit, H. Hofmeyer, Approximate large-deflection analysis of simply supported rectangular plates under transverse loading using plate post-buckling solutions, *Thin-Walled Struct.* 46 (2008) 1224–1235.
- [6] P. Bezier, Numerical Control: Mathematics and Applications, Wiley, New York, 1972.
- [7] E.N. Dvorkin, K.-J. Bathe, A continuum mechanics based four-node shell elements for general non-linear analysis, *Eng. Comput.* Vol. 1 (1984) 77–89.
- [8] I.M. El Aghoury, K. Galal, Design of rectangular industrial duct plates subjected to out-of-plane pressure considering nonlinear large deformations, *Thin-Walled Struct.* Vol. 77 (2014) 1–7.
- [9] O.M. Ibearugbulem, L.O. Ettu, J.C. Ezech, Direct integration and work principle as new approach in bending analyses of isotropic rectangular plates, *Int. J. Eng. Sci.* (IJES) 2 (3) (2013) 28–36.
- [10] C.E. Imrak, I. Gerdemeli, The problem of isotropic rectangular plate with four clamped edges, *Sadhand* 32 (3) (2007) 181–186.
- [11] S., Levy (1941a). Bending of rectangular plates with large deformations. Report No. 737-National Advisory Committee for Aeronautics.
- [12] Levy, S. (1941b). Square plate with clamped edges under normal pressure producing large deformations. Report No. 740, National Committee for Aeronautics.
- [13] Levy, S. (1942). Square plates with clamped edges under normal pressure. Report No.847, National Committee for Aeronautics.
- [14] G.H. Little, Efficient large deflection analysis of rectangular plates with general transverse form of displacement, *Comput. Struct.* 71 (1999) 333–352.
- [15] Z.J. Liu, Nonlinear analysis of flue gas ductwork for power plant, *Structures Congress, Structural Engineering Institute of ASCE*, Boston, MA, 2014.
- [16] Love, A. E. H. On the small free vibrations and deformations of elastic shells, *Philosophical trans. of the Royal Society (London)*, 1888, Vol. série A, N° 17 p. 491–549.
- [17] R.D. Mindlin, Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates, *ASME J. Appl. Mech.* 18 (1951) 31–38.
- [18] Ramberg, W., McPherson, A. E., and Levy, S. (1942). Normal-pressure tests of rectangular plates. Washington: Technical Note No.849, National Advisory Committee for Aeronautics.
- [19] E. Reissner, The effect of transverse shear deformation on the bending of elastic plates, *ASME J. Appl. Mech.* 12 (1945) 68–77.
- [20] T. Thanga, B. Halabieh, K.S. Sivakumaran, Strength of plates of rectangular industrial ducts, *Procedia Engineering*, 14, Elsevier 2011, pp. 622–629.
- [21] S. Timoshenko, S. Woinowsky-Kreiger, *Theory of Plates and Shells*, 2nd ed, McGraw-Hill, New-York, 1959.
- [22] A.C. Ugural, *Stresses in plates and shells*, 1st edition, Mc Graw-Hill, 1981.
- [23] Wang, C. T. (1948). Bending of rectangular plates with large deflections. Technical Note No.1462, National Advisory Committee for Aeronautics.
- [24] D. Wang, I. El-Sheikh, Large-deflection mathematical analysis of rectangular plates, *Journal of Engineering Mechanics*, 131, ASCE 2005, pp. 809–821.
- [25] W.C. Young, R.G. Budynas, *Roark's Formulas for Stress and Strain*, vol. 6, McGraw-Hill, New York, 2002.