

Nominal moment capacity of partially deteriorated AISC W-section beams



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ABSTRACT

The moment capacity of partially deteriorated beams is usually estimated based on the deteriorated section properties regardless of the deterioration length along the beam, which in many cases underestimates the overall capacity of the beam. Structural deterioration in steel bridge girders is usually caused by ageing, corrosion, fatigue, and higher load demands. Area losses in the steel beam cross-section due to this deterioration result in an overall reduction in the beam's flexural capacity. This loss in the flexural capacity varies significantly for different AISC W-sections.

In this paper, a new dimensionless section property named WSSR (W-Section Sensitivity Ratio) is proposed and used as a relative indicator of the sensitivity of the section to the loss of its moment of inertia due to the loss of part of its flange area. A new term is introduced for partially deteriorated beams named “equivalent yield moment”. Consequently, a parametric study was conducted using a numerical finite element model calibrated using experimental test results in order to correlate the WSSR to the loss in the nominal flexural capacity. Moreover, a new design (code independent) nominal capacity chart is generated to accurately determine the “equivalent yield moment” and corresponding deflection of the partially deteriorated steel girders using the deterioration in sectional inertia and the deterioration length along the beam.

1. Introduction

Structural deterioration of steel bridges is usually caused by weathering effects that leads to corrosion or by accidental damage of the lower flanges hit by trucks. According to a report by the Federal Highway Administration of the United States of America [1], more than 60,000 bridges were classified as deficient and more than 40% of these deficient bridges are steel structures. This paper focuses on developing an evaluation approach for structural deteriorations that can eventually lead to an overall loss in the flexural capacity of steel girders. The evaluation of residual capacity due to spread of deterioration or localized damage is practically challenging for engineers. There are many studies on enhancing the performance of steel beams such as [2,3], and deteriorated steel beams such as Fernando et al. [4] and Hmidan et al. [5], Sweedan et al. [6] and Gholami et al. [7]. It is important for a retrofit design engineer to quantify the amount of deterioration in the load carrying capacity and the amount of increase in deflection due to the deterioration of an existing bridge girder. This motivated previous researches to develop simple capacity evaluation models.

Kayser and Nowak [8] developed a corrosion damage model for simple-span steel girder bridges to predict the area loss due to corrosion. Kulicki et al. [9] provided guidelines for field inspection of existing bridges. Their report included the evaluation process

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Nomenclature			
A	cross-section area of the steel beam	M_w	partially deteriorated beam
A_d	cross-section area of the deteriorated steel beam	S	the moment corresponding to the actual deflection of the original beam
AASHTO	American Association of State Highway and Transportation	S_{bo}	elastic section modulus
AISC	American Institute of Steel Construction	S_{bd}	elastic section modulus to the bottom fiber of the non-deteriorated section
AREMA	American Railway Engineering and Maintenance-of-Way Association	$WSSR$	elastic section modulus to the bottom fiber of the deteriorated section
b_d	width of the deteriorated flange		W-Section Sensitivity Ratio (Relative indicator of the sensitivity of the section to the loss of its moment of inertia due to the loss of part of its flange area)
b_o	width of the original flange	Δ_{ao}	the actual deflection for the original non-deteriorated beam
E	modulus of elasticity	Δ_{ap}	the actual deflection for the partially deteriorated beam
I_d	moment of inertia of the deteriorated beam	Δ_{yield}	the deflection at the onset of yielding for the fully deteriorated beam
I_o	moment of inertia of the original beam	Δ_{yd}	the deflection at the onset of yielding for the fully deteriorated beam
k_1	load-flexural deflection coefficient (based on the load pattern)	Δ_{yo}	the deflection at the onset of yielding for the original non-deteriorated beam
k_2	2 (1 + ν)	Δ_{yp}	the deflection at the onset of yielding for the partially deteriorated beam
k_3	moment-load coefficient (based on the load pattern)	ψ	the beam deterioration level factor
L_d	length of the deteriorated beam	ν	Poisson's ratio
L_o	full length of the beam.	σ_{yield}	yield stress of the beam material
LTB	lateral torsional buckling.		
M_{yield}	the moment at the onset of yielding		
M_{yd}	the moment at the onset of yielding for the fully deteriorated beam		
M_{yo}	the moment at the onset of yielding for the original non-deteriorated beam		
M_{yp}	the moment at the onset of yielding for the		

for the load carrying capacity of a bridge and the elements affected by corrosion, defining the type of loads to be used, calculating the resulting member loads, and defining the required safety levels. van de Lindt et al. [10] presented the results of experimental and numerical analyses of the steel beam end zones with simulated damage of various sizes and shapes to develop a model that can provide structural analysts with a simplified approach for computing the reduced capacity of typical bridge steel sections. Based on the experimental results, design charts were developed to allow the engineer to directly read the deterioration factor ψ_d , the ratio of the reduced capacity to the original capacity, for a given damage depth and damage height of a beam.

Corrosion is one of the main forms of deterioration of steel bridges. Localized corrosion often occurs along the bottom flange of stringers and girders or in the webs of the bridge columns, hence leading to a substantial reduction in their flexural capacities. Rahgozar [11] carried out an analysis on two different deteriorated I-beam steel section models to obtain minimum curves for reliable estimation of the remaining shear, moment, and bearing capacity. Corrosion was simulated by a reduction in the thickness of the web and the flange. Alongside the analytical work, four corroded beams, 30 years old, were tested to evaluate the results of the analytical work. The analysis results showed that at uniform thickness losses equal to 50%, the remaining ultimate load capacity can be reduced to 30%. Besides the effect of corrosion on the carrying capacity, it can also change the classification of the element (e.g. class one to class two) and as a result of this it can change the mode of failure depending on the relative thickness loss in the various locations. In another approach, Zhang and Yuan [12] focused on the corrosion fatigue effects on life estimation of deteriorated bridges subjected to dynamic vehicle loads are evaluated based on a reliability-based approach.

On the other hand, some researchers focused on the section-level properties of the I-shaped beams. Tide [13] thoroughly studied the W-sections and investigated several reported incidents of brittle fracture of bridge girders in the k-area. Others classified I-sections based on the member's ductility based on the section's rotational capacity such as Shoukhouian [14]. Similarly, Gioncu et al. [15,16] focused on the European I-beam sections determining the available rotation capacity of wide-flange steel beams using the local plastic mechanism methodology. Most of the previous studies that were carried out on deteriorated steel beams simulated the deterioration by local notch at the middle-span such as Chen [17], Hmidan et al. [5], Ghafoori et al. [18], Zhou et al. [19], Colombi and Fava [20], Riccio et al. [21] and Deng et al. [22]. However, there are few studies that considered the effect of deterioration along the length of the beam. Amongst which, Galal et al. [23] investigated this effect experimentally where the results showed that the length of the deterioration has a major influence on the remaining flexural capacity of the deteriorated beams.

The current study combines several approaches from literature and extends them. Firstly, this study extends the authors' aforementioned experimental program (Galal et al. [23]) by conducting a parametric study to accurately investigate the effect of deterioration along the length for different beams. Also, on the section-level, the deteriorated W-section properties are thoroughly studied and several findings are presented. Moreover, the paper provides tools to simplify the analysis of laterally restrained beams that are locally deteriorated in their lower flange by developing simple yet practical nominal capacity design charts for engineers. Such nominal capacity design charts can be convenient in codes of practice (such as AASHTO [24] and AREMA [25]) using either the Load

and Resistance Factor (LRFD) or the Allowable Stress Design (ASD) design philosophies, accordingly.

2. Research significance, parameters investigated and limitations

The core of this study is divided into two parts: the first part focuses on sectional analysis of all AISC W-sections [26] to determine their sensitivity to deterioration, whereas the second part includes a parametric study to investigate the different deterioration factors along the beam length that can cause reduction in the flexural behavior. The findings are then used to create nominal design charts for evaluating the amount of deterioration in the flexural capacity of steel girders based on the loss in the area of the lower flange and its extent along the length of the element. It should be noted that the aforementioned findings are limited to the studied sections, support boundary conditions and load patterns. These charts can be adopted at some point in the roadway design standards (e.g. AASHTO [24]) or in the railway design manual (e.g. AREMA [25]) using the appropriate probability factors and factors of safety.

Based on the investigation of several researchers [10,11,23], the deterioration in girders is more common to happen in lower flanges rather than in the upper flanges. This is whether the deterioration is due to localized corrosion or due to direct lateral impact from traffic. Thus, the study will only cover cases of deterioration in the lower flange. Moreover, it is assumed for all beams in this study that the upper compression flange is continuously supported by the slabs. Thus, the effect of Lateral Torsional Buckling (LTB) is eliminated. This study can be further extended to cover other American or non-American I-beam cross-sections. Moreover, the effect of local buckling and lateral torsional buckling could also be investigated in future studies.

2.1. Section analysis

The aim of this part of the study is to understand the behavior of the commonly used I-beam sections. The AISC W-sections [26] are used in this study as a practical example that can be later extended to other standard sections and to determine their sensitivity to loss in their flange areas. This can simulate cases of deterioration such as localized corrosion or perforation of lower flanges as well as the damage of bridge girders being hit by vehicles on highways. In order to determine the range of deterioration in the inertia of all AISC W-sections due to the loss of a given percentage of the lower flange area, a parametric study is needed.

The AISC W-sections vary in dimensions, weights and heights to provide various axial and/or moment capacities based on the needs of the design engineer. It will be effective, from a design engineer point of view, if it is possible to correlate the sensitivity of different steel sections to the loss in their flange area. A quick exercise was made for all AISC W-sections where the moment of inertia of all sections about the major axis was calculated by reducing the width of the lower flange by 10%, 30%, 50% and 100% (i.e. case of

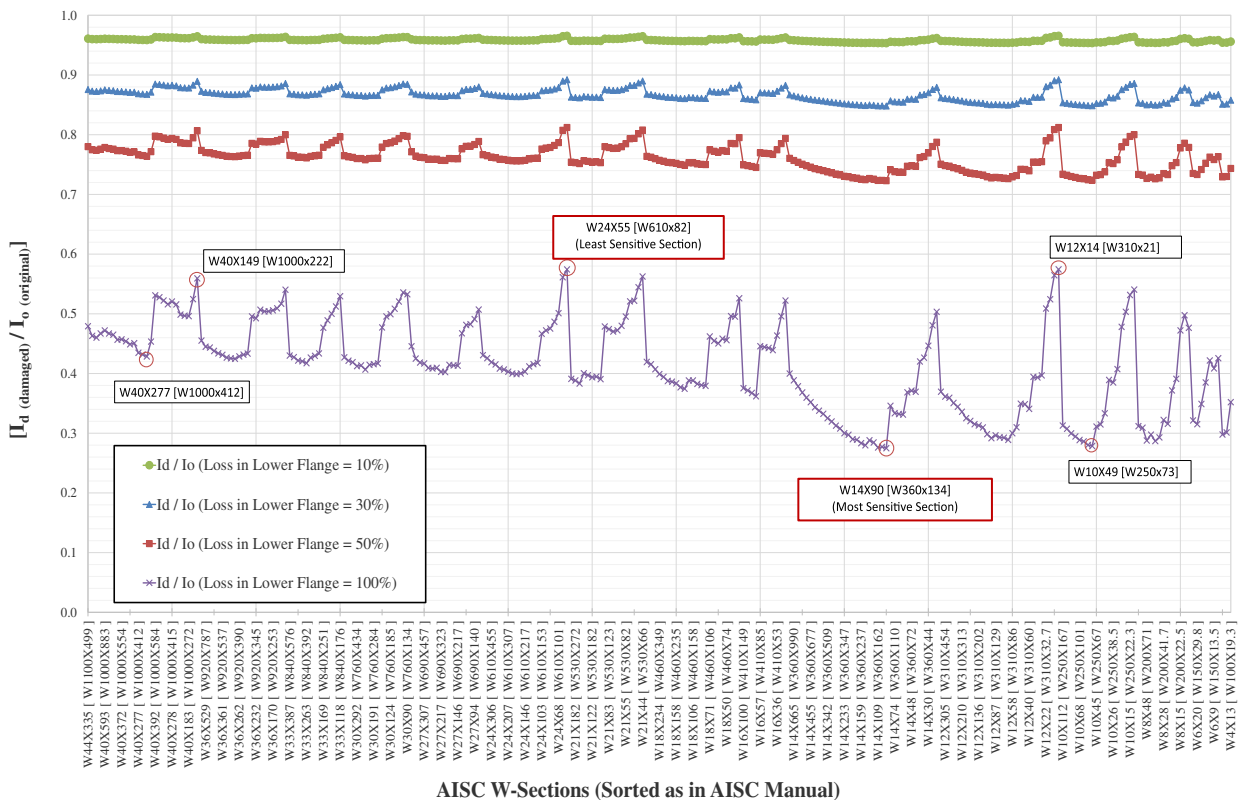


Fig. 1. Ratios of deteriorated to non-deteriorated moment of inertia of all AISC W-sections.

no flange representing the extreme case of complete loss due to corrosion or extreme damage of the bottom flange), respectively. This moment of inertia will be called the damaged inertia or “ I_d ” while the original undamaged moment of inertia will be called “ I_o ”. The ratio “ I_d/I_o ” is plotted for all sections in Fig. 1. A 10% loss in the lower flange roughly reduces all sections’ inertias to 96% of its original one. As the ratio of loss in lower flange increases, the difference in response of different sections becomes more significant.

The case of 100% loss in the lower flange area clearly shows the variation of the response of different AISC sections. The reduction in moment of inertia in this case was found to vary from 42.6% to 72.5%. This variation depends on the weight per unit length of the section and the section height. The maximum and minimum values of “ I_d/I_o ” for 100% flange deterioration for all sections are 0.57 and 0.27, respectively. A dimensionless parameter called the WSSR is introduced as an indicator of the sensitivity of the section to the loss of its moment of inertia due to loss of part/all of its tension flange area relative to other AISC W-sections and is given by Eq. (1). In other words, WSSR is a specific value for each AISC W-section that is calibrated to range from 0.0 to 1.0 corresponding to the minimum (i.e. 0.27) and maximum (i.e. 0.57) ratios of I_d/I_o (representing a 100% loss in the tension flange for all AISC W-section). A list of WSSR values for all AISC W-section is provided in Appendix A.

$$WSSR = \frac{(I_d/I_o)_{MAX} - (I_d/I_o)}{(I_d/I_o)_{MAX} - (I_d/I_o)_{MIN}} = \frac{0.57 - (I_d/I_o)}{0.57 - 0.27} = 3.33 \times (0.57 - (I_d/I_o)) \quad (1)$$

3. Experimental and numerical validation

3.1. Experimental tests

Before performing the parametric study, a validation with the experimental work conducted by the authors Galal et al. [23] is conducted. This section will go through the description of the experimental program. Four beams were tested by Galal et al. [23] to investigate the behavior of locally and uniformly deteriorated steel beams where the deterioration was represented as area loss in the tensile flange. Four beam IDs were assigned and are presented in Fig. 2. Out of the four beams, the BFO beam was designed to represent the non-deteriorated beam, identified as the control beam, whereas the other three steel beams were designed with two different levels of deterioration. Beams BF-H0.33 and BF-N0.33 have a 33% tension flange area reduction, while beam BF-N0.50 has a 50% area reduction. The letter “H” refers to beams with holes, local losses, while the letter “N” refers to notched beams. The reduction in area of the tension flange of beam BF-N0.33 was in form of two notches with a length of 450 mm located in the mid-span for simulating the effect of uniformly distributed deterioration, whereas the reduction in area of the tension flange of beam BF-H0.33 was concentrated in the mid-span to simulate the behavior of the beam with a locally deteriorated cross-sectional area.

3.2. Finite element model

In this study, a simple finite element (FE) model using frame elements was used to calibrate the numerical model with respect to the experimental test results described in the previous section. The ABAQUS 6.12 [27] standard solver was used for the analysis. The frame elements use a cubic formulation to take into account the shear deformations and provide more accurate load-deflection results. The beam elements representing the deteriorated section are offset such that the upper flange level will always be on the same level. The beam material has a yield stress of 300 MPa and the material model is idealized based on experimental dog-bone coupon tests. The load deflection curves for both the experimental and numerical simulation of the four beams are plotted in Fig. 3.

Despite the simplicity of the finite element model using frame elements, it is clear that there is a very good agreement between the behavior of the finite element model and the experimental test results. It can also be seen in Fig. 3 that there is a slight difference in the load-deformation behavior of the locally deteriorated beam and the control beam. For both beams BFO and BF-H0.33, almost the

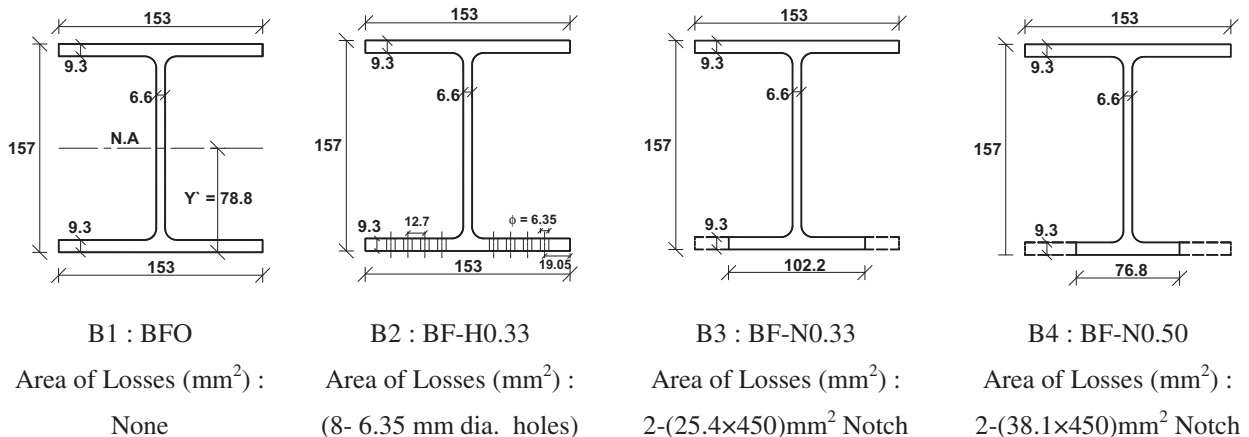


Fig. 2. Cross sectional areas of the four tested beams (Galal et al. [23]).

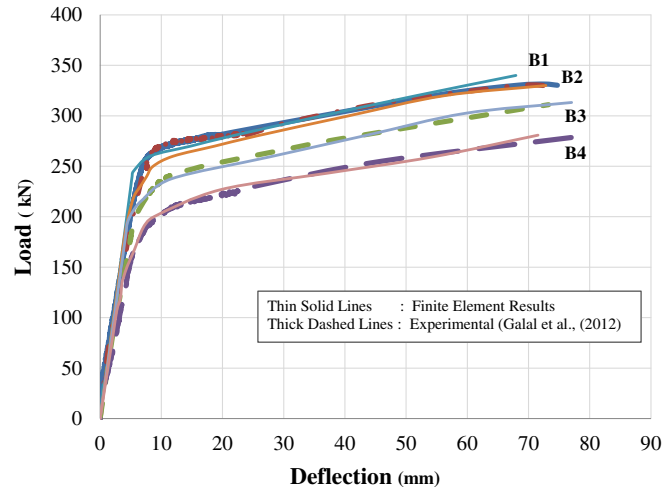


Fig. 3. Experimental versus finite element model load-deflection response at mid span of the four studied beams.

same yield and ultimate loads have been recorded in the test results at the same corresponding deflections. This shows that the corrosion localized at a single section of the beam has a local influence on the strains of the corroded deteriorated cross section only, whereas it has a negligible effect on the global load-deformation behavior of the whole beam. Although both beams BF-H0.33 and BF-N0.33 have the same loss in sectional area, but the difference in the deterioration length, L_d , had a significant impact on the load deflection behavior. Both beams BF-N0.33 and BF-N0.5 have the same deterioration length, L_d , but have different lower flange deterioration, which had a greater impact on the load deflection behavior. Moreover, in any beam with a uniform (unchanged) section, the yielding moment M_{yield} (at which first yielding occurs) can be directly derived from the elastic section modulus, S (which is a cross-sectional property). On the other hand, for beams with a deteriorated section in the middle, the behavior is different. For example, if we consider beams BF-H0.33 and BF-N0.33, they both have the same section modulus for the deteriorated section, but due to the spread of damage along a length, L_d , the load deflection behavior is different, hence, the equivalent yielding moment of the beam is different and needs to be determined. In the next section, these factors will be thoroughly investigated and quantified in a parametric study using the calibrated finite element model.

4. Numerical parametric study

4.1. Definitions and assumptions

The deflection of any simple beam with length “ L ” subjected to a certain loading pattern causing a moment “ M ”, having a cross-section with moments of inertia “ I ” and its elastic section modulus “ S ” and modulus of elasticity “ E ” can be expressed using Eq. (2). The first term in Eq. (2) represents the classical small-deformation theory deflection. The second term incorporates the effect of shear deformation, where α is the section's shape factor and ν is Poisson's ratio for the beam material. The yield stress of the material is σ_{yield} .

$$\Delta = \left(k_1 \frac{L^2}{I} + k_2 \frac{\alpha}{A} \right) \left(\frac{M}{E} \right) \quad (2)$$

where k_1 = Load-Flexural Deflection coefficient (based on the Load Pattern); $k_2 = 2(1 + \nu)$

Thus the deflection corresponding to the moment at which first yielding M_{yield} occurs will be called Δ_{yield} : and will be given by Eqs. (5) and (6).

$$\Delta_{yield} = \left(k_1 \frac{L^2}{I} + k_2 \frac{\alpha}{A} \right) \left(\frac{M_{yield}}{E} \right) \quad (3)$$

where $M_{yield} = S_{bottom} \sigma_{yield}$

$$M_{yield} = k_3 P_{yield} L \quad (4)$$

where k_3 = Moment-Load coefficient (based on load pattern)

Classification of cross-sections is a significant concept in the design of steel flexural members. One of the classification approaches is according to the member's ductility such as Gioncu and Mosoarca [15] and Shokouhian and Shi [14]. However, the current research does not focus on the ductility of members as it studies only the pre-yielding behavior which is adopted by many design standards. The yield point is one of the parameters that influence the member's ductility according to Gioncu et al. [16]. The goal of this research is to define an equivalent yielding moment capacity and deflection for other beams having different cross sections along

its length (for example: a beam with a deteriorated section in the middle).

Based on the assumptions of this study listed in Section 2, the compression flange of all beams is assumed to be laterally supported by the slab. Thus, the effect of Lateral Torsional Buckling (LTB) is eliminated from this study. Loss in flange area in this study is assumed to be in the tension flange; hence, the local buckling limits for the compression flange will not be significantly affected. This study is concerned with the overall behavior of the deteriorated beam, and does not cover effects of the change in the local web plate boundary condition that affects the local buckling critical stress.

In the next section, a parametric FE study will be conducted in order to plot the actual load deformation curves for different deterioration length ratios. A new definition for yielding load of beams with deteriorating cross-sections is proposed. As mentioned before, this study focuses on the AISC W-sections. Fig. 4 illustrates the parameters involved in the proposed deterioration scheme in I-beams considered in this study.

The deterioration of the steel beam section is simulated by removing a certain percentage of the lower flange area along a certain deterioration length, L_d , of the full beam length, L_o . Fig. 5 shows the load-deflection curves of both a non-deteriorated beam and a fully-deteriorated beam, where points A and B are the points corresponding to Eqs. (3) and (4) for both original and fully deteriorated beams. The yield moment for the original beam will be called “ M_{yo} ” and that of the fully deteriorated beam will be noted “ M_{yd} ”, and their corresponding deflections will be “ Δ_{yo} ” and “ Δ_{yd} ”, respectively.

In this paper, the locus of the point of equivalent yielding of any partially deteriorated beam will be assumed to be lying on the line AB. The intersection of the real moment-deflection curve with line AB is indicated by point C as shown in Fig. 5. The M_{yp} is the equivalent yield moment for the partially deteriorated beam and Δ_{yp} will be the corresponding deflection. In order to find this point C as a function in the deterioration length and level, a parametric study should be conducted to have a wide spectrum of beams and compare their behaviors and be able to predict the location of this point by simple equations. A new factor “ ψ ” (equal to the ratio of the distances AC/AB) is proposed to represent the beam deterioration level. This will be illustrated in the following sections.

4.2. Parametric study

The proposed dimensionless parameter $WSSR$ showed to be a good indicator of the sensitivity of the section to the loss of its moment of inertia due to loss of part of its flange area. The next step is to investigate the load-deflection behavior of such deteriorated beams with different lengths. In order to get the extreme cases depicted in Fig. 1, four beam sections were selected, two representing the category of large sections in the AISC W-sections (namely W40X277 [W1000x412] and W40X149 [W1000x222]) while the others two representing the category of small sections (namely W12X14 [W310x21] and W10X49 [W250x73]). Each pair of sections was chosen based on its $WSSR$ value, such that one would represent the highest $WSSR$ value in its category and the other would represent the least $WSSR$. In this parametric study, all beams are uniformly loaded and considered to be laterally supported. Practically short and long beam lengths are selected for each section. Table 1 lists the selected sections to be studied along with their $WSSR$ values and the studied beam lengths and values of L_d/L_o . The $WSSR$ provides a useful indicator to the engineer for the level of sensitivity of the AISC W-section to the loss of its bottom flange. This could be particularly useful for the engineer when choosing the most suitable

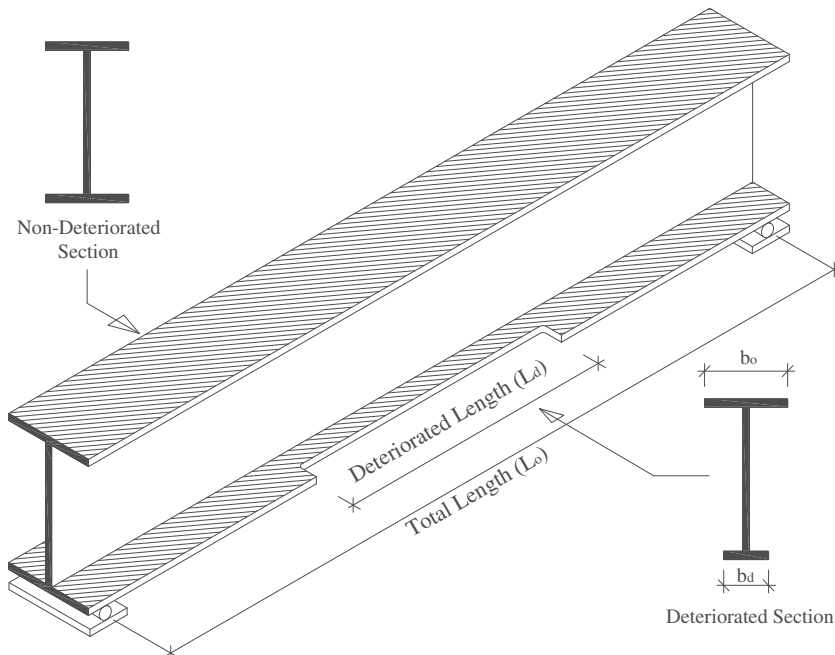


Fig. 4. Parameters and deterioration scheme in I-beams considered in this study.

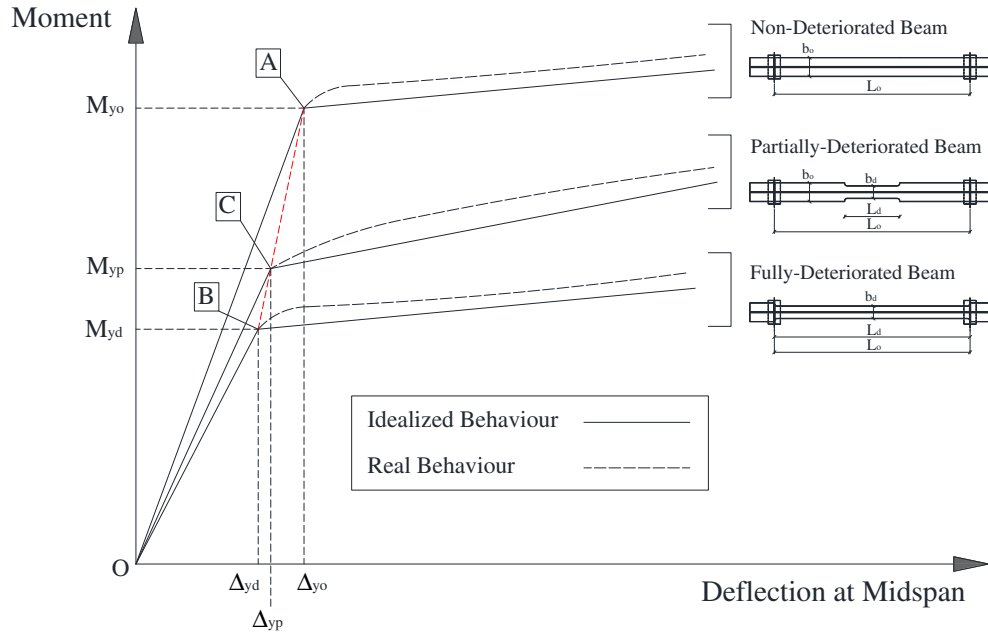


Fig. 5. Real versus Idealized load-deflection behavior of non-deteriorated, partially deteriorated and fully-deteriorated beams.

Table 1

The studied sections and their WSSR values.

Section	WSSR	Sensitivity	Studied lengths	Studied L_d/L_o
W40X277 [W1000x412]	47.0%	Average	6 m, 15 m	20%, 50%
W40X149 [W1000x222]	0.05%	Low	6 m, 15 m	20%, 50%
W10X49 [W250x73]	99.0%	Very high	3 m, 7 m	5%, 10%, 20%, 50%
W12X14 [W310x21]	0.0%	Very low	3 m, 7 m	20%, 50%

AISC W-section that satisfies the inertia and weight criteria. Moreover, the engineer should also consider choosing the section that has a lower sensitivity to loss in its capacity (i.e. a lower WSSR) in case of possible future deterioration in the beam's bottom flange.

For all studied beams, two deterioration length ratios L_d/L_o are selected, namely 20% and 50%. For the W10X49 [W250x73] beam, 5% and 10% were also studied. The load-deformation curves of all the studied beam combinations will be plotted in Fig. 6 (for the Low Sensitivity Sections) and plotted in Fig. 7 (for the High Sensitivity Sections). For uniformly loaded beams, Eq. (2) can be rewritten in Eq. (5) as:

$$\Delta_{yield} = \frac{S_{bottom} \sigma_{yield}}{E} \left(\frac{40}{384} \cdot \frac{L^2}{I} + 2.6 \frac{\alpha}{A} \right) \quad (\text{For case of uniform load}) \quad (5)$$

By inspecting Eq. (5), it only provides the deflection at yield initiation for beams with a uniform cross section. This means that Eq. (5) is more accurate for only the non-deteriorated and fully deteriorated beams. Whereas for the partially deteriorated beam, this equation will provide a conservative value of the yielding point as shown earlier in the experimental results. In this research, the value of the deflection at the onset of yielding for the partially deteriorated beam Δ_{yp} is found by the intersection of the line “AB” in Fig. 5 with their real load-deflection curves. These points of intersection are highlighted on the load-deflection curves presented in Figs. 6 and 7. The study was extended to cover different deterioration length ratios. It was found that when the deteriorated length passes roughly 60% of the span, the behavior is almost similar to a beam with deterioration along its full length.

4.3. Proposed equations

A computer script was generated to parametrically input the beam with variable cross sections to model the deteriorated section along a length, L_d , at the middle of the whole beam length, L_o , as shown in Fig. 4. All studied beams were assumed to be laterally supported, thus the effects of lateral torsional buckling are eliminated. Several W-sections from the AISC sections library [26] were used with different beam lengths. The points corresponding to the equivalent yielding moment for partially deteriorated beams “ M_{yp} ” and the corresponding deflection “ Δ_{yp} ” were plotted for all models as shown in Fig. 8. The curves in Fig. 8 represent the relation between the section deterioration ratio I_d/I_o and the beam deterioration level $\psi = AC / AB$ (shown in Fig. 5). The results plotted in

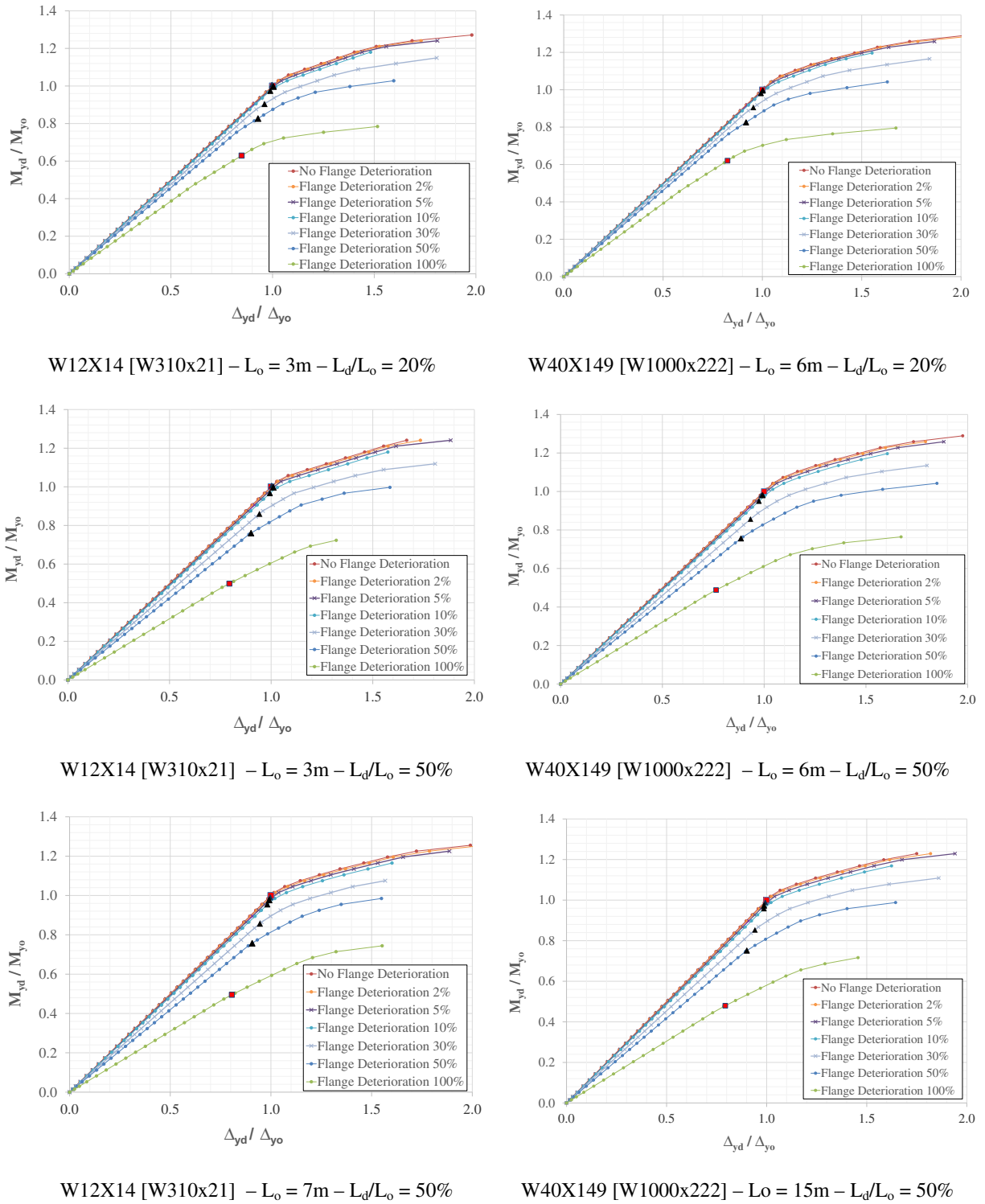


Fig. 6. Real versus Idealized load-deflection behavior of non-deteriorated, partially deteriorated and fully-deteriorated beams with low sensitivity sections.

Fig. 8 clearly show that the behavior depends on the sensitivity of the section, as each group of models having the same section group together based on their $WSSR$ values. Moreover, the effect of the deterioration length is clear in the results as it affects the bulge of each curve group. Large values of L_d/L_0 results in a stronger curve bulge while small values of L_d/L_0 results in, approximately, a straight line.

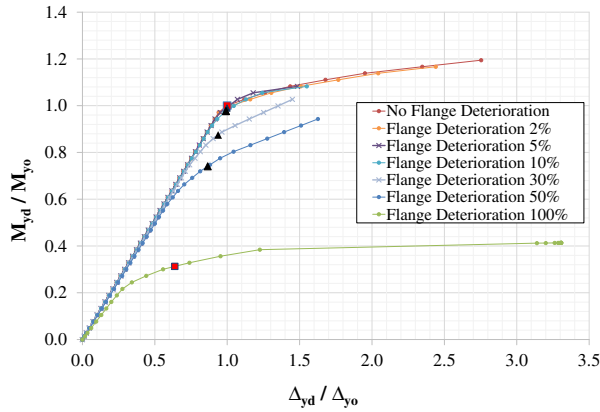
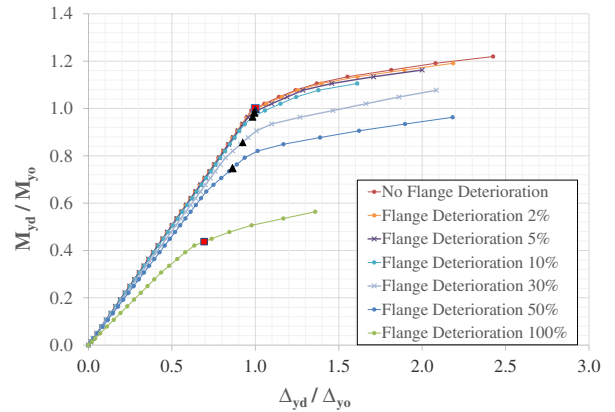
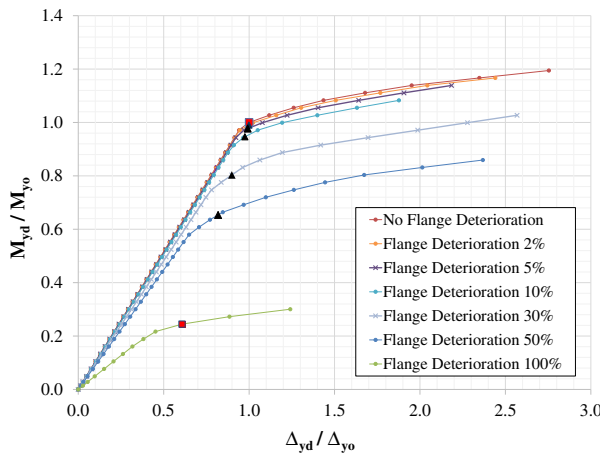
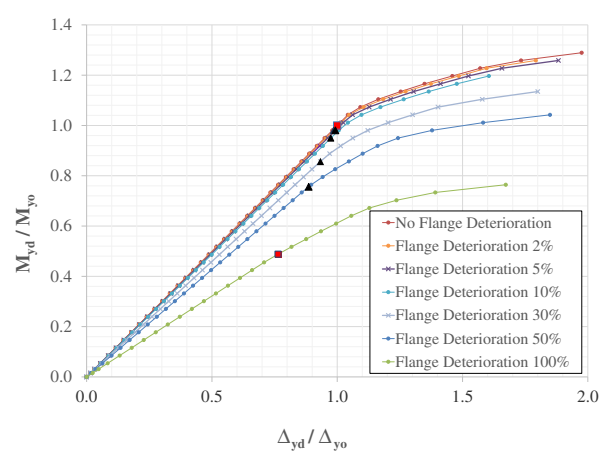
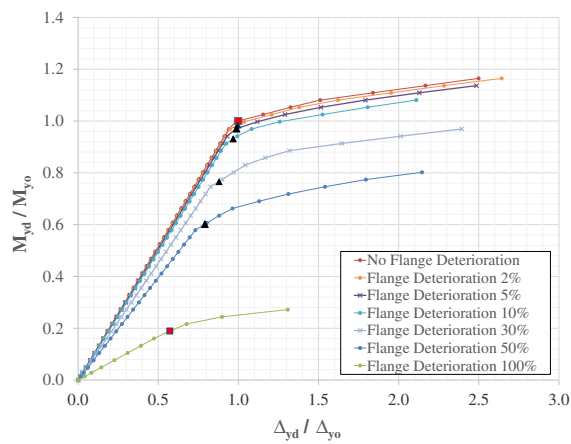
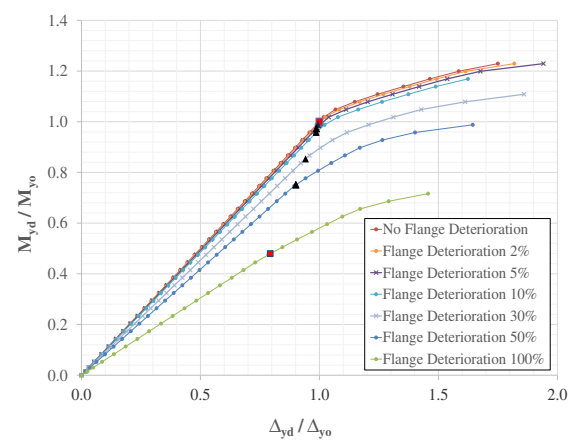
W10X49 [W250x73] – $L_0 = 3\text{m}$ – $L_d/L_0 = 20\%$ W40X277 [W1000x412] – $L_0 = 6\text{m}$ – $L_d/L_0 = 20\%$ W10X49 [W250x73] – $L_0 = 3\text{m}$ – $L_d/L_0 = 50\%$ W40X277 [W1000x412] – $L_0 = 6\text{m}$ – $L_d/L_0 = 50\%$ W10X49 [W250x73] – $L_0 = 7\text{m}$ – $L_d/L_0 = 50\%$ W40X277 [W1000x412] – $L_0 = 15\text{m}$ – $L_d/L_0 = 50\%$

Fig. 7. Real versus Idealized load-deflection behavior of non-deteriorated, partially deteriorated and fully-deteriorated beams with high sensitivity sections.

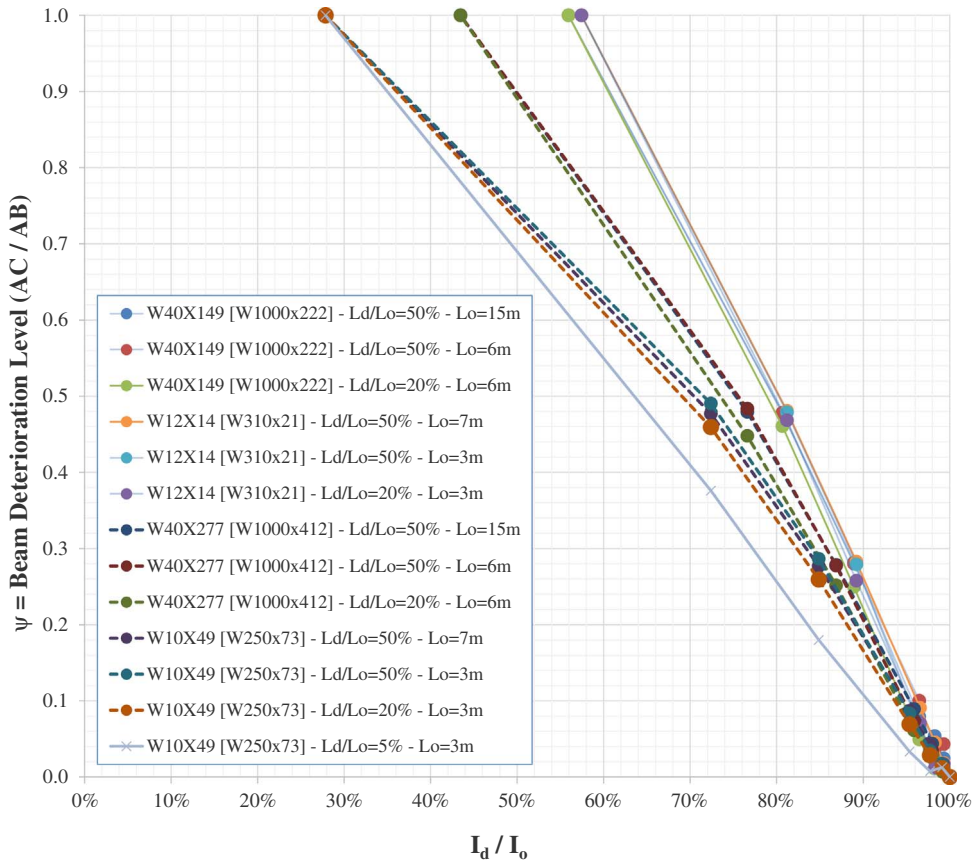


Fig. 8. Relationship between the moment of inertia deterioration ratio of the beam and the beam deterioration level ψ .

The values in Fig. 8 were traced and a new chart is generated in Fig. 9. The newly proposed nominal design chart is plotted to cover the full spectrum of sections with different WSSR values and different deterioration lengths “ L_d/L_o ”.

In order to be able to get the equivalent yielding point for any beam, the factor $\psi = AC / AB$ can be determined from the chart in Fig. 9. The beam deterioration level, ψ , represents the function of the rate of movement of point C on line AB. Therefore, it can easily be incorporated in a linear parametric Bezier equation as shown in Eqs. (6) and (7) to obtain the deteriorated beam's equivalent deflection and moment at yield initiation:

$$\Delta_{yp} = (1 - \psi) \cdot \Delta_{yo} + \psi \cdot \Delta_{yd} \quad (6)$$

$$\begin{aligned} M_{yp} &= (1 - \psi) \cdot M_{yo} + \psi \cdot M_{yd} \\ &= \sigma_{yield} ((1 - \psi) \cdot S_{bo} + \psi \cdot S_{bd}) \end{aligned} \quad (7)$$

The WSSR values (tabulated in Appendix A) are important in the proposed evaluation procedure. The engineer will enter the chart (Fig. 9) using the percentage of deterioration in inertia (I_d/I_o) and crosses the chart series with the specific value of WSSR (assigned to the beam under consideration) and deterioration length percentage (L_d/L_o) to obtain the Beam Deterioration Level (ψ). The value of ψ will be then used to determine the deflection, Δ_{yp} , at yield of the partially deteriorated beam, and its corresponding moment, M_{yp} , using Eqs. (6) and (7), respectively.

4.4. Worked example

A truck hit a railway steel multi-girder bridge passing over a highway underpass. The steel girders have a yield strength of 350 MPa. The railway bridge girder has a span of 10.0 m and its section is W36X487 [W920X725]. The lower flange was damaged along a length of 2.0 m in its mid-span, approximately 50% of the lower flange width is permanently deformed. It is required to determine the amount of reduction in the moment capacity and the increase in deflection in order to be able to take the decision to reduce the live loads until the strengthening of the bridge girder.

The procedure developed in this paper will be used. The girder section is W36X487 [W920X725] having a WSSR of 0.44, a form (shape) factor $\alpha = 2.8$, an area of $91,945 \text{ mm}^2$, a moment of inertia I_o of $14.83 \times 10^9 \text{ mm}^4$ and a bottom section modulus S_{bo} of $29.7 \times 10^6 \text{ mm}^3$. By calculating the damaged moment of inertia I_d due to the loss of 50% of the flange width, the moment of inertia I_d

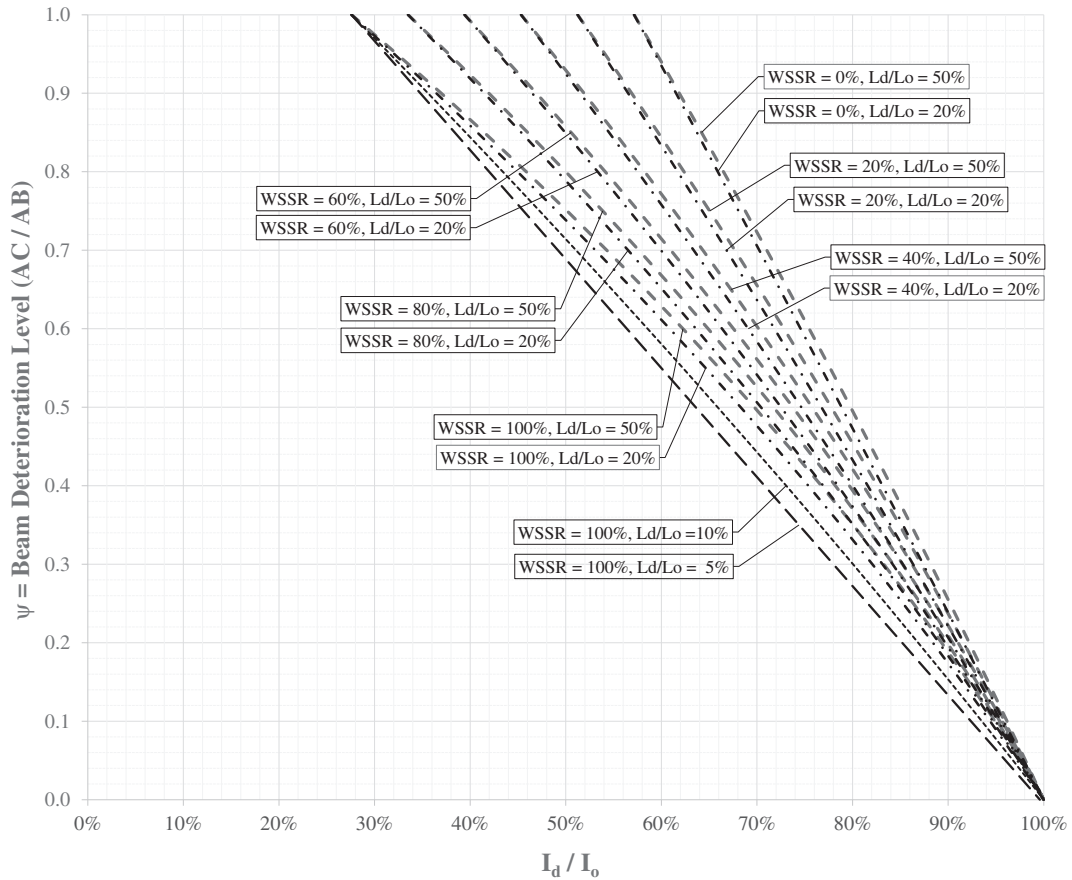


Fig. 9. Proposed nominal design charts relating the deterioration ratio of the moment of inertia of the beam to the beam deterioration level ψ .

becomes $11.41 \times 10^9 \text{ mm}^4$ and the bottom section modulus S_{bd} of $19.7 \times 10^6 \text{ mm}^3$, thus the estimated deterioration in inertia $I_d/I_o = 77\%$. The deteriorated length is 2.0 m in a span of 10.0 m, this gives a ratio of $L_d/L_o = 0.2$. Using these values in the newly proposed chart in Fig. 9, the deterioration level ψ is found to be $\psi = 0.45$. For simplicity, we can consider the loading pattern on the girder to be uniform especially that it is an edge girder. Now we can simply calculate the values Δ_{yo} and Δ_{yd} from the following equations:

$$\Delta_{yo} = \frac{S_{bo} \sigma_{yield}}{E} \left(\frac{40}{384} \cdot \frac{L^2}{I_o} + 2.6 \frac{\alpha}{A} \right) = 40 \text{ mm} \quad (8)$$

$$\Delta_{yd} = \frac{S_{bd} \sigma_{yield}}{E} \left(\frac{40}{384} \cdot \frac{L^2}{I_d} + 2.6 \frac{\alpha}{A_d} \right) = 34.2 \text{ mm} \quad (9)$$

Then we can calculate the equivalent deflection at yield onset of the partially deteriorated beam Δ_{yp} and the equivalent moment M_{yp} . The value of the moment at yield onset of the original beam $M_{yo} = 10,395 \text{ kN m}$.

$$\Delta_{yp} = (1 - \psi) \cdot \Delta_{yo} + \psi \cdot \Delta_{yd} = 37.4 \text{ mm} \quad (10)$$

$$M_{yp} = \sigma_{yield} ((1 - \psi) \cdot S_{bo} + \psi \cdot S_{bd}) = 8855 \text{ kN m} \quad (11)$$

As the hit bridge is a railway bridge, the AREMA [25] will be used, similarly the AASHTO [24] can be used in case of a roadway bridge. The Allowable Stress Design (ASD) method will be adopted for this example for simplicity. According to the AREMA [25], the working design stress $= 0.55 \sigma_{yield}$, therefore the original design moment should be $= 0.55 M_{yo} = 5717 \text{ kN m}$ which is less than the new equivalent moment $M_{yp} = 8855 \text{ kN m}$ and still in the elastic range as shown in Fig. 10. Now, the factor of safety becomes 1.5 instead of the original factor of 1.8. The remaining aspect to be checked is the increase in deflection. It is worth noting that the purpose of this example is to highlight the concept of the equivalent yield moment and to get the exact nominal capacity of partially deteriorated beams and is not specifically targeting a certain code of practice or a certain design philosophy as it can be easily adopted by codes in future by selecting appropriate factors of safety in case of using the ASD or appropriate loads and resistance factors in case of using the LRFD design philosophy.

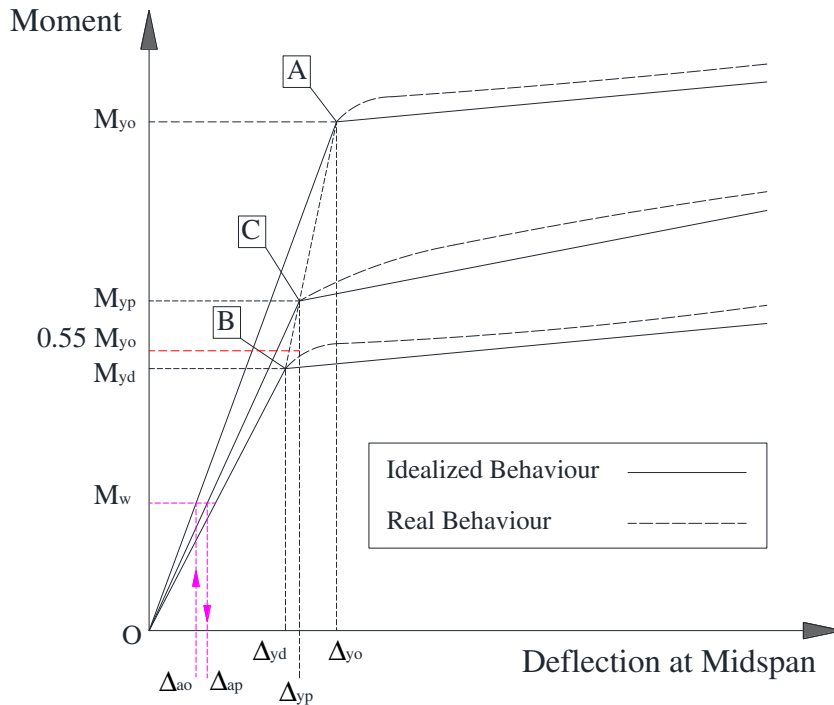


Fig. 10. Determining the allowable design moment and the increase in deflection in a deteriorated steel beam.

In order to determine the increase in the deflection under the same loading, the actual deflection of the non-deteriorated beam Δ_{ao} has to be obtained first, which depends on the real loading. From the moment-deflection diagram shown in Fig. 10, a working moment value corresponding to the actual deflection (at the intersection with line OA) can be obtained, it will be given a notation of M_w . The intersection of the horizontal line from M_w with the line OC gives the new deflection of the partially deteriorated beam Δ_{ap} , this deflection needs to be checked with the allowable deflection of the bridge girder under consideration.

From the above, it can be seen that the proposed method gives a time-effective procedure for engineers to check the capacity of deteriorated beams as well as the increase in their deflections. Usually, engineers estimate the moment capacity of partially deteriorated beams based on the weakest deteriorated section regardless of the deterioration level and length along the beam, which in many cases underestimates the overall capacity of the beam. The proposed method provides a more systematic procedure to estimate the equivalent yielding point of partially deteriorated beams, which helps in determining the moment capacity of the deteriorated beams.

5. Conclusions

In this study, a dimensionless section property called the *WSSR* is proposed, and it is shown to be a good indicator of the sensitivity of the AISC W-sections to the loss of its moment of inertia due to the loss of part of its flange area. This parameter can be extended in future to other types of standard I-Beam sections. This property was then used as an important parameter in a proposed design chart along with the deterioration in sectional inertia and the deterioration length to determine the moment capacity of partially deteriorated beams. Based on this study, the following conclusions are drawn:

- The amount of deterioration and its length along the beam have a significant effect on the remaining yielding moment capacity of the deteriorated steel girder. Once the deteriorated length passes roughly 60% of the span, the behavior is almost similar to a beam with uniform deterioration along its full length.
- Beam deterioration should always be presented in terms of loss in the moment of inertia (not by loss in area) in order to be able to normalize the representation of deterioration for any cross section.
- The proposed design chart provides a systematic procedure for engineers to check the yielding moment capacity of deteriorated beams as well as the decrease in their equivalent elastic moment of inertia. The generated charts are useful in assessment of many cases of partial deterioration such as cases of localized corrosion, perforation of webs and damaged lower flanges in I-beams.

It should be noted that the above findings are limited to the studied sections, support boundary conditions and loading patterns. The assumptions in this study eliminated the need to consider any lateral torsional buckling or local buckling effects.

Appendix A. Table of WSSR values for all AISC W-sections

Section	WSSR	Section	WSSR	Section	WSSR	Section	WSSR	Section	WSSR
W44X335 [W1100- X499]	0.32	W33X201 [W840X299]	0.47	W24X84 [W610X125]	0.33	W16X31 [W410X46.1]	0.26	W12X58 [W310X86]	0.92
W44X290 [W1100- X433]	0.37	W33X169 [W840X251]	0.33	W24X76 [W610X113]	0.29	W16X26 [W410X38.8]	0.17	W12X53 [W310X79]	0.88
W44X262 [W1100- X390]	0.38	W33X152 [W840X226]	0.29	W24X68 [W610X101]	0.24	W14X730 [W360X1086]	0.58	W12X50 [W310X74]	0.75
W44X230 [W1100- X343]	0.36	W33X141 [W840X210]	0.25	W24X62 [W610X92]	0.04	W14X665 [W360X990]	0.62	W12X45 [W310X67]	0.75
W40X593 [W1000- X883]	0.34	W33X130 [W840X193]	0.21	W24X55 [W610X82]	0.00	W14X605 [W360X900]	0.65	W12X40 [W310X60]	0.78
W40X503 [W1000- X748]	0.36	W33X118 [W840X176]	0.15	W21X201 [W530X300]	0.61	W14X550 [W360X818]	0.69	W12X35 [W310X52]	0.60
W40X431 [W1000- X642]	0.37	W30X391 [W760X582]	0.49	W21X182 [W530X272]	0.62	W14X500 [W360X744]	0.72	W12X30 [W310X44.5]	0.60
W40X397 [W1000- X591]	0.39	W30X357 [W760X531]	0.51	W21X166 [W530X248]	0.64	W14X455 [W360X677]	0.74	W12X26 [W310X38.7]	0.59
W40X372 [W1000- X554]	0.39	W30X326 [W760X484]	0.52	W21X147 [W530X219]	0.58	W14X426 [W360X634]	0.77	W12X22 [W310X32.7]	0.22
W40X362 [W1000- X539]	0.40	W30X292 [W760X434]	0.54	W21X132 [W530X196]	0.59	W14X398 [W360X592]	0.79	W12X19 [W310X28.3]	0.17
W40X324 [W1000- X483]	0.42	W30X261 [W760X389]	0.54	W21X122 [W530X182]	0.60	W14X370 [W360X551]	0.81	W12X16 [W310X23.8]	0.03
W40X297 [W1000- X443]	0.41	W30X235 [W760X350]	0.56	W21X111 [W530X165]	0.60	W14X342 [W360X509]	0.83	W12X14 [W310X21]	0.00
W40X277 [W1000- X412]	0.47	W30X211 [W760X314]	0.53	W21X101 [W530X150]	0.61	W14X311 [W360X463]	0.85	W10X112 [W250X167]	0.87
W40X249 [W1000- X371]	0.48	W30X191 [W760X284]	0.53	W21X93 [W530X138]	0.32	W14X283 [W360X421]	0.87	W10X100 [W250X149]	0.89
W40X215 [W1000- X321]	0.49	W30X173 [W760X257]	0.53	W21X83 [W530X123]	0.33	W14X257 [W360X382]	0.89	W10X88 [W250X131]	0.92
W40X199 [W1000- X296]	0.40	W30X148 [W760X220]	0.32	W21X73 [W530X109]	0.35	W14X233 [W360X347]	0.92	W10X77 [W250X115]	0.93
W40X392 [W1000- X584]	0.15	W30X132 [W760X196]	0.26	W21X68 [W530X101]	0.34	W14X211 [W360X314]	0.93	W10X68 [W250X101]	0.95
W40X331 [W1000- X494]	0.16	W30X124 [W760X185]	0.25	W21X62 [W530X92]	0.32	W14X193 [W360X287]	0.95	W10X60 [W250X89]	0.96
W40X327	0.17	W30X116 [W760X173]	0.22	W21X55 [W530X82]	0.26	W14X176 [W360X262]	0.95	W10X54 [W250X80]	0.98

[W1000-X486]									
W40X294	0.20	W30X108	0.18	W21X48	0.18	W14X159	0.97	W10X49	0.99
[W1000-X438]		[W760X161]		[W530X72]		[W360X237]		[W250X73]	
W40X278	0.18	W30X99	0.13	W21X57	0.18	W14X145	0.98	W10X45	0.88
[W1000-X415]		[W760X147]		[W530X85]		[W360X216]		[W250X67]	
W40X264	0.20	W30X90	0.14	W21X50	0.10	W14X132	0.96	W10X39	0.87
[W1000-X393]		[W760X134]		[W530X74]		[W360X196]		[W250X58]	
W40X235	0.25	W27X539	0.43	W21X44	0.04	W14X120	0.97	W10X33	0.80
[W1000-X350]		[W690X802]		[W530X66]		[W360X179]		[W250X49.1]	
W40X211	0.26	W27X368	0.50	W18X311	0.52	W14X109	1.00	W10X30	0.62
[W1000-X314]		[W690X548]		[W460X464]		[W360X162]		[W250X44.8]	
W40X183	0.26	W27X336	0.52	W18X283	0.53	W14X99	0.99	W10X26	0.63
[W1000-X272]		[W690X500]		[W460X421]		[W360X147]		[W250X38.5]	
W40X167	0.17	W27X307	0.53	W18X258	0.56	W14X90	1.00	W10X22	0.56
[W1000-X249]		[W690X457]		[W460X384]		[W360X134]		[W250X32.7]	
W40X149	0.05	W27X281	0.55	W18X234	0.59	W14X82	0.76	W10X19	0.32
[W1000-X222]		[W690X419]		[W460X349]		[W360X122]		[W250X28.4]	
W36X652	0.40	W27X258	0.55	W18X211	0.60	W14X74	0.80	W10X17	0.24
[W920X970]		[W690X384]		[W460X315]		[W360X110]		[W250X25.3]	
W36X529	0.43	W27X235	0.55	W18X192	0.62	W14X68	0.81	W10X15	0.14
[W920X787]		[W690X350]		[W460X286]		[W360X101]		[W250X22.3]	
W36X487	0.44	W27X217	0.57	W18X175	0.63	W14X61	0.81	W10X12	0.11
[W920X725]		[W690X323]		[W460X260]		[W360X91]		[W250X17.9]	
W36X441	0.46	W27X194	0.57	W18X158	0.64	W14X53	0.69	W8X67	0.88
[W920X656]		[W690X289]		[W460X235]		[W360X79]		[W200X100]	
W36X395	0.47	W27X178	0.53	W18X143	0.66	W14X48	0.68	W8X58	0.89
[W920X588]		[W690X265]		[W460X213]		[W360X72]		[W200X86]	
W36X361	0.48	W27X161	0.54	W18X130	0.67	W14X43	0.68	W8X48	0.96
[W920X537]		[W690X240]		[W460X193]		[W360X64]		[W200X71]	
W36X330	0.49	W27X146	0.54	W18X119	0.62	W14X38	0.52	W8X40	0.92
[W920X491]		[W690X217]		[W460X177]		[W360X57.8]		[W200X59]	
W36X302	0.50	W27X129	0.36	W18X106	0.62	W14X34	0.49	W8X35	0.96
[W920X449]		[W690X192]		[W460X158]		[W360X51]		[W200X52]	
W36X282	0.50	W27X114	0.31	W18X97	0.64	W14X30	0.43	W8X31	0.94
[W920X420]		[W690X170]		[W460X144]		[W360X44]		[W200X46.1]	
W36X262	0.49	W27X102	0.31	W18X86	0.65	W14X26	0.31	W8X28	0.84
[W920X390]		[W690X152]		[W460X128]		[W360X39]		[W200X41.7]	
W36X247	0.48	W27X94	0.28	W18X76	0.65	W14X22	0.24	W8X24	0.86
[W920X368]		[W690X140]		[W460X113]		[W360X32.9]		[W200X35.9]	
W36X231	0.47	W27X84	0.22	W18X71	0.38	W12X336	0.68	W8X21	0.68
[W920X344]		[W690X125]		[W460X106]		[W310X500]		[W200X31.3]	
W36X256	0.26	W24X370	0.48	W18X65	0.40	W12X305	0.71	W8X18	0.61
[W920X381]		[W610X551]		[W460X97]		[W310X454]		[W200X26.6]	
W36X232	0.27	W24X335	0.5	W18X60	0.41	W12X279	0.72	W8X15	0.34
[W920X345]		[W610X498]		[W460X89]		[W310X415]		[W200X22.5]	
W36X210	0.23	W24X306	0.52	W18X55	0.39	W12X252	0.75	W8X13	0.26
[W920X313]		[W610X455]		[W460X82]		[W310X375]		[W200X19.3]	
W36X194	0.23	W24X279	0.53	W18X50	0.40	W12X230	0.77	W8X10	0.33
[W920X289]		[W610X415]		[W460X74]		[W310X342]		[W200X15]	
W36X182	0.23	W24X250	0.55	W18X46	0.26	W12X210	0.80	W6X25	0.85
[W920X271]		[W610X372]		[W460X68]		[W310X313]		[W150X37.1]	
W36X170	0.23	W24X229	0.56	W18X40	0.26	W12X190	0.83	W6X20	0.87

[W920X253]		[W610X341]		[W460X60]		[W310X283]		[W150X29.8]	
W36X160	0.22	W24X207	0.57	W18X35	0.16	W12X170	0.85	W6X15	0.75
[W920X238]		[W610X307]		[W460X52]		[W310X253]		[W150X22.5]	
W36X150	0.19	W24X192	0.58	W16X100	0.66	W12X152	0.87	W6X16	0.63
[W920X223]		[W610X285]		[W410X149]		[W310X226]		[W150X24]	
W36X135	0.11	W24X176	0.59	W16X89	0.68	W12X136	0.87	W6X12	0.51
[W920X201]		[W610X262]		[W410X132]		[W310X202]		[W150X18]	
W33X387	0.48	W24X162	0.58	W16X77	0.69	W12X120	0.88	W6X9	0.55
[W840X576]		[W610X241]		[W410X114]		[W310X179]		[W150X13.5]	
W33X354	0.49	W24X146	0.57	W16X67	0.71	W12X106	0.92	W6X8.5	0.50
[W840X527]		[W610X217]		[W410X100]		[W310X158]		[W150X13]	
W33X318	0.51	W24X131	0.54	W16X57	0.43	W12X96	0.94	W5X19	0.92
[W840X473]		[W610X195]		[W410X85]		[W310X143]		[W130X28.1]	
W33X291	0.51	W24X117	0.53	W16X50	0.43	W12X87	0.93	W5X16	0.91
[W840X433]		[W610X174]		[W410X75]		[W310X129]		[W130X23.8]	
W33X263	0.52	W24X104	0.52	W16X45	0.44	W12X79	0.94	W4X13	0.74
[W840X392]		[W610X155]		[W410X67]		[W310X117]		[W100X19.3]	
W33X241	0.49	W24X103	0.36	W16X40	0.45	W12X72	0.94		
[W840X359]		[W610X153]		[W410X60]		[W310X107]			
W33X221	0.48	W24X94	0.34	W16X36	0.37	W12X65	0.95		
[W840X329]		[W610X140]		[W410X53]		[W310X97]			

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