

INCREMENTAL MODIFIED PUSHOVER ANALYSIS

HOSSEIN AZIMI, KHALED GALAL* AND OSKAR A. PEKAU

Department of Building, Civil and Environmental Engineering, Concordia University, Montréal, Québec, Canada

SUMMARY

The recently developed Incremental Dynamic Analysis (IDA) requires nonlinear time history analyses at different levels of intensity of an ensemble of ground motions, which is time-consuming due to the high computational efforts involved. In the current study, a simplified method named as Incremental Modified Pushover (IMP) is developed and evaluated. In this method, the response of the structure is obtained using one pushover analysis at any specified level of ground motion intensity. The associated higher mode effects are explicitly considered when determining target roof displacement and lateral load pattern. In the bilinear idealization of the pushover curves, a new approach has been used. Moment resisting steel frames with 4, 8, 12 and 16 stories, as well as their corresponding soft-story models are used for verifying the proposed method. The studied frames were subjected to seventeen different scaled earthquake ground motions. The results of the presented method, IMP, are verified in terms of maximum roof displacement, maximum inter-story drift and maximum plastic hinge rotation at different ground motion intensities. The results show that the IMP method gives higher response values compared with IDA, which can be viewed as being more conservative. Copyright © 2008 John Wiley & Sons, Ltd.

1. INTRODUCTION

The estimation of seismic demands and capacity of structures during severe earthquakes is important in performance based seismic design. However, few researchers have done valuable works to estimate the response of a structure when subjected to different levels of earthquake intensities (Vamvatsikos and Cornell, 2002). Having the structure's demand–capacity relationship facilitates the estimation of the probability of occurrence of different limit states during an earthquake event. However, because of intrinsic uncertainty in earthquake ground motions, the determination of the level of reliance has been always a challenge.

One of the most noticeable methods for estimation of probability of different limit states is Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002). IDA is a useful nonlinear dynamic technique that estimates the response of the structure in different levels of intensities of a specified earthquake record from linear behaviour to the complete instability of the building. Therefore, using this method, the index of damage measure versus intensity measure can easily be established. To satisfy the seismic performance of the structure, one can implement any reasonable upper-bound for damage and/or intensity measure that has direct influence on the final collapse of the structure. In the IDA method, a series of nonlinear time history analyses based on different scaled earthquake records are performed to obtain the damage–capacity curve. On each curve, the *X*-coordinate represents a damage measure and the *Y*-coordinate is the intensity measure. Commonly used damage measure is inter-story drift, although other parameters like roof displacement, plastic hinge rotation and damage

* Correspondence to: Khaled Galal, Department of Building, Civil and Environmental Engineering, Concordia University, 1515 St Catherine W. Montréal, Québec, Canada H3G 2W1. E-mail: galal@bcee.concordia.ca

index (Park and Ang, 1985a, 1985b) could be used. This curve clearly guides the designer towards better understanding of the expected performance of a structure when subjected to different levels of intensity of ground motions.

On the other hand, the backbone of the IDA procedure is the non-linear dynamic analysis, which is computationally expensive and time consuming. Consequently, there was a need for a simpler method compared with IDA, which can be used in the seismic performance-based design. Recently, modal pushover analysis (MPA) (Chopra and Goel, 2002) has been developed to estimate structural demands based on nonlinear static analysis considering higher mode effects. The MPA method was applied to several structural systems and was shown that it gives reasonable response prediction (Chintanapakdee and Chopra, 2003, 2004; Chopra and Goel, 2004). Using MPA instead of nonlinear dynamic analysis (in IDA procedure) in order to estimate demand in each level of intensity leads to a simpler and faster method called Modal Incremental Dynamic Analysis (MIDA) (Mofid *et al.*, 2005).

In the current paper, a faster method compared with MIDA, named as Incremental Modified Pushover (IMP), is developed and evaluated. In the MIDA method, which is based on modal pushover analysis, one needs to determine at least two equivalent SDOF systems. The number of such equivalent SDOF would increase up to five or more with the increase in the height of the building. Using a lateral load pattern which represent the effect of higher modes is expected to serve that objective. As a result, a lateral load pattern was applied to consider the second mode effect by recognizing that the effect of the third and higher modes can be neglected in regular buildings (Jan *et al.*, 2004).

In addition, a new method of idealization of the pushover curve is considered in this study. This method is expected to result in a better representation of the real pushover curve compared with the conventional method proposed by Applied Technology Council 2001 (ATC-40 report), with respect to the amount of work done by the system.

2. STRUCTURAL SYSTEMS AND GROUND MOTIONS

In this investigation, four moment-resisting steel frames 4, 8, 12 and 16 stories were considered. The bay width and story height are 5 and 3 m, respectively. The height-wise distribution of stiffness is defined to achieve equal inter-story drifts in all stories under the lateral forces specified in the Uniform Building Code 1997 (UBC-97) code except for the first story. The design base shear provided by that code was applied for designing the studied frames. The frames are designed according to the strong column–weak beam philosophy; therefore, plastic hinges form only at beam ends and at the base of the first-story columns. Columns in other stories are assumed to remain elastic. Design has been done using standard W-sections, and final design summary is presented in Table 1.

In addition to the studied regular moment resisting frames, frames with soft first story have also been investigated. The stiffness of the first story was modified by changing the stiffness of the columns in that story and the beam they support. The stiffness was changed with a local modification factor equal to 0.2. To ensure reasonable comparison of seismic demand on regular and soft frames, their fundamental vibration period, yield base shear and damping properties were kept the same. To maintain the same period as for the ‘regular’ frame, all story stiffness were scaled uniformly by global modification factor. All story strengths were scaled uniformly to obtain the same yield base shear as the regular frame. Table 2 shows the dynamic characteristics of regular frame models and soft frame models for the four studied buildings. The software used for all analyses in the present paper is Drain 2DX (Allahabadi and Powell 1988).

Seventeen ground motion records from the California region were used in this study. All records are from strong earthquakes with magnitudes bigger than six and Peak Ground Acceleration (PGA) more than 0.1 g. The ground motions could be classified as far-field as the closest distance to fault rupture is almost 15 km or longer. The ratio of PGA over Peak Ground Velocity (PGV) which refers as A/V ratio,

Table 1. Member details for the four studied frames

	Inner columns	Outer columns	Beams		Inner columns	Outer columns	Beams
Four story				Eight story			
4	W16X40	W16X40	W14X30	8	W21X44	W21X44	W14X30
3	W21X44	W16X40	W14X30	7	W21X44	W21X44	W18X40
2	W21X55	W21X55	W14X43	6	W24X94	W21X55	W18X40
1	W21X55	W21X55	W14X30	5	W24X94	W21X55	W18X40
16 story				4	W24X94	W21X55	W21X50
16	W21X44	W16X40	W14X30	3	W27X94	W24X94	W21X50
15	W21X55	W21X55	W14X43	2	W27X94	W24X94	W21X50
14	W21X83	W21X55	W18X50	1	W27X94	W27X94	W18X40
13	W21X83	W21X83	W21X50	12 story			
12	W24X94	W21X83	W21X57	12	W21X44	W21X44	W14X30
11	W27X94	W24X94	W21X57	11	W21X55	W21X44	W16X40
10	W27X94	W24X94	W21X68	10	W21X55	W21X55	W18X40
9	W27X94	W27X94	W21X68	9	W24X94	W21X55	W21X50
8	W27X94	W27X94	W21X68	8	W24X94	W24X94	W21X50
7	W30X116	W27X94	W24X68	7	W27X94	W24X94	W21X50
6	W30X116	W27X94	W24X68	6	W27X94	W24X94	W21X68
5	W30X132	W30X116	W21X68	5	W27X94	W27X94	W21X68
4	W30X147	W30X132	W21X68	4	W30X116	W27X94	W21X68
3	W30X211	W30X147	W21X68	3	W30X132	W30X116	W21X68
2	W30X211	W30X147	W21X50	2	W30X132	W30X116	W21X50
1	W30X211	W30X211	W14X30	1	W30X132	W30X132	W16X40

Table 2. Dynamic characteristics of the frames

Frame	Regular model		Soft model		
	T1 (sec)	T2 (sec)	GMF	T1 (sec)	T2 (sec)
Four story	0.7814	0.2456	2.020	0.7817	0.2153
Eight story	1.0840	0.3816	1.480	1.0828	0.3667
12 story	1.4315	0.5255	1.340	1.4309	0.5121
16 story	1.8067	0.6493	1.260	1.8076	0.6419

GMF = Global Modification Factor.

as a representative of frequency content, is around 1 for most of selected records, so these records can be classified as intermediate A/V ratio. The characteristics of the 17 records are shown in Table 3.

There are several available methods proposed by different researchers for the estimation of the duration of strong motion records. Each method is suitable for one purpose (Naeim, 2001). Bracketed duration method, proposed by Bolt (1973), is the time interval between the first and the last acceleration greater than 0.05 g. Trifunac and Brady (1975) defined the duration of strong motion as the time interval between the 5% and the 95% contribution to the integral of the square of acceleration. McCann and Shah (1979) considered cumulative root mean square acceleration (rms)² of the accelerogram as the governing parameter. Two cut-off points are determined such that the rate of change of the cumulative rms is changed.

3. IDEALIZING THE PUSHOVER CURVE

Figure 1 shows a regular bilinear idealization of the pushover curve as proposed by ATC-40. In this method, the initial stiffness slope is taken as the elastic stiffness of the structure, while the determina-

Table 3. Ground motion properties

No.	Earthquake	Abbreviation	Date	Closest distance to fault rupture (km)	PGA (g)	M	A/V ratio (g/m/s)	Strong motion duration (sec)		
								McCann	Trifunac	Bolt
1	Cape Mendocino	CM	04/25/92	18.5	0.385	7.1	0.88	9.4	15.4	16.8
2	Coalinga	CO	05/02/83	25.5	0.227	6.4	0.96	13.6	12.4	12.6
3	Chalfant Valley	CV	07/21/86	18.7	0.447	6.2	1.21	6.3	6.2	16.0
4	Imperial Valley	EL4	05/18/40	12.3	0.341	7.1	1.02	14.5	24.4	29.2
5	Imperial Valley	EL	10/15/79	23.6	0.309	6.5	0.85	15.4	8.9	9.9
6	Kern County	KC	07/21/52	41.0	0.178	7.4	1.02	15.5	28.8	15.6
7	Landers	LD	06/28/92	23.2	0.171	7.3	0.85	35.5	31.8	34.2
8	Loma Prieta	LP	10/18/89	14.4	0.367	6.9	0.82	8.0	11.4	16.6
9	San Fernando	SF	02/09/71	21.2	0.174	6.6	1.17	7.6	11.2	8.6
10	Imperial Valley	IV2	10/15/79	28.7	0.254	6.5	0.84	19.5	22.1	29.7
11	Imperial Valley	IV3	10/15/79	32.6	0.186	6.5	1.34	16.9	21.7	22.0
12	Loma Prieta	LP2	10/18/89	28.2	0.159	6.9	0.90	20.1	21.4	12.5
13	Loma Prieta	LP3	10/18/89	21.4	0.240	6.9	1.30	11.4	10.9	11.5
14	Loma Prieta	LP4	10/18/89	25.8	0.269	6.9	0.61	14.5	12.4	13.8
15	N-Palm	NP	07/08/86	39.6	0.117	6.0	0.95	18.6	13.4	8.2
16	Superstition Hills (A)	SH1	11/24/87	24.7	0.134	6.3	1.00	14.3	15.1	8.8
17	Superstition Hills (B)	SH2	11/24/87	24.4	0.207	6.7	0.60	36.1	28.8	32.5

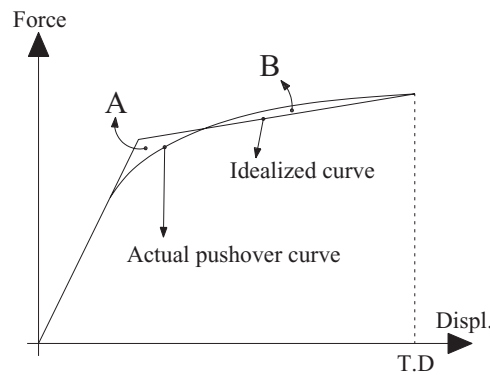


Figure 1. Actual and conventional idealized pushover curves (ATC-40, 1996)

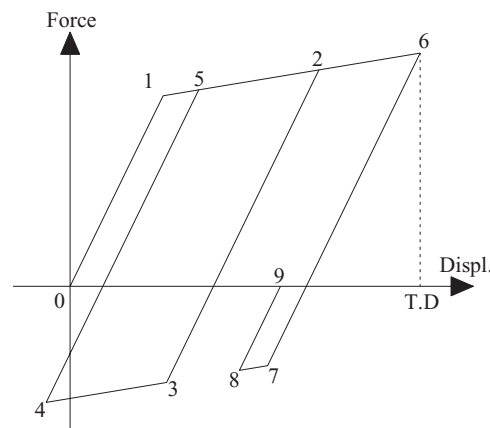


Figure 2. Sample hysteresis loops using idealized curves

tion of the yield displacement and the yield stiffness slope are defined such that areas A and B are equal for a given target roof displacement, T.D. The target roof displacement is estimated using formulas given by ATC-40. By equating the two areas, the actual and the idealized pushover curves would have the same work done (Lawson *et al.*, 1994).

The work done by the system using the idealized curve will be equal to that of using the actual pushover curve only when the roof displacement is equal to the target displacement. In other cycles where the roof displacement is less than the target roof displacement, the work done using the idealized curve will not be equal to that using the actual pushover curve (Figure 2). This shortcoming will be more noticeable in long-duration earthquakes, where there are more cycles. To reduce this limitation, a new procedure for idealizing the pushover curve is suggested.

The criterion for the new procedure to obtain the idealized curve is shown graphically in Figure 3. In this procedure, the yield stiffness slope is determined such that two conditions are satisfied: first, area B equals to the summation of areas A and C, and second, summation of these three areas takes the minimum value among relevant values for different yield stiffness slopes. From the geometry of the curves, by varying the yield stiffness line vertically without any change in the line slope, there is only one location in which $B = A + C$. For this slope, the summation of the three areas is calculated.

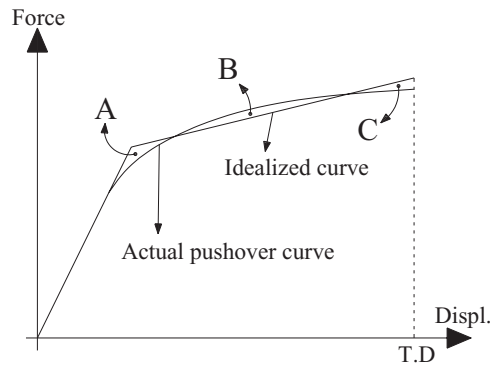


Figure 3. Proposed procedure for Idealization

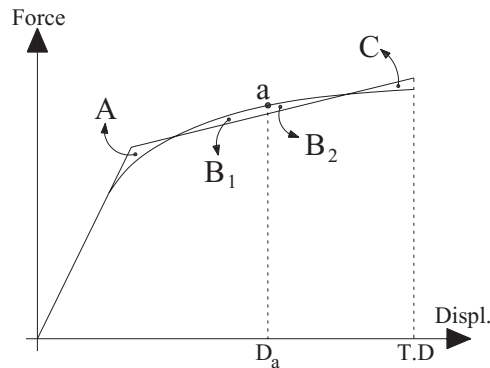


Figure 4. Advantages of the new procedure

This procedure will be repeated for different slopes until it reaches the minimum value for the summation. The slope, which produces the minimum summation, is the desired slope of the yield stiffness line.

This new approach has two advantages. First, the new idealized two-line curve is in the nearest possible location from the actual curve for different displacement values. The second benefit is the fact that in addition to having the condition of equality of work done at the end point of the curve, or T.D., there is another point, D_a (see Figure 4), in which the same condition is satisfied. This means that in the point represented as 'a', the area A is equal to B_1 . Therefore, in the hysteresis loops where the maximum displacement is equal to D_a , there will be no error in the amount of work (compared with the actual pushover curve). Satisfying the condition of equality of work done at two points along the curve is important due to the fact that during earthquakes, it is likely that the structure could experience a hysteretic response that does not necessarily reach the maximum displacement level, T.D. Therefore, with this new approach, the sum of the whole work done by the SDOF has a better approximation than the conventional procedures.

Although changing the yield stiffness slope in the proposed idealization will lead to varying the yield displacement point, this will have insignificant effect in calculation of target roof displacement. This fact is completely reasonable, because for actual pushover curves, especially for code-based designed models, alteration in second-line slope is rather small. On the other hand, for curves with positive second-line slope, applying this procedure always results in greater

second line slope which may not be desirable for structures experiencing high level of non-linearity.

This new procedure is demonstrated only for curves with positive slope of yield stiffness line, but can be used by the same algorithm for curves with negative slope.

4. EVALUATION OF THE PROPOSED IDEALIZATION OF THE PUSHOVER CURVE

In order to assess the proposed idealization procedure, eight frames have been studied, four regular frames and four frames with a high degree of softness in the first story. Figure 5 shows the differences between the pushover curves for the soft and regular models of the analyzed frames. These curves have been achieved neglecting P-delta effects.

Each curve has been idealized twice, first using the conventional method presented in the ATC-40, and second by using the proposed method. Changes in the main parameters of the two-line force-deformation relationship such as strain-hardening ratio and yield strength have been evaluated and compared for the two methods of idealization as shown in Table 4.

In nonlinear static methods, pushover curve is computed and idealized in order to be used as force-displacement relationship of an equivalent SDOF system. This SDOF system is mainly useful for calculation of target roof displacement. Therefore, the effect of the proposed idealization procedure is verified for this parameter only.

For demonstration purposes, the accuracy of the proposed method in estimation of the roof displacement, the 8- and 12-story regular frames and corresponding soft models are shown in Tables 5 and 6, respectively. Time history analysis was performed on the studied frames for all ground motions and

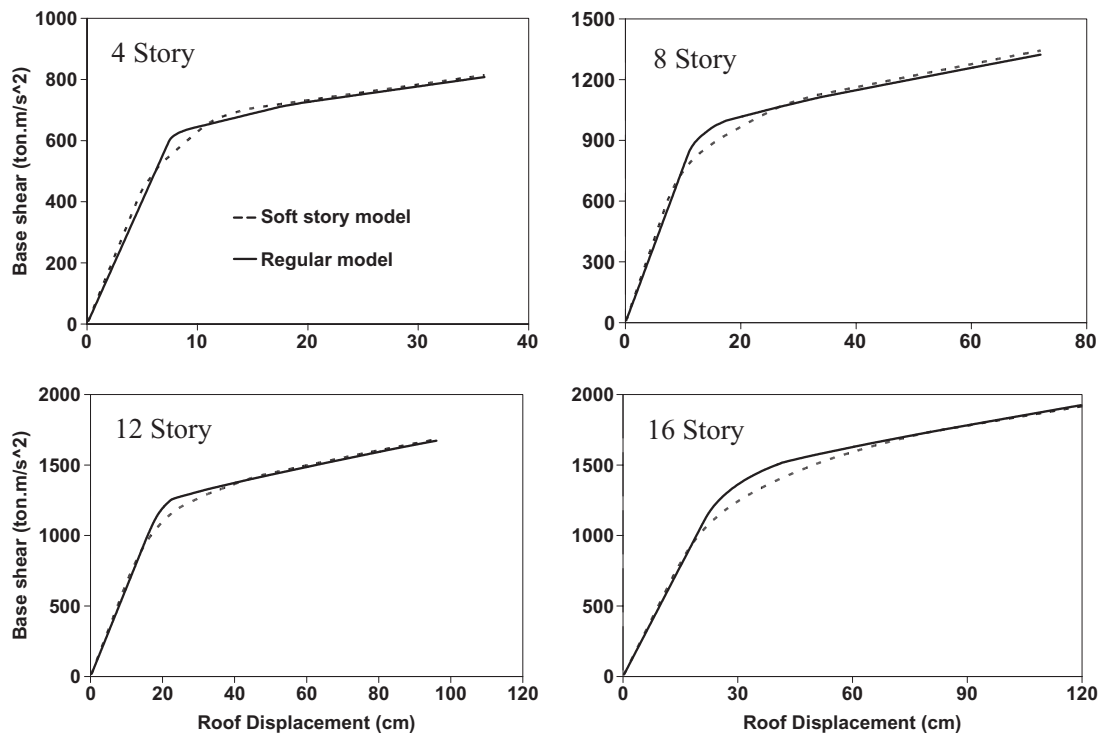


Figure 5. Pushover curves

Table 4. Main pushover parameters for the two methods

Model name	Strain-hardening ratio		Yield strength (tonne.m/s ²)	
	Conventional method	New method	Conventional method	New method
Four-story soft	0.0695	0.0780	636.0	624.3
Four-story regular	0.0698	0.0779	650.9	640.9
Eight-story soft	0.0802	0.0888	941.8	918.3
Eight-story regular	0.0752	0.0781	982.6	975.4
12-story soft	0.0900	0.1008	1212.6	1181.4
12-story regular	0.0852	0.0886	1263.4	1254.3
16-story soft	0.1127	0.1331	1335.1	1274.6
16-story regular	0.1011	0.1070	1439.1	1423.0

noted as exact method. Results have been calculated for two levels of intensity for each earthquake, first, a real earthquake with real PGA representing actual situation, and second, with the same earthquake but scaled uniformly to have a PGA twice of the original ground motion.

5. INCREMENTAL MODIFIED PUSHOVER ANALYSIS

The main purpose of the MIDA procedure (Mofid *et al.*, 2005) is to reduce the computational effort required for IDA by estimating the seismic demands for the structure by MPA, an approximate procedure (Chopra and Goel, 2001), instead of non-linear response history analysis. Therefore, in the MIDA procedure, each nonlinear response history analysis required in IDA is replaced by an MPA.

In the current study, a faster and simpler method compared with MIDA, named as IMP, is developed and evaluated. In MIDA method, which is based on modal pushover analysis, it is needed to determine at least two equivalent SDOF systems to incorporate the effect of higher modes. On the other hand, in the proposed IMP method, a lateral load pattern that accounts for the effects of the first and second modes is used. Third and higher modes have minor effects on the response of regular buildings (Jan *et al.*, 2004). Therefore, the IMP method will have one pushover analysis compared with two or more pushover analysis in the MIDA.

Similar to other nonlinear static methods, IMP procedure has some basic governing properties such as method of developing the equivalent SDOF system for estimating the target roof displacement and the lateral load pattern.

Qi and Moehle (1991) proposed a procedure for defining the equivalent SDOF system which is simple and results in good estimation of roof displacement. In the proposed IMP method, this procedure is used; however, the new method for idealization of the pushover curve is adopted. Appendix 1 describes the governing equations for the equivalent SDOF system that is based on the study done by Qi and Moehle (1991). Lateral load pattern and target roof displacement as proposed by Jan *et al.* (2004) are defined in Appendix 2.

6. WORKING PROCEDURE OF IMP

The IMP procedure is a method to generate approximate IDA curves. The main steps of this method are:

- (1) *Calculate elastic structural characteristics*: Determine the elastic structural characteristics for the studied models. These characteristics are period, mode shape and modal participation factor for only two first modes. Normalized modal shapes are used in the calculation of modal participation factors, while normalization is such that $\phi_r = 1$.

Table 5. Accuracy of new and conventional method to estimate roof displacement for eight-story frames

Record name.	Eight-story regular model						Eight-story soft model					
	Up to PGA			Up to 2PGA			Up to PGA			Up to 2PGA		
	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %
CM	20.3	-6.2	-6.1	37.7	-2.9	-3.1	19.3	-0.7	0.0	34.7	-0.3	-1.5
CO	13.5	3.9	2.7	19.9	6.5	6.0	12.2	6.1	6.4	17.7	13.3	11.0
CV	15.1	-8.1	-8.1	19.2	15.4	15.7	13.2	-5.6	-5.5	18.2	19.2	20.5
EL	11.8	2.7	2.7	18.6	25.2	24.9	10.0	18.7	18.8	17.6	26.6	25.8
EL4	14.6	-6.7	-6.7	22.2	5.6	5.6	11.7	15.1	15.4	18.7	20.9	20.3
IV2	10.1	2.1	2.1	28.5	0.1	0.8	9.9	0.1	0.1	31.6	-5.5	-3.3
IV3	6.2	-3.1	-3.1	12.4	-3.1	-3.1	6.0	-2.8	-2.8	12.0	-3.2	-3.1
KC	6.3	-8.6	-8.6	12.6	-8.6	-8.6	6.1	-10.2	-10.2	12.1	-9.9	-9.9
LD	6.3	-0.4	-0.4	12.2	2.7	2.7	6.3	-1.4	-1.4	10.3	21.1	21.5
LP	15.2	9.7	9.5	32.7	25.5	25.3	15.6	4.6	4.1	34.3	15.6	15.0
LP2	5.5	-5.0	-5.0	11.0	-5.0	-5.0	5.4	-4.6	-4.6	10.7	-4.6	-4.6
LP3	5.3	-5.8	-5.8	10.5	-4.4	-4.4	5.3	-5.0	-5.0	9.7	2.8	2.8
LP4	13.1	4.7	4.8	18.8	42.3	41.5	11.2	16.7	17.0	20.4	21.2	18.8
NP	8.6	-14.1	-14.1	14.8	0.6	0.6	8.4	-10.1	-10.1	12.7	6.8	3.0
SF	4.9	-0.6	-0.6	9.7	-0.6	-0.6	4.7	1.0	1.0	8.8	8.5	8.5
SH1	4.1	-1.2	-1.2	8.2	-1.2	-1.2	4.0	-3.7	-3.7	7.9	-3.5	-3.5
SH2	16.1	-7.1	-7.1	25.1	7.4	6.7	13.9	4.0	7.4	23.5	6.5	4.7

Table 6. Accuracy of new and conventional method to estimate roof displacement for 12-story frames

Record name.	12-story regular model						12-story soft model					
	Up to PGA			Up to 2PGA			Up to PGA			Up to 2PGA		
	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %	Exact (cm)	Conventional method error %	New method error %
CM	22.3	-6.2	-7.5	39.3	2.8	2.9	18.7	8.4	5.8	34.2	17.2	17.7
CO	14.2	-10.1	-10.1	26.7	-10.4	-10.4	13.8	-9.5	-9.5	24.5	-4.2	-4.0
CV	24.0	6.8	5.7	30.8	3.4	3.7	21.5	7.3	3.9	28.1	13.3	14.2
EL	13.9	-6.1	-6.1	26.6	21.4	21.3	12.9	-1.4	-1.4	22.8	38.8	38.9
EL4	12.9	8.6	8.6	28.2	3.3	3.4	13.0	4.7	4.7	28.7	-5.6	-8.3
IV2	15.8	-7.8	-7.8	39.7	7.2	8.0	16.1	-12.1	-12.1	43.1	3.5	6.0
IV3	9.8	-4.0	-4.0	19.7	-3.9	-3.9	9.5	-3.5	-3.5	19.1	-3.6	-3.6
KC	9.3	6.5	6.5	18.7	6.4	6.4	9.0	8.4	8.4	15.5	27.9	29.7
LD	10.8	-9.1	-9.1	20.2	-3.1	-3.1	10.1	-5.0	-5.0	15.8	20.8	20.9
LP	26.5	1.9	2.0	51.4	8.7	8.4	23.9	11.9	12.7	50.4	5.7	4.8
LP2	8.4	-1.6	-1.6	16.8	-1.6	-1.6	8.1	-1.0	-1.0	16.3	-0.9	-0.9
LP3	12.1	-16.1	-16.1	21.5	-5.6	-5.6	11.0	-9.5	-9.5	17.5	13.7	13.8
LP4	20.6	4.2	4.3	29.1	10.7	10.9	18.8	11.6	11.9	32.1	-0.2	0.5
NP	11.2	-17.5	-17.5	21.2	-13.1	-13.1	10.4	-13.1	-13.1	17.9	0.8	0.8
SF	4.4	4.0	4.0	8.9	4.0	4.0	4.3	5.8	5.8	8.5	5.8	5.8
SH1	8.9	-9.4	-9.4	17.6	-8.4	-8.4	8.2	-3.8	-3.8	15.9	-0.7	-0.7
SH2	25.8	-12.2	-12.2	38.1	15.9	15.7	23.5	-5.2	-4.8	40.3	4.2	4.0

- (2) *Calculate UB (Upper-Bound) factor and $\mathbf{f}_s$ lateral load pattern*: Calculate spectral displacement corresponding to the first two modes, then using modal participation factors found in step (1), *UB* factor as a measure of second-mode contribution in response is computed by applying Equation (A2.6), and also, $\mathbf{f}_s$ lateral load pattern is calculated using Equation (A2.7).
- (3) *Get equivalent SDOF structure characteristics*: Equivalent SDOF system is developed using the method proposed by Qi and Moehle (1991) described in this paper. Use an inverse triangular lateral load pattern and perform a nonlinear static procedure to obtain the pushover curve. Idealize this curve using the new proposed method of idealization into a bilinear curve and transfer that to the force-displacement relationship of SDOF structure. Using this transformed curve, find required SDOF characteristics, which are strain-hardening ratio (α), yielding strength $(r/m)_y$, yielding displacement (u_y) and elastic frequency (ω^*), which can be found using Equation (1):

$$\omega^* = \sqrt{\frac{(r/m)_y}{u_y}} \quad (1)$$

Damping is determined using Rayleigh method.

- (4) *Analyze equivalent SDOF structure*: Since the goal in this method is to find the response of the structure at different levels of intensity, there is needed to choose a parameter for scaling the ground motion intensity. There are several single-parameter measures of intensity of the ground motion available in the literature, such as PGA, spectral acceleration at the fundamental frequency of the structure ($S_a(T_1)$), PGV and spectral pseudo-velocity. For one level of scaled intensity, maximum displacement of the developed equivalent SDOF structure is computed using a nonlinear time history procedure or nonlinear spectrum. Maximum displacement of SDOF structure must be transformed by Equation (2) into corresponding maximum displacement of the roof's DOF knowing that γ is calculated using Equation (A1.4):

$$\text{Target roof displacement} = (Max\ Dis.)_{SDOF} \times \gamma(1 + UB) \quad (2)$$

Maximum hysteretic energy (E_H) dissipated during an earthquake event, which is an important factor for damage estimation in structures, can be estimated using equivalent SDOF structure. The relationship between maximum hysteretic energy of MDOF structure and maximum hysteretic energy of equivalent SDOF system, considering the effect of the equivalent mass of SDOF, is given by (Rodriguez, 1994):

$$(E_H)_{MDOF} = \gamma^2 (E_H)_{SDOF} \quad (3)$$

- (5) *Perform a pushover analysis*: In this step, MDOF structure is pushed with $\mathbf{f}_s$ lateral load pattern up to maximum roof displacement obtained in the previous step. At this level, desired structural demands such as maximum inter-story drift and plastic hinge rotation can be estimated.
- (6) *Generate 'IMP-based' IDA curves*: Repetition of steps (4) and (5) for each intensity level finally results in a curve with coordinates intensity measure versus damage measure. The obtained curve is called an individual "IMP based" IDA curve which is only for an earthquake ground motion. Using different earthquake records, fragility curves can be constructed.

As a summary, the working flowchart of the IMP method is presented in Figure 6.

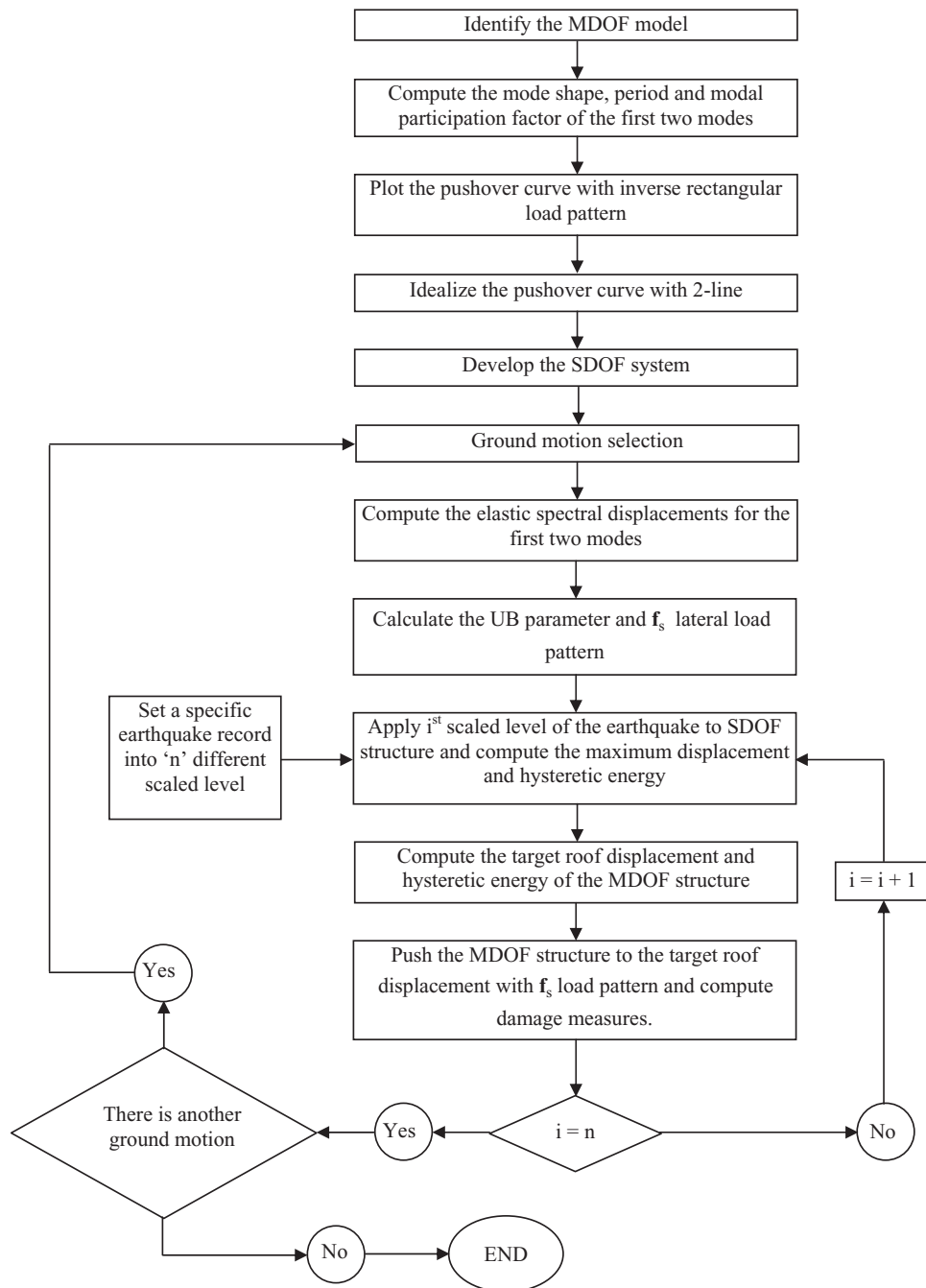


Figure 6. Working flow chart of the IMP method

7. VERIFICATION OF IMP METHOD

Four three-bay moment-resisting steel frames with 4, 8, 12 and 16-story are studied when subjected to 17 ground motions to verify the IMP method. In general, there are several ground motion scaling methods. Shome and Cornell (1997, 1998) demonstrated that the spectral acceleration at the fundamental frequency ($S_a(T_f)$) of the structure is a good single-parameter measure of intensity of the ground motion for building structures that are not shaken by earthquakes having near-field effects. However, in this research, PGA is considered in scaling the ground motions as it has been widely used as an identifier of the intensity of the ground motion (Kennedy and Ravindra 1984; Mofid *et al.*, 2005).

The results of IMP are compared with IDA and MIDA methods. IDA is considered as an exact response and MIDA is an approximate method based on MPA. For each model, 16, 50 and 84% fractile curves are presented in Figures 7–9 for peak roof drift ratio (θ_{roof}), maximum inter-story drift ratio (θ_{max}) and maximum plastic hinge rotation.

In the calculation of the target roof displacement, UB parameter is the representative of the effect of the second mode as presented in Appendix 2. This parameter is determined using modal participation factor and also spectral displacement; both of them are based on elastic characteristics of the frames in the IMP method. In some cases, such as SF record (see Table 7), the calculated UB parameter is not realistic, and this fact could be seen for some other records. The reason for these scattered responses arises from the spectral displacement, which is calculated based on the non-smooth shape of the response spectrum, and may cause unreasonable differences. On the other hand, UB parameter is intended to result in upper-bound value for roof displacement; in general, this parameter satisfies this purpose (see Figure 7).

Comparison between results for maximum roof displacement in Figure 7 with maximum inter-story drift (Figure 8) shows that the errors for θ_{max} is greater than θ_{roof} . In general, it is accepted that the estimation of inter-story drifts in non-linear static analyses has more difficulties and uncertainties compared with roof displacement since the calculation of θ_{max} is based on the calculated value for

Table 7. UB parameter for analyzed frames and selected earthquake ground motions

No.	Earthquake	Abbreviation	UB parameter			
			Four story	Eight story	12 story	16 story
1	Cape Mendocino	CM	0.055	0.065	0.127	0.277
2	Coalinga	CO	0.040	0.074	0.179	0.290
3	Chalfant Valley	CV	0.060	0.182	0.188	0.278
4	Imperial Valley	EL4	0.051	0.079	0.288	0.247
5	Imperial Valley	EL	0.063	0.099	0.199	0.157
6	Kern County	KC	0.046	0.120	0.156	0.140
7	Landers	LD	0.082	0.162	0.146	0.203
8	Loma Prieta	LP	0.058	0.070	0.113	0.072
9	San Fernando	SF	0.113	0.106	0.308	0.153
10	Imperial Valley	IV2	0.021	0.075	0.137	0.328
11	Imperial Valley	IV3	0.085	0.185	0.233	0.199
12	Loma Prieta	LP2	0.064	0.108	0.108	0.132
13	Loma Prieta	LP3	0.051	0.201	0.190	0.354
14	Loma Prieta	LP4	0.027	0.097	0.100	0.145
15	N-Palm	NP	0.067	0.043	0.050	0.107
16	Superstition Hills (A)	SH1	0.069	0.131	0.079	0.114
17	Superstition Hills (B)	SH2	0.032	0.104	0.088	0.102

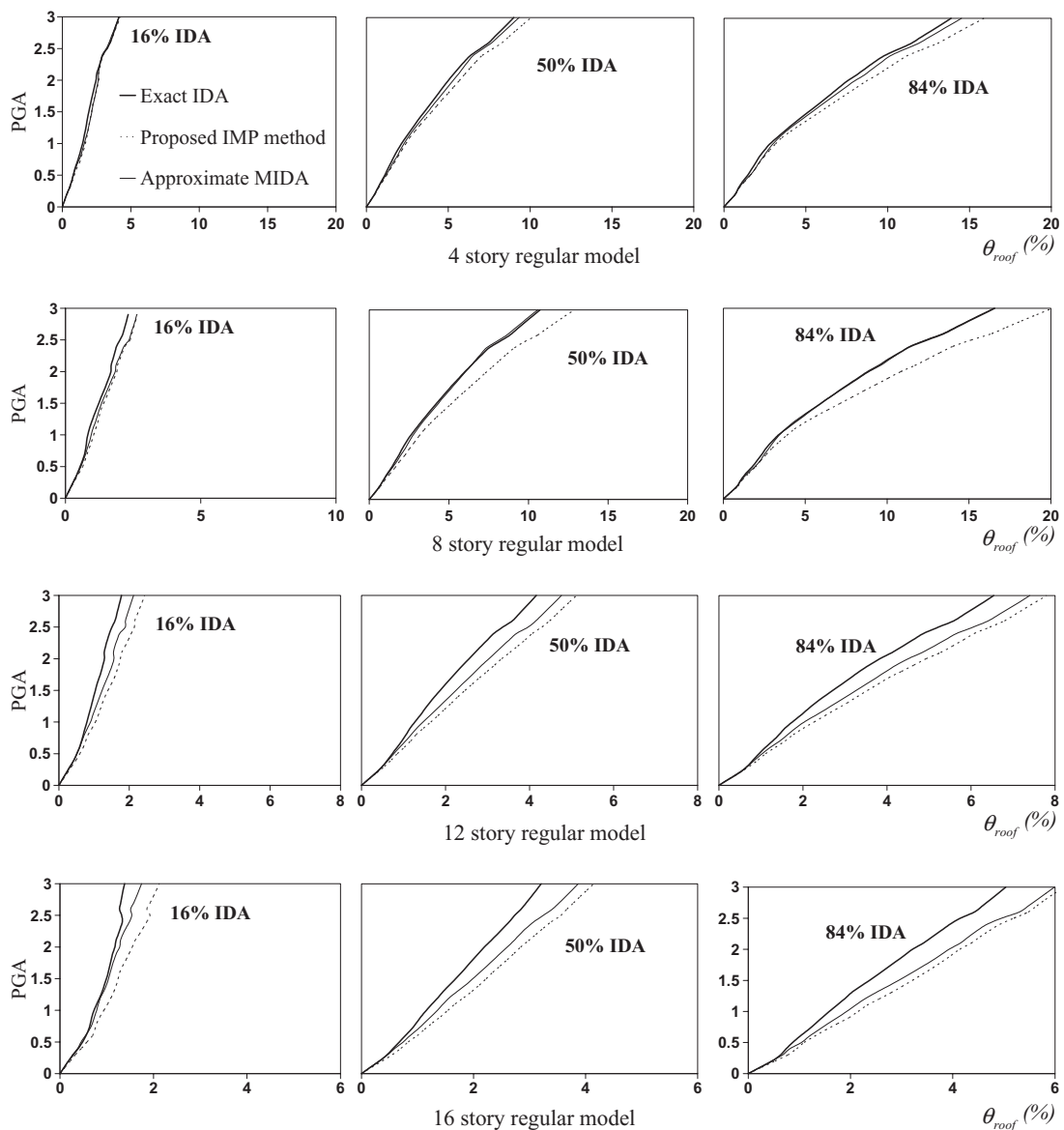


Figure 7. Sixteen, 50, and 84% fractile IDA curves for peak roof drift ratio, $\theta_{roof}(\%)$, from exact IDA, IMP as new method and MIDA as an approximate method

θ_{roof} . In this procedure, one pushover analysis is performed up to θ_{roof} using $\mathbf{f}_s$ lateral load pattern. This pushover analysis has some errors; therefore, it could add some extra errors in the calculation of θ_{roof} which means that the resultant error might be more in the calculation of θ_{max} . However, using mentioned lateral load pattern results in high and unacceptable errors. Therefore, using another lateral load pattern that has the advantage of $\mathbf{f}_s$, such as considering the effect of higher modes, could reduce the amount of the error considerably.

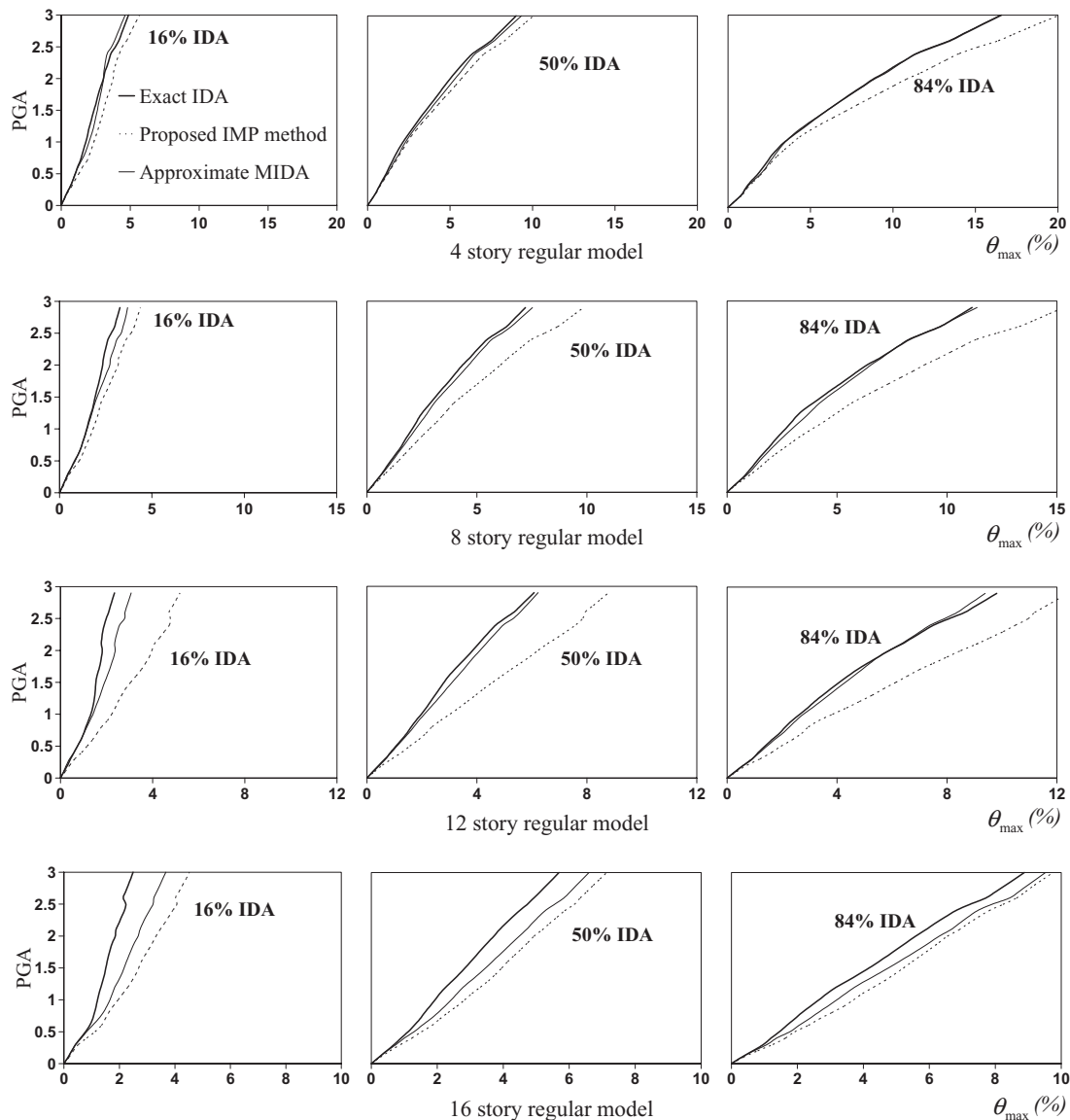


Figure 8. Sixteen, 50, and 84% fractile IDA curves for maximum inter-story drift ratio, $\theta_{\max}(\%)$, from exact IDA, IMP as new method and MIDA as an approximate method

8. DISCUSSION ON SOURCES OF ERRORS AND LIMITATIONS

There are several sources for errors in nonlinear static procedures compared with nonlinear time history analysis, which is assumed to be more accurate. In this section, some of these sources that are related more to the procedure proposed in this paper are mentioned.

The assumption of constant lateral deformation in the construction of the SDOF structure has some intrinsic errors.

In the $\mathbf{f}_s$ lateral load pattern, only the effects of the first two modes are considered. Consequently, it may introduce inaccuracy, especially in taller or irregular buildings.

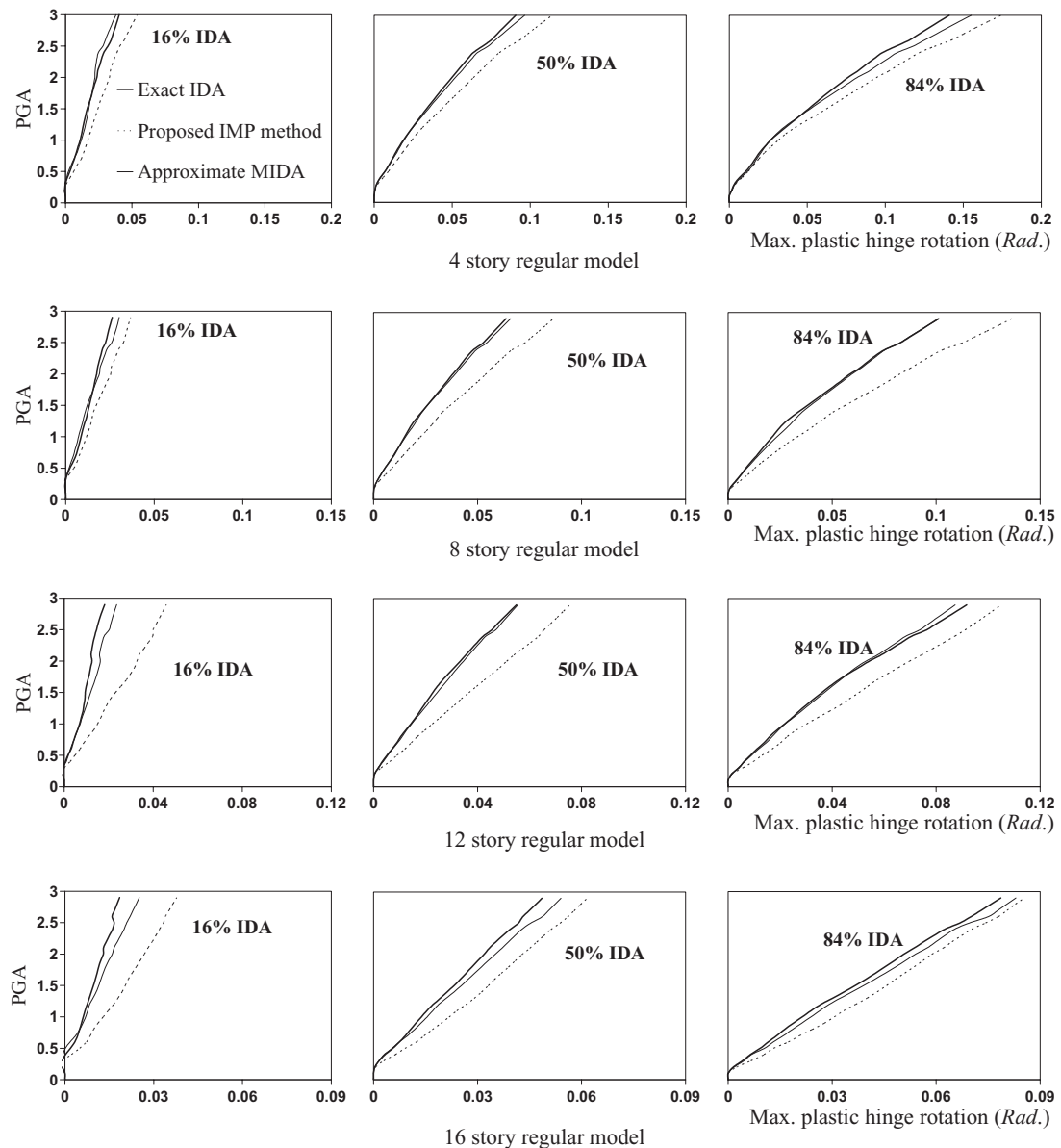


Figure 9. Sixteen, 50, and 84% fractile IDA curves for maximum plastic hinge rotation (Rad.) from exact IDA, IMP as new method and MIDA as an approximate method

Assuming constant values for parameter γ , Φ vector, participation factor, spectral displacement and other parameters calculated in the IMP method has an inherent inaccuracy, as in the nonlinear region, all of these parameters are changing continuously. Using an adaptive pushover procedure includes these changes and leads towards better results.

Approximating the base shear versus roof displacement curve with a bilinear curve will also cause some errors in the determination of target roof displacement.

While the combination of different records is desired, the intensity scaling method of the records, e.g. PGA, $S_a(T_I)$, PGV, etc., always introduces inevitable dispersion in results. The effect of scaling methods, other than using PGA, could be investigated to demonstrate the resultant effect on the proposed IMP method.

UB parameter is very sensitive to the selected record. This could be attributed to the non-smooth shape of the response spectrum, which is used for determining the spectral displacement. Therefore, significant unreasonable differences may appear for different periods.

9. CONCLUSIONS

Estimation of earthquake demands in order to draw IDA curves is useful to determine the seismic performance of the structure. Recently, MIDA has been proposed to draw these curves with less computational effort using approximate MPA instead of time history analysis. In the current study, first, a new method for bilinear idealization of pushover curve was investigated, and second, a new method named as IMP was proposed to achieve IMP-based IDA curves. IMP method uses the SDOF structure once. Consequently, it has less computational effort compared with MIDA, which itself is faster than IDA. Using four moment-resisting steel frames with 4, 8, 12 and 16 stories, as well as their corresponding soft-story models, subjected to 17 strong ground motions, the following conclusions were drawn.

The new proposed bilinear idealization procedure showed better results compared with the conventional idealization method, especially for structures having high values of roof displacement. The predicted results for roof displacement using the new procedure is reliable; however, the difference between the conventional and new procedure is small.

The estimated error in MIDA method is almost constant with respect to IDA technique in the linear region. However, that while this gets more unpredictable, when the calculations are carried on into the non-linear region. The IMP method follows the same trend.

In all the studied cases, the estimation of the response of the structure using IMP method is higher than that of using IDA or MIDA methods. In other words, IMP method is the upper-bound one compared with the other two methods. This means that the estimated response is on the conservative side, which is safer for design purposes.

The f_s lateral load pattern results in better estimation of demands in taller buildings because the effect of the second mode is more considerable. However, as this lateral load pattern resulted in unaccepted errors, using another load pattern able to consider the effect of higher modes is necessary.

It is important to emphasize that the drawn results have been obtained for the studied cases and the selected earthquakes. To generalize the conclusions, more diverse ground motions and models are needed to be analyzed.

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APPENDIX 1: EQUIVALENT SDOF SYSTEM FOR ESTIMATING TARGET ROOF DISPLACEMENT

If multi-degree-of-freedom frames are simplified with concentrated mass at each floor, the equation of motion for such system can be written in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{R}(t) = -\mathbf{M}\mathbf{1}\ddot{u}_g(t) \quad (\text{A1.1})$$

in which $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{R}(t)$ is the resistance vector, $\dot{\mathbf{U}}(t)$ and $\ddot{\mathbf{U}}(t)$ are the first and second derivatives of the relative displacement vector, $\mathbf{U}(t)$, with respect to the ground and $\mathbf{1}$ is the unit vector. For transforming an MDOF system to an equivalent SDOF system, lateral

deformation pattern Φ is assumed constant. For the determination of Φ , nonlinear static pushover is used with an inverse triangular load pattern, which is incrementally increased up to 1% of the height of the building. In the studied frames, the yield drift of the frames ranged from 0.6 to 0.9%. Therefore, the limit of 1% roof displacement drift was chosen to incorporate the effect of nonlinear deformed shape in the calculation of Φ vector.

For ease in calculations, Φ is normalized such that the roof DOF is set to unity ($\phi_r = 1$). Knowing Φ , the lateral displacement vector with respect to the ground, $U(t)$, can be found in terms of the roof displacement (Q_i and Moehle, 1991):

$$U(t) = \Phi \delta(t) \quad (A1.2)$$

where $\delta(t)$ is the roof displacement in time t . Using (A1.2) and other basic steps in the determination of SDOF systems finally results in (Q_i and Moehle, 1991):

$$\ddot{u}(t) + 2\xi\omega^*\dot{u}(t) + \frac{r(t)}{m} = -\ddot{u}_g(t) \quad (A1.3)$$

in which ξ is the damping ratio and ω^* is the natural frequency of the SDOF system. $\frac{r(t)}{m}$ can be defined using another parameter γ , which is (Q_i and Moehle, 1991):

$$\gamma = \frac{\{\phi\}^T M \{1\}}{M^*} \quad (A1.4)$$

$$\frac{r(t)}{m} = \frac{1}{\gamma} \frac{\{\phi\}^T \{R(t)\}}{M^*} \quad (A1.5)$$

In Equation (A1.5), $\frac{r}{m}$ can be calculated implicitly using a pushover analysis. In pushover analysis, the relationship between base shear and roof displacement is obtained, which needs to be transformed to force displacement of Equation (A1.3).

For this purpose, assuming that lateral displacement pattern follows the load pattern, $\{R(t)\}$ can be shown as (Q_i and Moehle, 1991):

$$\{R(t)\} = \{P\} R(t) \quad (A1.6)$$

Vector $\{P\}$ is constant and the factor of lateral load, $R(t)$, shows the variation of $\{R(t)\}$. Knowing $R(t)$ in every time, base shear (V_b) is calculated as:

$$V_b = \{1\}^T \{R(t)\} = \{1\}^T \{P\} R(t) \quad (A1.7)$$

Using Equations (A1.5), (A1.6) and (A1.7), the relationship between (V_b) and $\frac{r}{m}$ is found:

$$\frac{r}{m} = \left(\frac{\{\phi\}^T \{P\}}{\{1\}^T \{P\}} \times \frac{1}{\{\phi\}^T [M] \{1\}} \right) V_b \quad (A1.8)$$

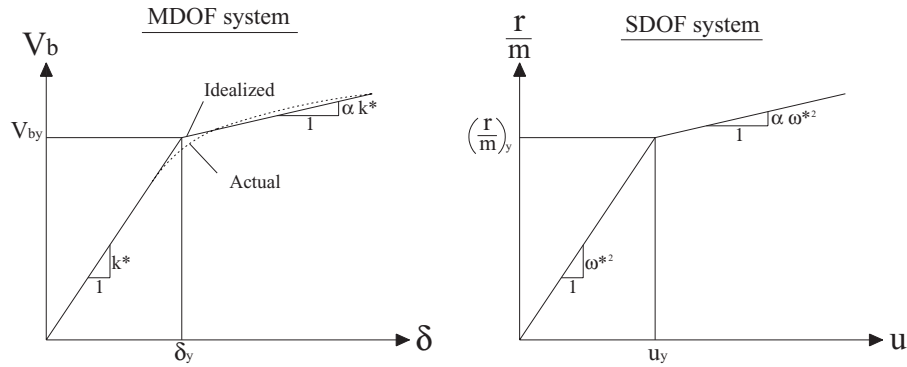


Figure A1. Transformation of the pushover curve to the equivalent SDOF system

Therefore, pushover curve, which has base shear versus roof displacement, can be transformed into $\frac{r}{m} - u$ of the equivalent SDOF system. Brief resulting curves are shown in Figure A1.

APPENDIX 2: LATERAL LOAD PATTERN AND TARGET ROOF DISPLACEMENT BASED ON JAN *ET AL.* (2004)

If we consider $q_n(t)$ as modal coordinate and also Γ_n as modal participation factor, both in the n th mode of vibration, roof displacement can be obtained using a combination of modes:

$$u_r(t) = \sum_{n=1}^N \varphi_{rn} q_n(t) = u_{r1}(t) \left(1 + \sum_{n=2}^N \frac{\varphi_{rn} q_n(t)}{\varphi_{r1} q_1(t)} \right) \quad (\text{A2.1})$$

in which $u_{r1}(t) = \varphi_{r1} q_1(t)$ is the roof displacement in the first mode. If $\{\phi\}_n$ which is the n th structural mode shape, is normalized such that $\varphi_{rn} = 1$, Equation (A2.1) can be rewritten as:

$$u_r(t) = u_{r1}(t) \left(1 + \sum_{n=2}^N \frac{q_n(t)}{q_1(t)} \right) \quad (\text{A2.2})$$

Knowing that $[\mathbf{K}]\{\phi\}_n = \omega_n^2 [\mathbf{M}]\{\phi\}_n$, equivalent static load $\{f_s(t)\} = [\mathbf{K}]\{U(t)\}$ is:

$$\{f_s(t)\} = \sum_{n=1}^N \omega_n^2 [\mathbf{M}]\{\phi\}_n q_n(t) \quad (\text{A2.3})$$

In conventional pushover procedure, which is defined in codes, nonlinear static procedure is performed with a constant lateral load pattern that is incrementally increased to reach the predetermined roof displacement. Both lateral load pattern and target roof displacement are based on the assumption that the response of the structure is governed by the first mode of free vibration; in addition, modal shape does not change in nonlinear region. It is shown that using only the first two modes results in a good estimation of elastic response of structure subject to earthquake excitation and contribution of higher modes could be neglected (Jan *et al.*, 2004). Therefore, if one consider only the two first modes and

choose absolute sum as the superposition method, equivalent static load and roof displacement are simplified such that:

$$\{f_s\} = q_1 \left(\omega_1^2 [M] \{\phi\}_1 + \omega_2^2 [M] \{\phi\}_2 \frac{q_2}{q_1} \right) \quad (\text{A2.4})$$

$$u_r = u_{r1} \left(1 + \frac{q_2}{q_1} \right) \quad (\text{A2.5})$$

in which $\frac{q_2}{q_1}$ is:

$$UB = \frac{q_2}{q_1} = \left| \frac{\Gamma_2 D_2}{\Gamma_1 D_1} \right| \quad (\text{A2.6})$$

and $D_i = D(T_i, \xi_i)$ is spectral displacement for period T_i and damping ratio of ξ_i . As only the shape of the lateral load pattern is important, equation A2.4 can be simplified as:

$$\{f_s\} = \omega_1^2 [M] \{\phi\}_1 + \omega_2^2 [M] \{\phi\}_2 UB \quad (\text{A2.7})$$