

Experimental and analytical behavior of haunched thin-walled RC girders and box girders

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ABSTRACT

This paper presents an experimental and analytical investigation on the behavior of haunched thin-walled reinforced concrete (RC) girders and box girders subjected to centric and eccentric loads. Five tests were conducted on medium-scale RC girders and box girders to study the effect of load eccentricity and the influence of bottom slabs on their ultimate load-carrying capacities, failure mechanisms and load–deformation relationships. Accordingly, the tested girders were modeled using ADINA, and a non-linear finite element analysis was conducted. The investigation showed that load eccentricities considerably reduced the ultimate loads and the ductility of the girders with open sections. However, when the open section was converted to a full-span box section, the behaviors of the girders were significantly improved.

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1. Introduction

For bridge spans that have continuity from both ends, haunched thin-walled girders are thought of as efficient structural systems in view of rigidity and strength requirements, economic consideration and self-weight reduction. Haunched thin-walled girders have advantages of effective use of material in flexure. However, due to the considerable effects of torsional warping, distortion and distortional warping, transverse stress, and shear lag in thin-walled members under torque loads, the structural analysis and calculation of haunched thin-walled sections are obviously more complicated than that of solid sections. The complex loading effects cannot be determined only by the engineers' beam theory. More advanced mechanical theories are necessary to be applied to determine the combined effects of bending, shear and torsion actions on haunched thin-walled structures.

Extensive theoretical investigations on straight and curved thin-walled structures have been carried out during the past several decades. Vlasov [1] developed the theory of thin-walled elastic beams, which became the basis of modern theoretical models to analyze box girders. Since then, several approaches were developed and adopted to analyze thin-walled structures.

Boswell [2] reported several methods for the analysis of thin-walled structures, such as: thin-walled beam theory, matrix progression and matrix transfer analysis, equivalent beam method, grillage method, folded plate, finite strip analysis and the finite element technique. Boswell [3] conducted an analytical and experimental study on the ultimate behavior of reinforced concrete (RC) box girder bridges. The experimental program was conducted on four box girders. Based on the assumption of sufficient ductility in RC box girders, a modified yield criterion that involved stress combination of bending, shear and axial forces, was developed to predict collapse loads. Possible collapse mechanisms were considered on the ultimate load predictions.

Analytical and numerical approaches about the problems of haunched thin-walled girders under symmetrical and asymmetrical loading have been carried out by many researchers. Dilger and Langohr [4] analytically proved that, for haunched box girders, the conventional shear design, which considers the contribution of the sloped compression force, is on the unsafe side. Based on variable angle truss model, more generic equations for the design of shear force resistance in haunched box girders were developed, and this approach was used to check the shear design of the Confederation Bridge in Canada. Li [5] extended the finite strip method to the analysis of continuous haunched slab-on-girder bridges and box-girder bridges, where his study includes linear and non-linear analysis. Li [6] developed a finite element method on the basis of linear elasticity thin-walled theory and proposed generic models to estimate the effects of

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extension, flexure, torsional warping, distortion, distortional warping and shear lag for haunched box girders. However, such numerical and analytical solutions need to be verified by experimental investigations, which is not widely available in literature. Similar to the analysis models of Li [5] and Li [6], many researches of thin-walled beams under eccentric loads have focused on closed sections. Nevertheless, open sections, such as double tees sections are also frequently used in concrete construction practices.

Several experimental works have been conducted to study the elastic and plastic behavior of RC thin-walled girders, but most previous investigations have been focused on the behavior of straight beams. Spence and Morley [7] experimentally examined 13 straight girders with deformable box sections under different combinations of symmetrical and anti-symmetrical loads, and they observed that the failure mode types depend on the proportion of symmetrical load to anti-symmetrical load. Rasmussen and Baker [8] experimentally studied ultimate load-carrying capacity and failure mechanisms in deformable thin-walled box section beams with constant cross-section subjected to eccentric loads. The test results show that the ultimate capacity of a RC box girder will be over-evaluated based on the space truss theory, which does not involve the effects of distortion; therefore, the modification of the space truss theory is necessary. Rasmussen [9] analyzed the plastic behavior of deformable box sections and found that the level of anti-symmetrical loading and the ratio of bending to in-plane capacity of the walls in the box section influence the development of plastic corner hinges and the spread of plasticity. In the experimental work of Rasmussen and Baker [8], deformable box sections with various degrees of reinforcement were tested under different load eccentricity to verify Rasmussen's [9] theories about the plastic behavior of box sections. The post-cracked behavior of concrete box girders is thoroughly studied and described; however, the investigations have been limited to straight girders with closed sections.

A survey of the available literature shows that the plastic behaviors of haunched girders with double tees (without bottom slabs) and box sections (with bottom slabs) are still not well understood due to the lack of experimental investigation. It can be proved, by applying elastic thin-walled theory, that the flexural and torsional stiffness of box sections (closed sections) is much

higher than that of double tees sections (open sections) with same wall thickness and girder height. Nevertheless, the significance of the bottom slabs' effects on the plastic behavior of haunched RC box girders, the prediction of cracking loads, the ultimate loads and their ductility is still obscure. In order to obtain a better understanding about the linear and non-linear behavior of haunched RC girder with both open (double tees) and closed (box) sections, as shown in Fig. 1, five middle span haunched thin-walled girders with various lengths of bottom slab were tested under centric and eccentric vertical loads up to failure. The spread of plasticity and the development of plastic hinges are observed.

Additionally, numerical analysis based on finite element modeling was also conducted to present a practical approach for studying the behavior of haunched thin-walled RC girders. Modeling thin-walled RC structures exposed to eccentric loads poses modeling challenges due to the inherent non-linear properties of concrete, the composite effect of the steel and concrete, as well as the 3D stress condition in thin-walled structures. ADINA [10], a computer program for finite element analysis, has the capability to accommodate the above-mentioned complex problems [11–16]. Therefore, it was selected for the non-linear finite element modeling of the tested girders. Finite element models were established to simulate the five tested specimens. 3D solid elements in concrete material model were used to represent concrete part, while truss elements in bi-linear plastic material model to simulate steel rebars in the girders. A comparison of the experimental and theoretical results was conducted to provide a careful validation of the numerical models. The analytical results are very useful for thorough understanding of the concepts of failure mechanics of haunched thin-walled girders that are subjected to centric and eccentric loads.

As such, the objective of the current experimental and analytical study is to investigate the effect of the presence of and the length of bottom slabs on the behavior of haunched thin-walled RC girders subjected to centric and eccentric monotonic load at mid span. The study evaluates the difference in the response of the five tested specimens through their crack propagation pattern, cracking and failure load capacities, vertical and lateral load–deflection relationships as well as strain in steel rebars.

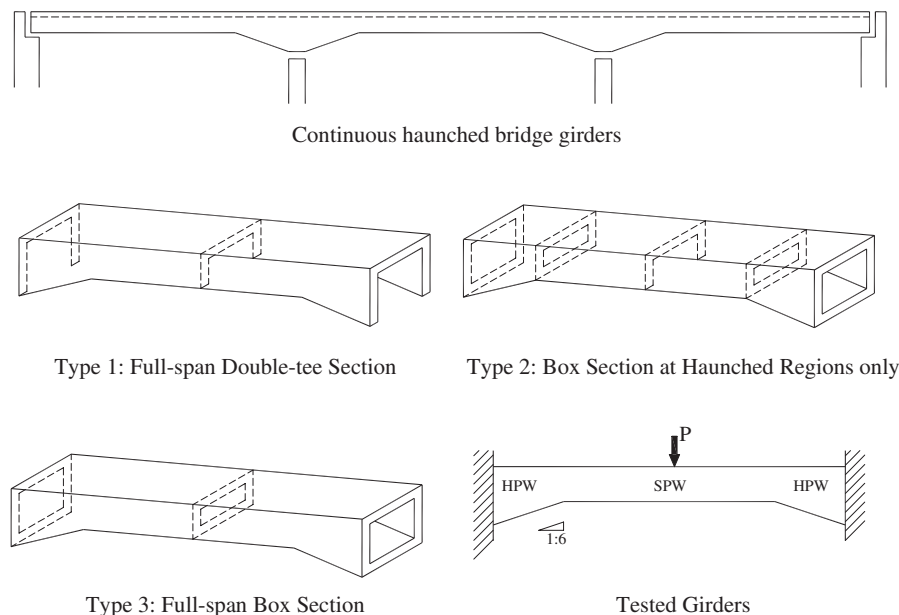


Fig. 1. Continuous thin-walled haunched girders and the three studied types.

2. Experimental study

2.1. Experimental test program

2.1.1. Design and development of test specimens

Five RC girders were tested to observe the behavior of thin-walled haunched girders under centric and eccentric loads. The experimental program was designed to investigate the influence of bottom slabs on the girders' behaviors when thin-walled haunched girders are individually subjected to two load cases: a centric load case and an eccentric load case resulting in combined bending, torsion and shear.

In order to evaluate the effects of bottom slabs, five specimens with same section dimensions were fabricated into three type models with varied length of bottom slabs. The reinforced thin-walled elements were assembled in double-tee sections and box sections. The three girder types schematically shown in Fig. 1 are denoted as Type 1, a full-length double tees section model; Type 2, a partial closed section model; and Type 3, a full-length box section. Type 2 has two sections along its span: with bottom slabs to form box sections only at haunched parts, and without bottom slabs in the straight part to form double tees sections.

The outer dimensions and reinforcement of the girder models were determined as a scale of approximate 1:6.67 of conventional RC bridge girders. The five tested specimens with 4.00 m total

length were fixed at two supports during the experimental program. The clear spans were 3.0 m, while a 0.5 m was used for fixation set-up at the two ends. The tested girders were haunched with a 1:6 slope at the bottoms of webs at two ends. The haunched part of the web and straight part of the web are denoted as HPW and SPW, respectively, as shown in Fig. 1. The wall thickness of all cross-sections was 50 mm, and the width of the cross-section was chosen as 400 mm. The depth and span ratio was chosen as 1/15, lying in the range of common depth/span ratios used in thin-walled bridge girders according to the survey of Soliman and Mirza [17]. Thus, the depth of the tested girders at SPW was chosen as 200 mm, and the depth of HPW varied from 300 to 200 mm. A cross diaphragm was used at the mid-span of each specimen in order to assist in distributing the eccentric load amongst the two webs.

The positive bending moments are expected to occur at SPW, whereas the negative bending moments at HPW. Therefore, 4 $\varnothing$ 10 were arranged at tension zone, bottom of SPW and top of HPW, while 4 $\varnothing$ 8 were placed at compression zone, bottom of HPW and top of SPW. 2 $\varnothing$ 8 were distributed at top flanges, and same steel bars were placed in the bottom flanges if there were bottom slabs. Plain round bars of $\varnothing$ 6 were used to form two layers of closed stirrups in each flange and web as shear and torsional reinforcement. The spacing of the stirrups is 200 mm throughout the spans.

Fig. 2 shows the dimensions of the three types of tested girders in elevation, plan and cross-section; Fig. 3 shows the layout of

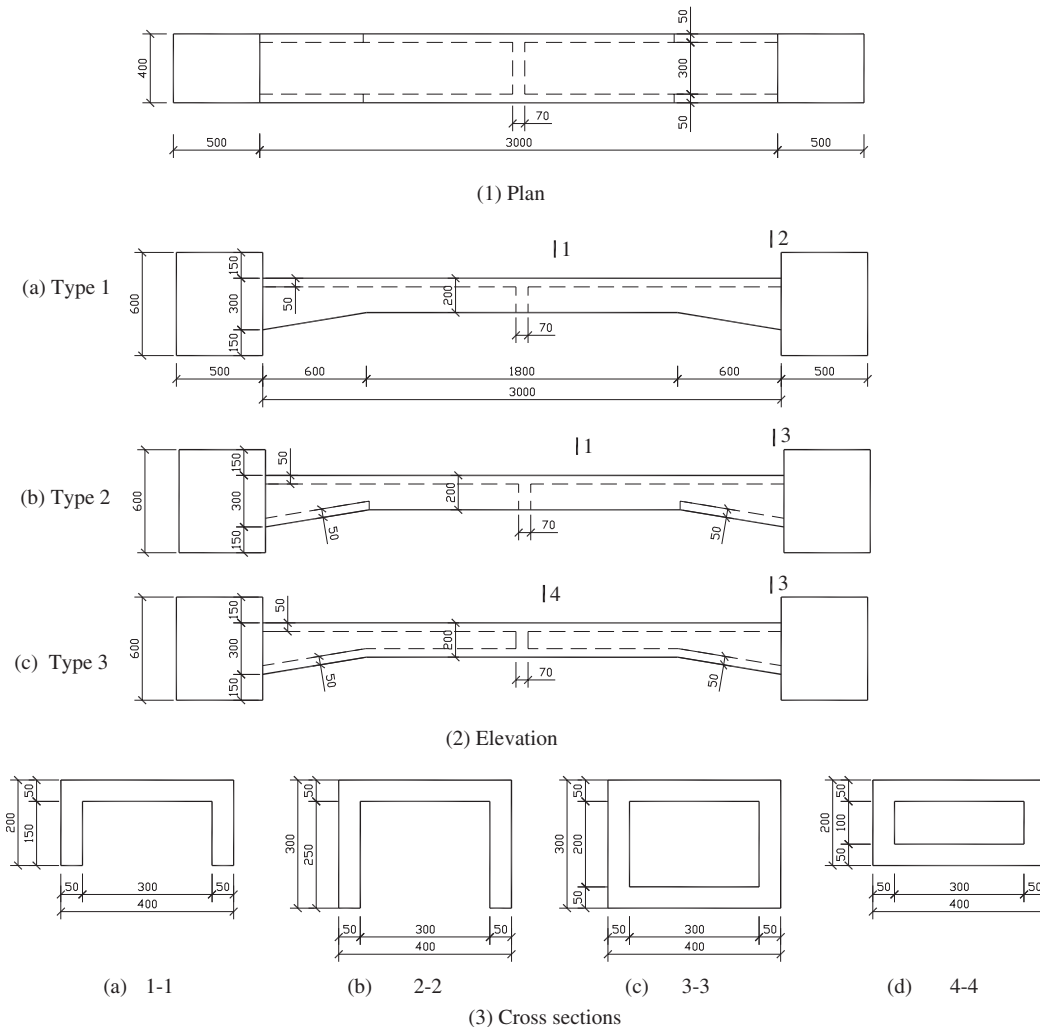
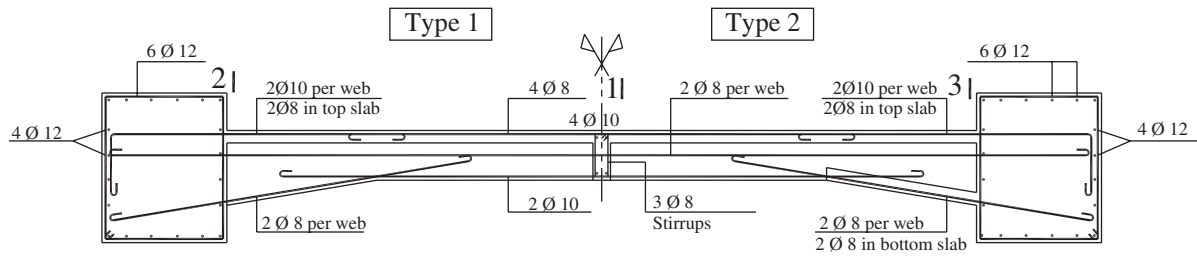
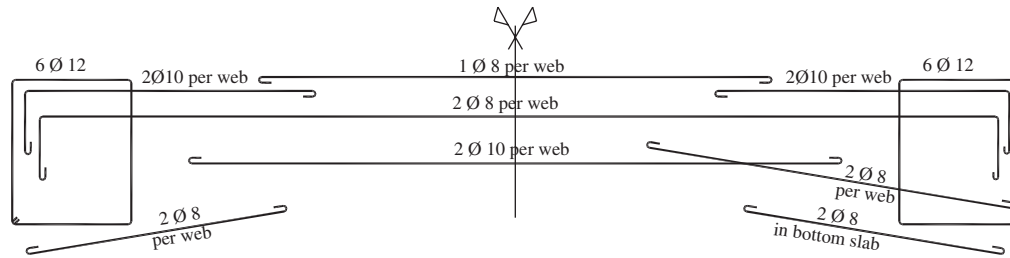


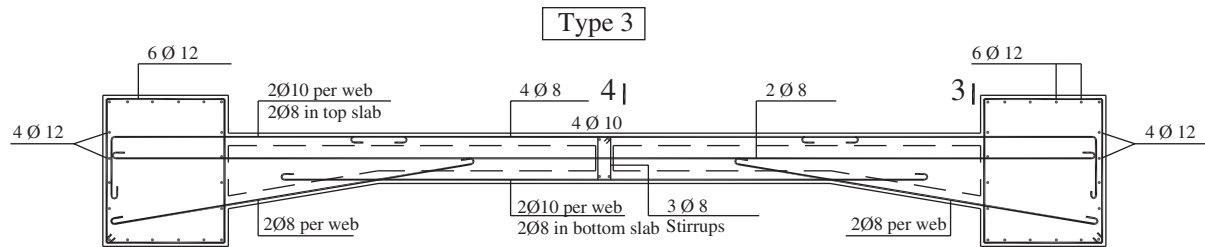
Fig. 2. Concrete dimensions of the three types of girders.



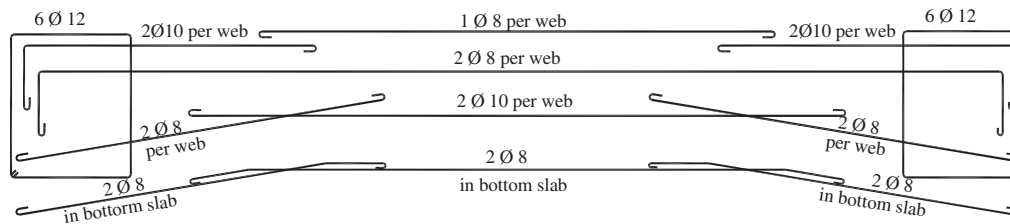
(1) Elevation of Type 1 and Type 2



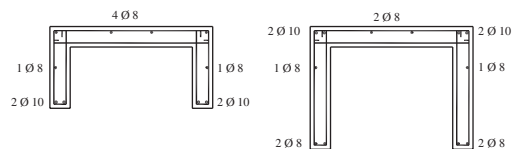
(2) Details of reinforcement in Type 1 and Type 2



(3) Elevation of Type 3

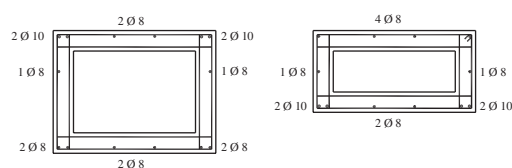


(4) Details of reinforcement in Type 3



(a) 1-1

(b) 2-2



(c) 3-3

(d) 4-4

(5) Cross sections (Stirrup: $\phi 6 @ 200$)**Fig. 3.** Details of reinforcement of the five girders.

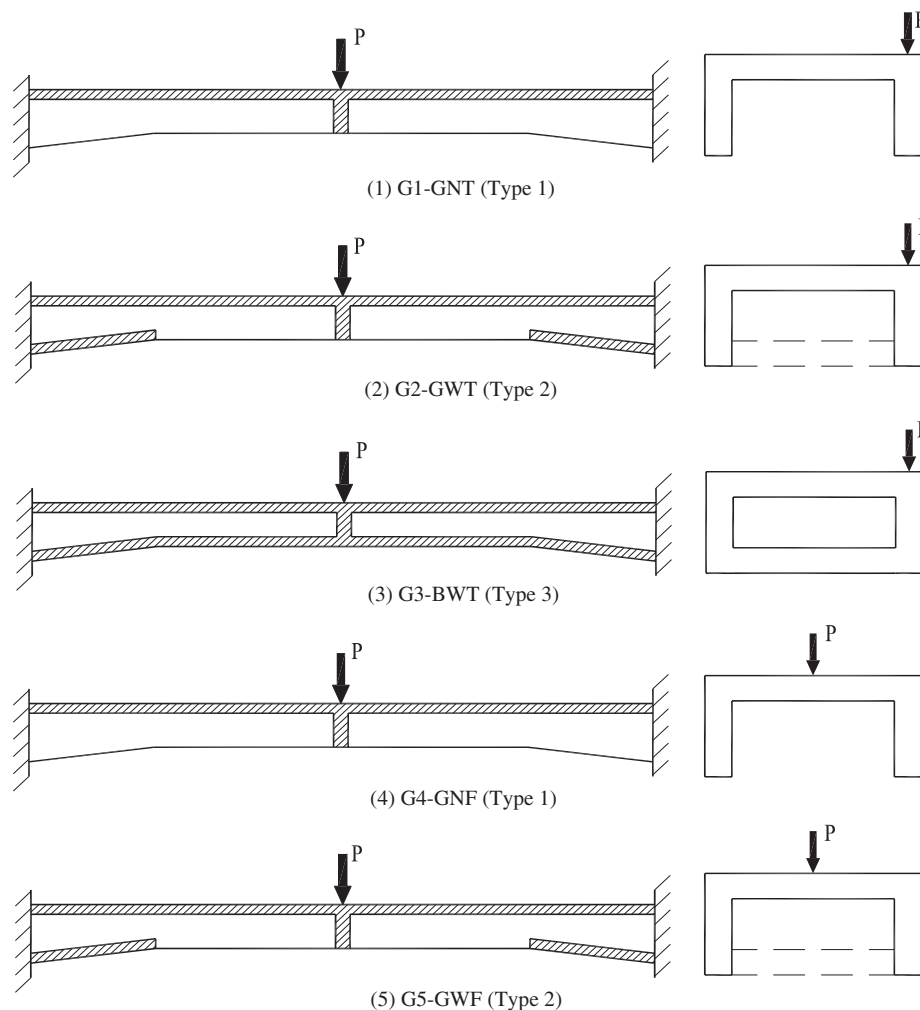


Fig. 4. Loading cases of the five tested girders.

reinforcement; and Fig. 4 shows the load cases of the five tested girders.

The five specimens are denoted as G1-GNT, G2-GWT, G2-BWT, G4-GNF and G5-GWF. The three letters after the hyphen denote whether the section is girder (G) or box (B), without (N) or with (W) bottom slabs, and if it is loaded in flexure (F) or torsion (T). Three specimens, G1-GNT, G2-GWT and G3-BWT, fabricated as Type 1, Type 2 and Type 3, respectively, were subjected to eccentric loads to explore the girder behaviors and the effects of bottom slabs in the eccentric load case. These three girders were subjected to combined bending moment, shear and torsion. Two specimens, G4-GNF and G5-GWF, fabricated as Type 1 and Type 2, respectively, were subjected to centric loads. These two were subjected to bending moment and shear force.

Polystyrene blocks were used as void former to create a core when developing a thin-walled girder. After the reinforcement cage was positioned in the mold, mechanically well-mixed concrete was cast on the form and high vibration was applied to guarantee the uniform distribution of concrete and avoid air voids. The moisture curing stage of specimens lasted 7 days.

2.1.2. Material properties

Same materials were used in the five specimens. Concrete used was made of a mix of natural sand, natural gravel, ordinary Portland cement and tap water. The maximum nominal size for gravel was 10 mm. Superplasticiser was used as water reducer to obtain a suitable workability and required concrete strength

Table 1

Average cube compressive strength f_{cu} and equivalent cylinder compressive strength f_c at the day of testing of the girders (in MPa)

Girder	Average f_{cu} (MPa)	Average f_c (MPa)
G1-GNT	35.6	34.8
G2-GWT	35.2	34.5
G3-BWT	36.1	35.6
G4-GNF	36.5	35.9
G5-GWF	35.5	34.5

Table 2

Characteristics of used steel rebars (tension test)

Nominal diameter (mm)	Actual diameter (mm)	Yield stress (MPa)	Tensile stress (MPa)
6	6.25	381.9	545.0
8	7.85	343.5	505.6
10	10.1	343.5	505.6

after 28 days similar to the actual concrete strength in real bridges.

Nine standard cubes, $158 \times 158 \times 158 \text{ mm}^3$, were cast and tested to measure the cube strength of concrete f_{cu} , to determine the stress–strain curve of concrete, the modules of elasticity and

the tensile strength. The control samples were taken from the concrete at regular intervals during casting of the girders and were cured under same situation as tested girders. Table 1 shows the average cube compressive strength f_{cu} and equivalent cylinder compressive strength f_c at the day of testing the girders (in MPa).

Plain round bars of $\varnothing 8$ and $\varnothing 10$ were used as longitudinal reinforcement, while plain round bars of $\varnothing 6$ worked as stirrup reinforcement in the fabrication of specimens. Three specimens randomly selected for each bar size were tested to determine the steel properties including yield and ultimate strength, modules of elasticity and ductility. Table 2 specifies the properties of the steel rebars carried out by simple tension test. All rebars had a modulus of elasticity of 200,000 MPa.

2.1.3. Test instrumentation and test procedure

Each test specimen was clamped at the two ends to the test bed. They were flexurally and torsionally restrained at both supports. The deformation and deflection of test girders were measured at each load increment by dial-gauges placed at different positions. Concentrated loads were applied at the mid-span of the five specimens through a 300 kN capacity hydraulic jack. The point loads act on the top of the webs and the center-line of the top flanges to incur eccentric and centric loads, respectively. The loads were applied in a 5 kN increment till failure.

2.2. Test results

Comparison between the results of the five tests was carried out to evaluate the significance of impact of load eccentricity and length of bottom slab. The following sections discuss the results of cracking and failure loads, crack patterns, and vertical and lateral deflections.

2.2.1. Cracking and failure loads

The experimentally observed cracking and failure loads of the five girders are shown in Table 3. From the table, it could be seen that load eccentricities have different degrees of impact on the cracking and failure loads according to the girder configuration, i.e. Types 1, 2 or 3. For all five tests, the first observed cracks were vertical flexure cracks. For G1-GNT and G2-GWT, the first crack was observed at the lower fibers of loaded webs near mid-span, whereas for G4-GNF and G5-GWF, cracks simultaneously started at two lower fibers of webs at mid-span and top slab near the fixed supports. For G3-BWT, positive moment cracks on the lower edge of loaded web and negative moment cracks on the top flange also started at the same load. On the other hand, as bottom slabs were added at haunch zones only in G2-GWT, the high torsional and flexural stiffness ratio of the end zones compared to the mid-

span section resulted in the highest cracking load in the top flange.

For the eccentric load case, it could be seen from Table 3 that the presence of bottom slabs postpones the onset of cracks. The positive moment cracking loads of G2-GWT and G3-BWT on the loaded web are 1.25 and 1.33 times the cracking load of G1-GNT, respectively. Nevertheless, it also shows that full-length bottom slab does not significantly increase the first cracking load of the girder compared to that with the partial bottom slabs at haunched parts. In fact, the cracking loads of G2-GWT and G3-BWT on loaded webs are very close, and the cracking loads of G2-GWT on unloaded webs and top flanges are even higher than that of G3-BWT.

Load eccentricity induced different cracking loads between loaded web and unloaded web of girders G1-GNT, G2-GWT and G3-BWT. However, the bottom slabs reduce the difference of cracking loads between the two webs. For G1-GNT, the difference is 20 kN, 66.67% of the cracking load at the loaded web; as a result of the bottom slabs at haunched parts in G2-GWT, the difference is reduced to 10 kN, only 26.67% of the cracking load at the loaded web; additionally, due to a full-span box section in G3-BWT, the difference of cracking loads between the two webs is 5 kN, only 12.5% of the cracking load at the loaded web.

In general, load eccentricity decreased the cracking load and load capacities of G1-GNT by approximately 25% and 16.2%, respectively, compared to G4-GNF, which was subjected to centric load. On the other hand, the cracking load and load capacities of G2-GWT are decreased 16.6% and 7.9%, respectively, compared to G5-GWF. Therefore, it could be said that the presence of bottom slabs at haunched part reduces the rate of decrease of cracking loads and failure loads due to load eccentricities.

It was also observed that bottom slabs play relatively more important role for girders under eccentric loads compared to those under centric loads. The increase in failure load of G2-GWT is about 13% of the failure load of G1-GNT; however, the increase of the failure load of G5-GWF is only 3% of that of G4-GNF. Furthermore, extending the bottom slab to the full span increases the failure load of G3-BWT, compared to G1-GNT and G2-GWT, by about 26% and 12%, respectively.

Though G3-BWT was subjected to eccentric loads, the failure load of G3-BWT reached the highest magnitude among the five test girders. This result can be an exposition of that a full-length bottom slab greatly enhances the load-carrying capacities of bridge girders under eccentric loads. It should be noted that it was anticipated that the failure load of a box girder, with bottom slab along the whole length, subjected to centric load would not have a significant increase compared to G4 and G5. This is further verified in the analytical phase of this study.

Table 3
Experimental and analytical cracking, yielding and failure loads (in kN)

	Experimental				Analytical		
	Cracking load			Failure load	Yielding load		Failure load
	At mid-span		At support		Loaded web	Unloaded web	
	Loaded web	Unloaded web	Top flange				
G1-GNT	30	50	40	77.5	46	65	76.8
G2-GWT	37.5	47.5	51.5	87.5	49	71	88.3
G3-BWT	40	45	40	97.5	79	83	109.3
G4-GNF	40	40	40	92.5	58	58	103
G5-GWF	45	45	45	95	65	65	115
G3'-BWF					78	78	122

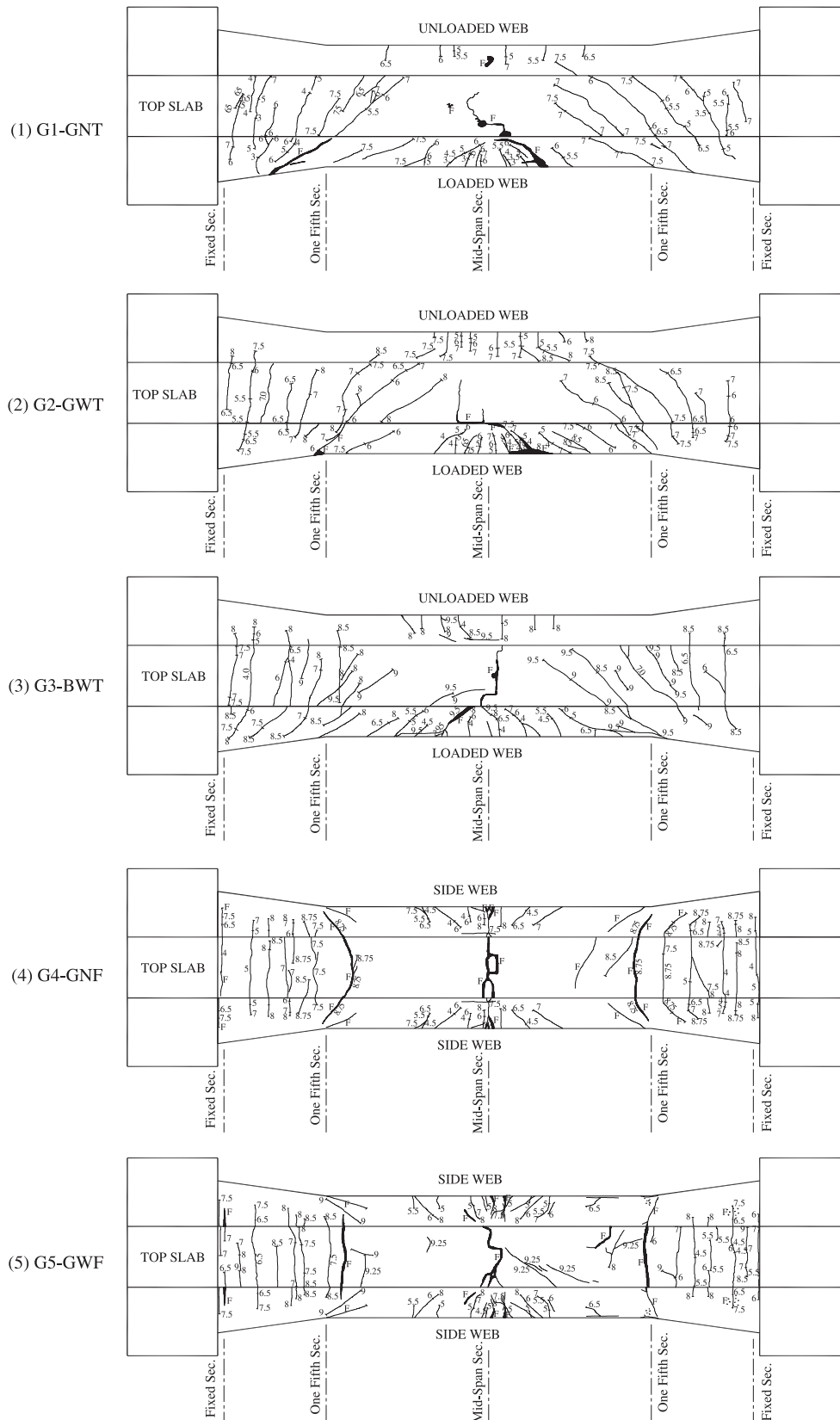


Fig. 5. Crack patterns of the tested girders at failure.

2.2.2. Crack patterns

Fig. 5 shows the crack patterns of the five tested girders and the effects of load eccentricities and bottom slabs on the crack distribution and the positions of the crushing hinges. Concrete

crushing is observed in all five specimens close to failure. Additionally, due to the configurations of the girders, restrained conditions and load cases were symmetrical about mid-span of the models; the crack patterns were also approximately symmetrical about mid-span.

Same as the foregoing assessment regarding the significance of bottom slabs, the bottom slabs have less effect on G4-GNF and G5-GWF (centrically loaded) in respect of the development of cracks and plastic hinges than on G1-GNT, G2-GWT and G3-BWT (eccentrically loaded).

For G4-GNF and G5-GWF, a large number of parallel flexural cracks, perpendicular to the girder's longitudinal axis, mainly appear at the top flange at haunched parts and develop downward with load increase. A few vertical flexure cracks initiating from the bottoms of two webs and developing upward can also be observed near mid-spans. A few diagonal shear cracks were observed to initiate from the bottom side of two webs and the cracks develop upward with 45° in zones close to mid-span and with 60° at the junction of HPW and SPW. The extent of these diagonal cracks is limited in the straight part of the two girders.

Contrary to the centric load case, significant influences of bottom slabs can be found among G1-GNT, G2-GWT and G3-BWT. G3-BWT had, on the top slab, the most parallel negative bending cracks of the three girders, while G2-GWT had more cracks than those of G1-GNT.

Unlike G4-GNF and G5-GWF, eccentric loads induced larger number of diagonal cracks mainly on the top slabs and loaded

webs in G1-GNT, G2-GWT and G3-BWT. The diagonal cracks appearing at loaded webs, due to combined action of bending, torsion and shear, were inclined with $40\text{--}50^\circ$ toward mid-spans. Torsional cracks, typically inclining at 45° from loaded webs toward unloaded webs, developed on the top flanges of G1-GNT, G2-GWT and G3-BWT at the load levels 55, 60 and 70 kN, respectively. The cracks start at the end of the haunched parts (one-fifth section), after the one-fifth section by about 1/2 girder depth toward mid-span, and before the one-fifth section by about 1/2 girder depth toward girder ends in G1-GNT, G2-GWT and G3-BWT, respectively.

For the unloaded SPW, the cracks mainly appear as vertical bending cracks; only a few diagonal cracks are found in this zone. Also, G2-GWT had larger number of cracks at unloaded web compared to G1-GNT. No observable cracks appeared at the unloaded HPW of G1-GNT, and only a few negative bending cracks extended from top slab of G2-GWT at high load levels. However, in G3-BWT, more flexural cracks extending to the unloaded HPW reflects more even stress distribution than in G1-GNT and G2-GWT. Hence, it could be said that bottom slabs increase the girders' torsional stiffness that results in more uniform stress distribution in the girder's cross-sections.

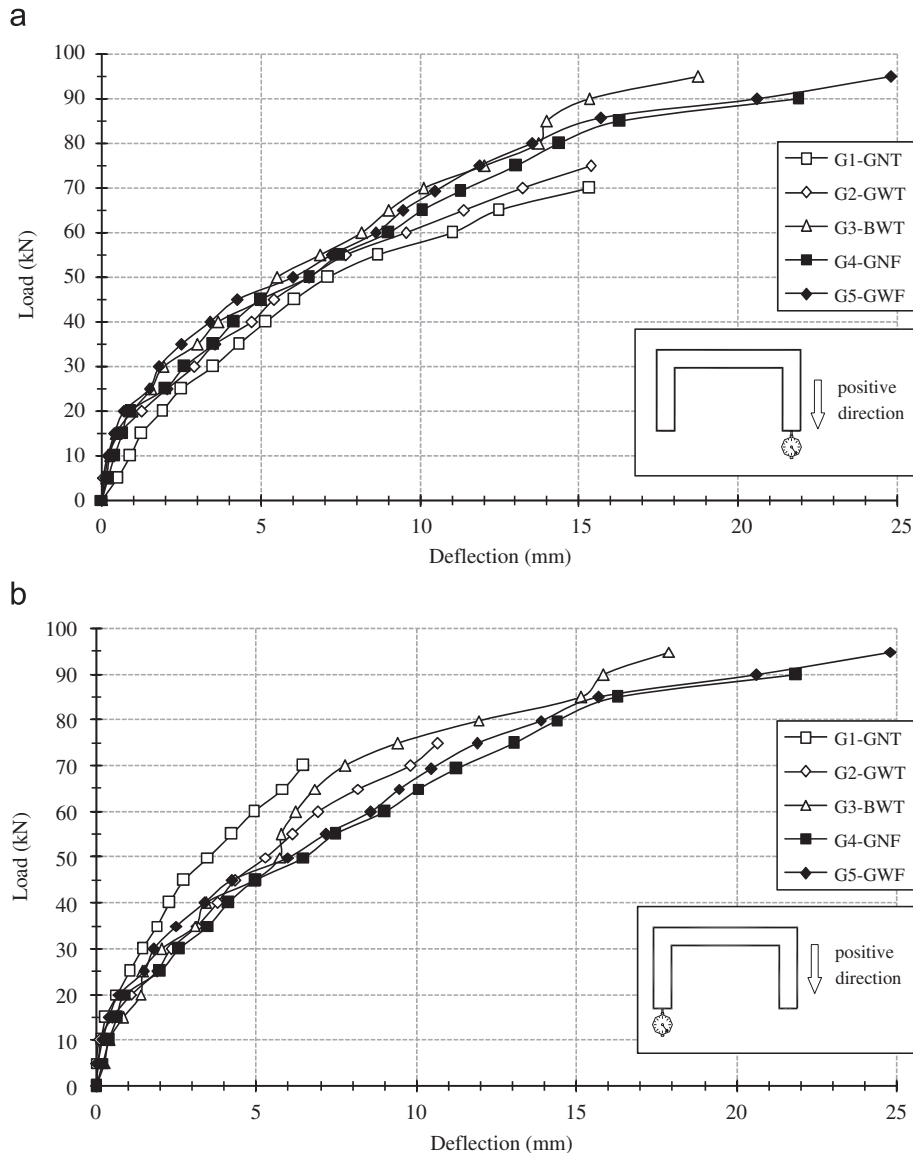


Fig. 6. Load-vertical deflection relationships at mid-span. (a) Under the loaded web and (b) under the unloaded web.

An identical failure mode can be found in G4-GNF and G5-GWF; that is the formation of three plastic hinges at straight parts that resulted in a collapse mechanism when the failure loads were reached: one of the three plastic hinges occurs at mid-spans and the other two appeared near the sections where the girder depth begins to change. Nevertheless, for a fixed-fixed box girder with a constant section, the plastic hinges are expected to appear at the mid-span and the two ends near the supports. The tests in this study shows that the haunches at two ends of the girder

shifted the two-ends plastic hinges towards the mid-span up to the intersection of the SPW and the HPW.

Failure modes of the three specimens also differed from each other. For G1-GNT and G2-GWT, it is found that the specimens fail by the shear mechanism rather than flexural mechanism. Diagonal crushing hinges appeared at the loaded side webs on G1-GNT and G2-GWT. Both G1-GNT and G2-GWT had an inclined crushing hinge on the loaded web near mid-spans. While the shear-yielding regions for G1-GNT started in the loaded

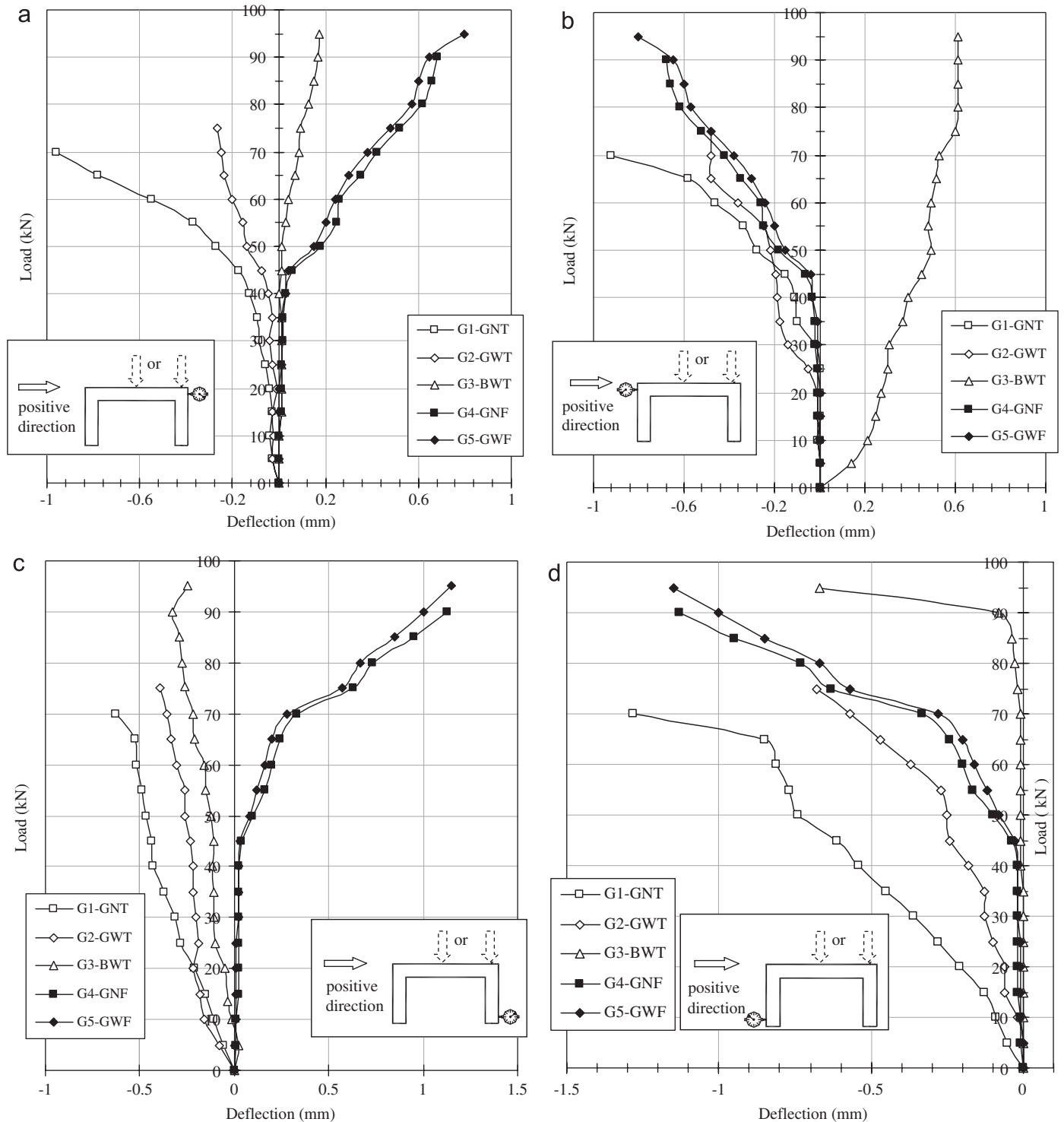


Fig. 7. Load-lateral deflection relationships at mid-span. (a) At the upper side of the loaded web, (b) at the upper side of the unloaded web, (c) at the lower side of the loaded web, and (d) at the lower side of the unloaded web.

HPW, the shear-yielding zones in G2-GWT were shifted to be in the SPW. This could be attributed to the torsional stiffening effect of the bottom slab in the HPW of G2-GWT. On the other hand, combined flexure, shear and torsion also caused shear yielding in G3-BWT, yet G3-BWT had a tendency towards a flexure-dominated failure mode due to the contribution of the full-length bottom slab.

2.2.3. Vertical and lateral deflection

Figs. 6(a) and (b) show the load versus vertical deflection relationships at mid-span section under the loaded and unloaded webs of the five tested girders, respectively. All five girders had a linear load–deflection response up to onset of the first cracks, after which a non-linear response was observed upon increasing the load; a significant increase in the vertical deflections was recorded, mainly due to the big reduction in stiffness caused by excessive cracking and yielding.

Comparing the vertical deflections of two girders of the same types, Types 1 and 2, but subjected to different load cases, it was found that the deflections of G1-GNT and G2-GWT on the loaded webs were higher than those of G4-GNF and G5-GWF, respec-

tively. In contrary, the deflections of G1-GNT and G2-GWT on the unloaded webs were lower than those of G4-GNF and G5-GWF, respectively. This indicates that the superposition of torsional displacement results in uneven deflections between the two webs.

For the centric load case, the load–deflection curves of G4-GNF and G5-GWF are almost identical and reflect that these two girders failed in approximately pure flexure-dominated failure mode with obvious horizontal plateaus after yielding. Compared to G4-GNF, it could be seen that the bottom slabs at haunches of G5-GWF resulted in a slightly better performance; less vertical deflection, slightly higher load-carrying capacity, and a longer plateau after yielding.

For the eccentric load case, bottom slabs significantly influenced the deformations of the girders. Compared with G1-GNT at the section of the mid-span, the deflection of G2-GWT at the loaded webs reduced 33% and 15% at load levels of 0.3 and 0.6 of the girders' failure loads, respectively. Similarly, due to the full-length bottom slab, the deflections of G3-BWT reduced 55% and 27% at load levels of 0.3 and 0.6 of the girders' failure loads, respectively. In contrary, for unloaded webs at mid-span, the

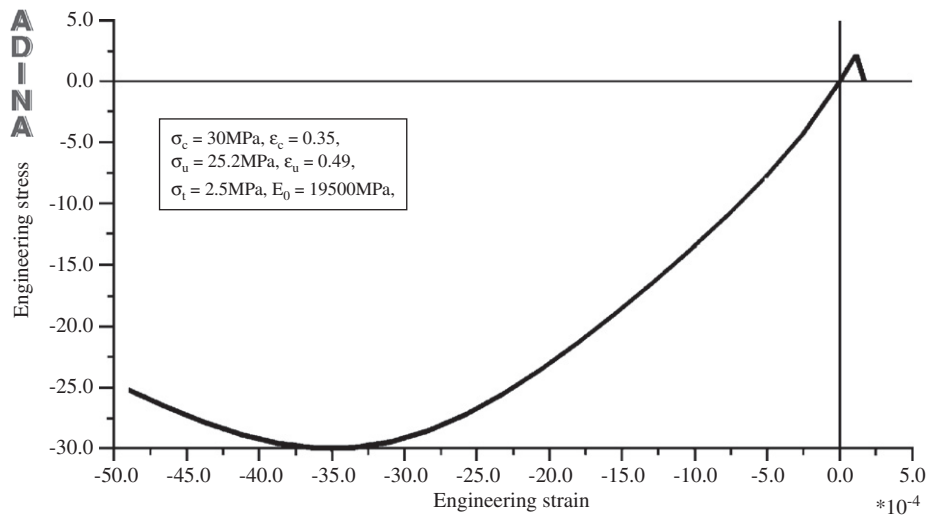


Fig. 8. Analytical stress–strain relationship of concrete [11].

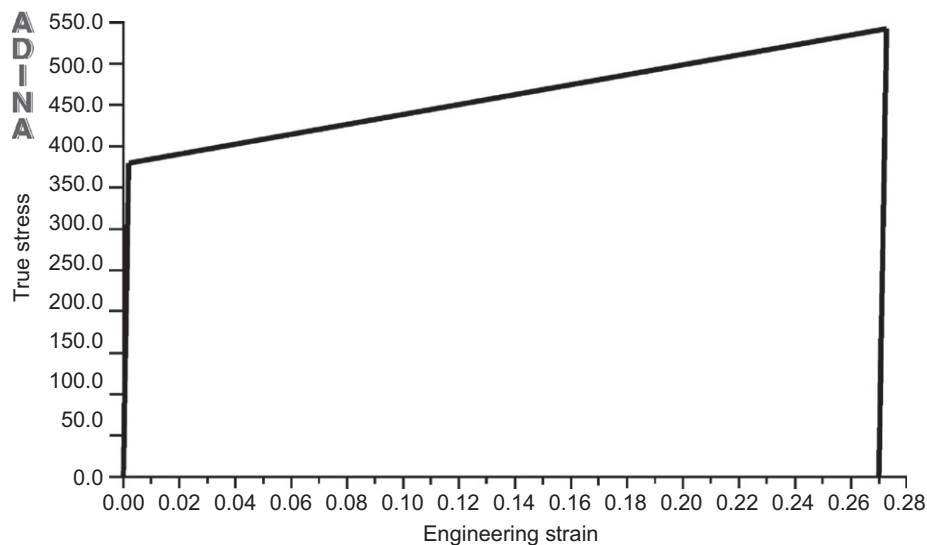


Fig. 9. Stress–strain relationship of steel rebars. (a) Stirrup rebars: $F_y = 380$ MPa, $E = 200,000$ MPa and (b) longitudinal rebars: $F_y = 340$ MPa, $E = 200,000$ MPa.

deflection in G1-GNT was relatively less than in G2-GWT and G3-BWT under same loading.

At high load levels (close to failure), it could be seen that the mid-span vertical deflections at loaded webs are higher than at unloaded webs by about 136%, 44% and 5% for G1-GNT, G2-GWT and G3-BWT, respectively. This means that there is less uneven deflection between the two webs in G2-GWT and in G3-BWT (compared to the case of G1-GNT). This improvement could be attributed to the increase in torsional stiffness caused by the bottom slabs of G2-GWT and G3-BWT.

From the load–deflection curves of G1-GNT and G2-GWT, it is noticed that no clear plateau can be observed, which indicates that eccentric loads caused brittle failure in G1-GNT and G2-GWT.

However, the curve of G3-BWT implies that the full-length bottom slab enables G3-BWT to tend to have a plastic failure even though the plateau in this curve is not as clear as G4-GNF and G5-GWF.

Figs. 7(a)–(d) show the mid-span load versus lateral deflection relationship at the upper and lower sides of the loaded and unloaded webs of the five girders. As expected, for centric load case, no obvious lateral deflection occurred in G4-GNF and G5-GWF before cracking. Only after the loading exceeded 45 kN, lateral deflection occurred, and the two webs were observed to deflect outwards. Also, G4-GNF and G5-GWF have identical load–lateral deflection curves, where the deflections in G5-GWF were slightly lower under each load level that could be attributed to the presence of bottom slabs.

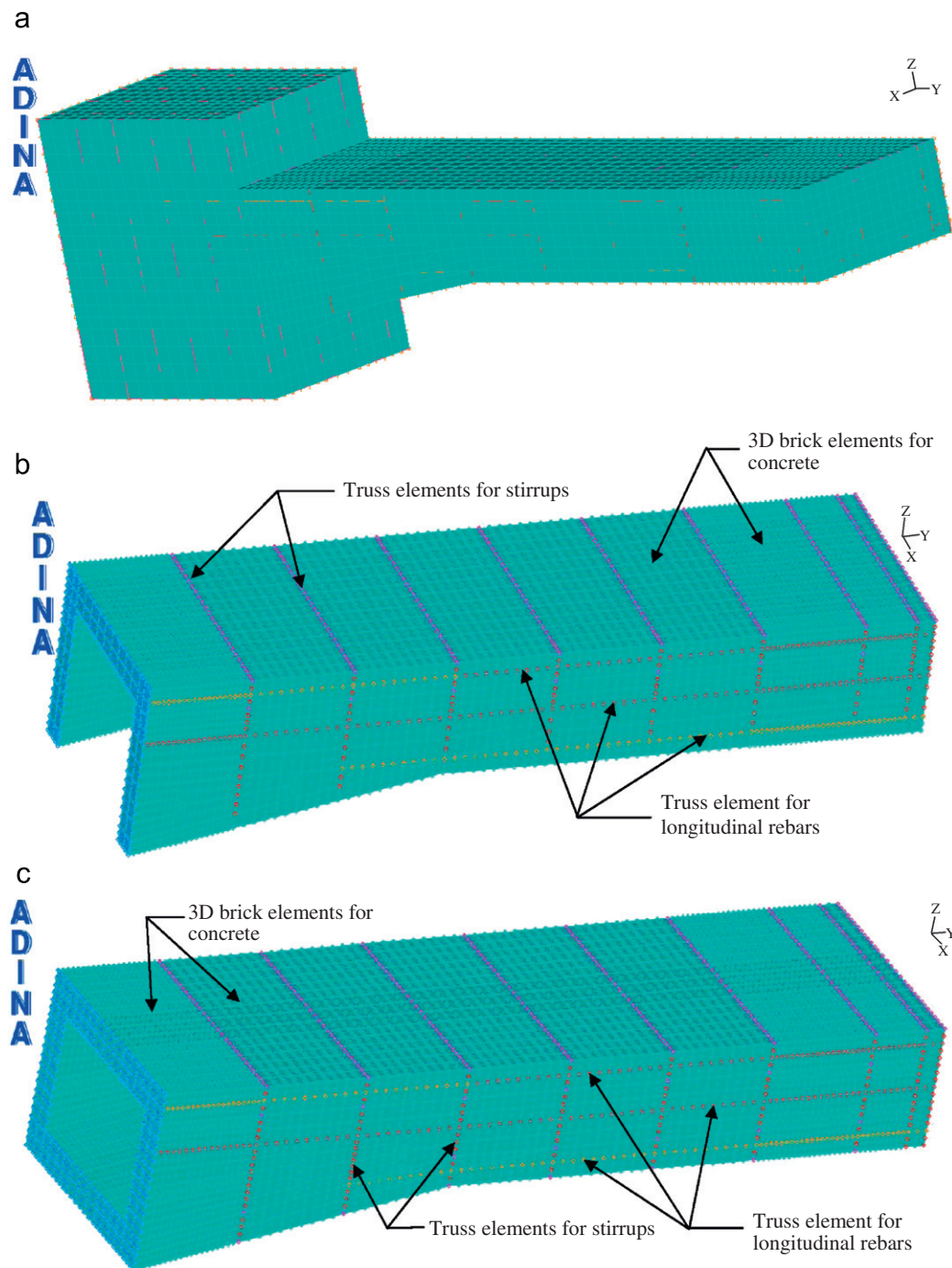


Fig. 10. Finite element model mesh using ADINA. (a) Model mesh with end-block; (b) G1-GNT and G4-GNF; and (c) G2-GWT, G3-BWT, G3'-BWF and G5-GWF.

For eccentric load case, the difference among the load-lateral deflection curves may be accounted for by the effects of bottom slabs. At the mid-span of G1-GNT and G2-GWT, it could be seen that the whole sections were observed to deflect laterally towards the unloaded side. The lateral deflections in G1-GNT and G2-GWT may be attributed to the shear centers moving away from the vertical axis of the section as a result of unsymmetrical cracking, as described earlier, of the two webs. Also, the upper edge of the section horizontally deflected more than the lower edge did. Moreover, upon loading, G2-GWT had less lateral deformation compared to G1-GNT, due to the presence of bottom slabs at haunched parts. However, the load-deflection curve of G3-BWT differs from the others, and a twisting deformation is observed: the upper sides horizontally move toward the loaded side, while the lower edges deflected toward opposite direction.

3. Numerical analysis

3.1. Non-linear finite element analysis model

For conducting the analytical phase of this study, the finite element computer program ADINA (2005) [10], which accommodates both geometrical and material non-linear analysis, was used. The choice of element geometry and material properties in this analysis was based on the literature review about non-linear FEA (finite element analysis) for RC structures and a careful review of the finite element types available in the rich library of the ADINA program [11–15]. ADINA has formulated a non-linear concrete material model that can be employed with 3D solid elements. In the current study, static non-linear analysis was adopted.

In the non-linear static analysis, Full Newton Iterations and Automatic Step Incrementation were selected. As the predominant source of non-linearities of RC beams under vertical loads results from material properties, rather than from large displacement/rotation, therefore, small displacement was selected in kinematic assumptions.

In order to simplify the model, the effects of bond slip and initial imperfections were not considered in modeling the girders, though these factors could have an effect on the experimental results.

3.1.1. Material models

The composite effects of the steel and concrete should be considered in the analysis of thin-walled RC girders. Both

materials, concrete and steel, experience different inelastic behaviors. The finite element approach applies two different ADINA material models, concrete material model and plastic-bilinear material model to simulate the different behaviors of concrete and steel rebar, respectively.

In ADINA, the concrete material model developed by Bathe et al. [11] offers the full range of linear and non-linear analysis. The concrete model represents basic concrete characteristics, such as tensile failure, compression crushing failure and strain softening from compression crushing failure to an ultimate strain, at which the material totally fails [10]. Material failures are governed by tensile failure (cracking) and compression crushing failure (crushing) envelopes [10]. For the failure surface, the constants of normal and shear stiffness reduction factors were taken as 0.0001 and 0.5, respectively. Constant Poisson's ratio was 0.167. Principal stress ratio was 0.75, and Kupfer failure envelope was specified. Fig. 8 shows the non-linear constitutive stress-strain relation of concrete material and the material properties.

The plastic-bilinear material model based on Von Mises yield condition is employed for steel reinforcement. Fig. 9 shows the stress-strain curve for the used steel. The properties of the bilinear model of the rebars were taken as those of the experimental specimens.

3.1.2. Finite element model

Although ADINA provides wide options for element types, most of them have severe limitations with regard to the structural arrangements and complex responses for thin-walled structures under eccentric loads. For thin-walled girders subjected to eccentric loads, 3D stress/strain state is required in the FE analysis due to the complex interactions of torsional warping, cross-section distortion, in addition to flexural and torsional actions. Only 3D solid elements could offer the most comprehensive alternative that could successfully represent the complex geometry and capture the requirement of 3D stress/strain conditions. As such, the 27 nodes 3D solid elements with concrete material properties proved to be most suitable for modeling concrete parts. High-order ($4 \times 4 \times 4$) Gauss integration is used. For better compatibility of the finite element mesh, three-node truss elements are employed to model longitudinal and transverse reinforcement, and the Gauss integration order is 4. Fig. 10 shows the meshed analytical models. In the model, the truss elements and 3D solid elements shared the same nodes. As such, the steel truss elements and concrete brick elements are perfectly connected, and bond slip was not considered.

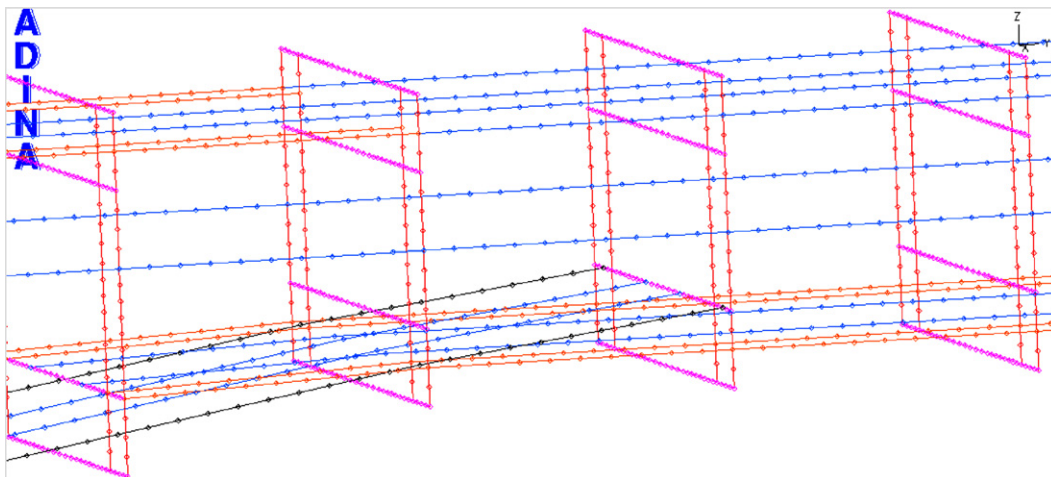


Fig. 11. Truss elements representing the steel rebars in the FE model.

Another advantage of using 3D solid elements is reflected on being able to model the exact geometrical details of the five tested girders. Additionally, in order to actually simulate the layout of steel rebar and concrete cover, two layers of geometric volumes are defined in each of the webs and flanges. All truss elements, which represent stirrups and longitudinal reinforcements are assigned at the boundaries of the volumes. As shown in Fig. 11, the truss elements representing the bottom longitudinal rebars of the haunched webs and straight webs, respectively, are placed at volume boundaries of the different layers to avoid joining at the same nodes. The truss elements coming from the two different directions only share same nodes with the concrete brick elements. The two lines of truss elements of the straight webs are attached at the boundaries of the two outside surfaces, while the bottom rebars of the haunched webs become one line of truss elements placed at the inside boundary. The section area of the

truss elements at the haunched webs equals the sum of section area of two rebars in the experimental specimens. Same strategy is used to model the longitudinal rebars of bottom flange in G3-BWT.

Due to symmetry in geometry, loading and support conditions in the longitudinal direction of the girders, only half of each tested girders was modeled. Also, based on engineering judgment and the objectives of the analysis, finer meshes are applied at the vicinities of the mid-span and the fixed-ends of the five girders, while the other regions are meshed coarsely to save on computational time.

The finite element models developed and used throughout this investigation simulate the five tested girders and their load cases as described in the above sections. For each tested specimen, the concentrated loads are applied on the four points of one volume. The load centers or load eccentric to the section centroids

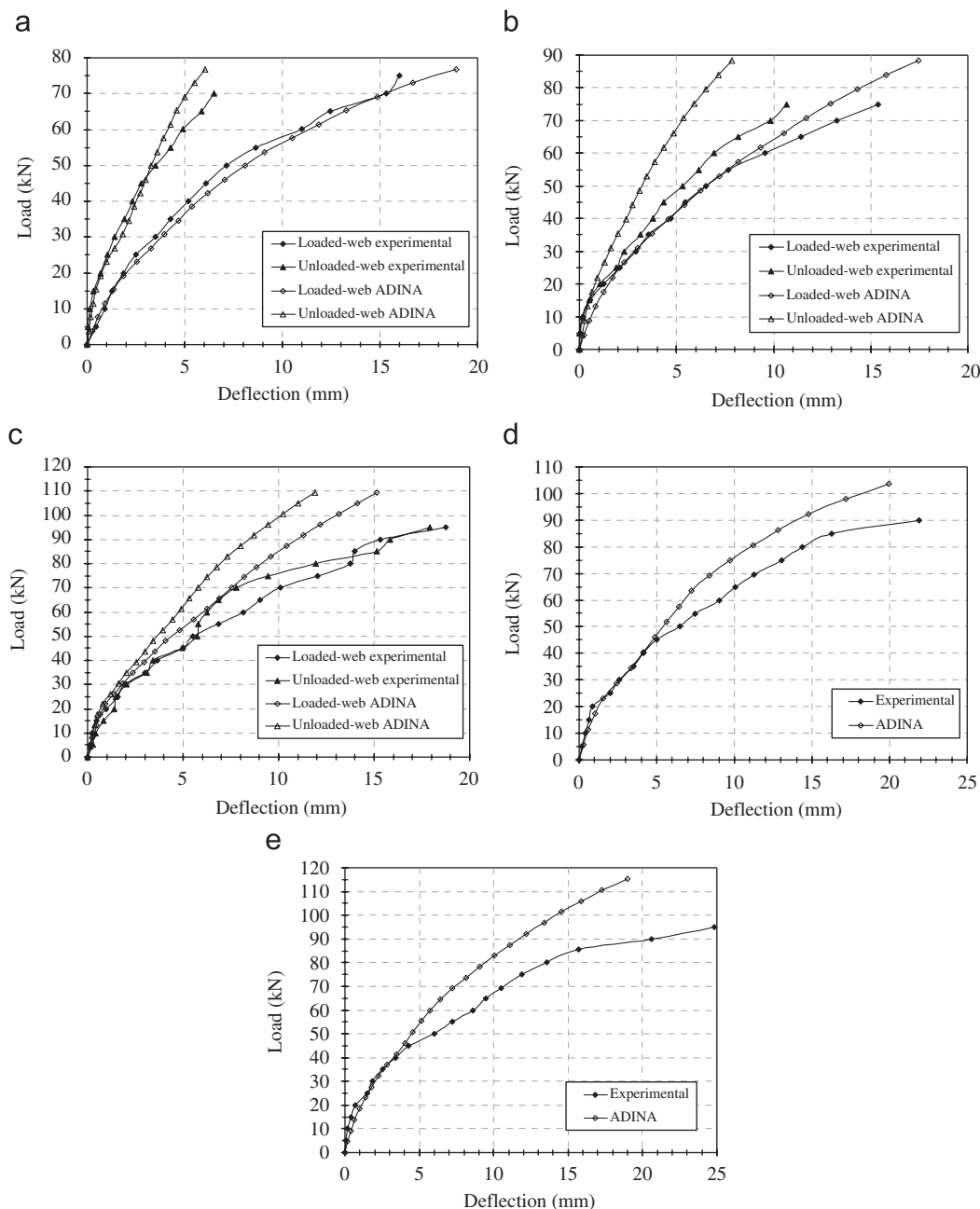


Fig. 12. Comparison between experimental and analytical load-vertical deflection relationships at mid-span. (a) G1-GNT, (b) G2-GWT, (c) G3-BWT, (d) G4-GNF, and (e) G5-GWF.

of the girders coincides with that of experimental tests. In addition, a full-length box section girder subjected to centric load, G3'-BWF, was also modeled to evaluate the relative performance of box girder under centric load, which was not evaluated experimentally.

The outputs of the finite element analysis and the comparison with experimental results are discussed in the following sections.

3.2. Analytical results and the comparison with experimental results

For each analysis performed, vertical deformations and steel strains at loaded and unloaded webs at mid-spans are calculated and saved. Cracking patterns and the deformation figures at the last load steps were also stored. All calculations are under same energy convergence criterion.

In order to accurately model the tested girders, the support conditions have to be carefully represented. Although the tested girders were designed to have fixed-end conditions at the test machine through the end-blocks and the clamps, it is obvious that it is difficult to completely avoid small rotations or displacements at the fixed supports during testing. Therefore, the two end-blocks were included in the finite element models, where the dimensions of the concrete blocks and the layouts of reinforcements were taken as those of the tested girders.

3.2.1. Failure loads and load–displacement relationship

The performances of FE models under respective load cases are checked against experimental observations. The comparisons of the load–displacement curves between numerical calculations and experimental results are shown in Fig. 12. As can be seen from the figure, the mid-span load-vertical displacement of the FE models showed more rigid response compared to the experimental measurements. This could be due to the fact that bond slip and initial imperfections were not represented in the FE models.

Through incremental loading of the girders, all specimens showed growth of cracks and increase in deflection and tensile strains in the rebars until they reach yield strain, followed by crushing of concrete in the top flanges at mid-span sections. Upon continuing the load increments on the girders, concrete crushing begins to occur at the negative moment region (i.e. at the bottom sides of the girders). At this stage, it is assumed that the failure load is reached. As shown in Fig. 12, a good agreement between the analytical and experimental failure loads is noticed.

Fig. 13 shows the load-vertical deflection curves calculated at the mid-spans for loaded and unloaded webs, respectively, of the six analyzed girders. As expected, the load eccentricities significantly reduced the stiffness and load-carrying capacity of the girders. Nevertheless, the stiffness and strength of girders were increased whenever the bottom slabs were added. From the figure, it could be seen that the full-length bottom slab (i.e. haunched box girder) greatly improved the girder's behavior under eccentric loads in terms of load-carrying capacity and deformations. However, for girders that were subjected to centric load, the effects of the bottom slabs are not as significant as those subjected to eccentric load.

Table 3 shows the analytical failure loads of the six analyzed girders. From the table, it could be seen that the load-carrying capacities of G2-GWT and G3-BWT are higher than that of G1-GNT by about 15% and 42%, respectively, due to adding bottom slabs. On the other hand, the increase in the load-carrying capacities of girders G5-GWF and G3'-BWF capered to G4-GNF is only about 12% and 18%, respectively. This result supports the experimental observations that a girder with

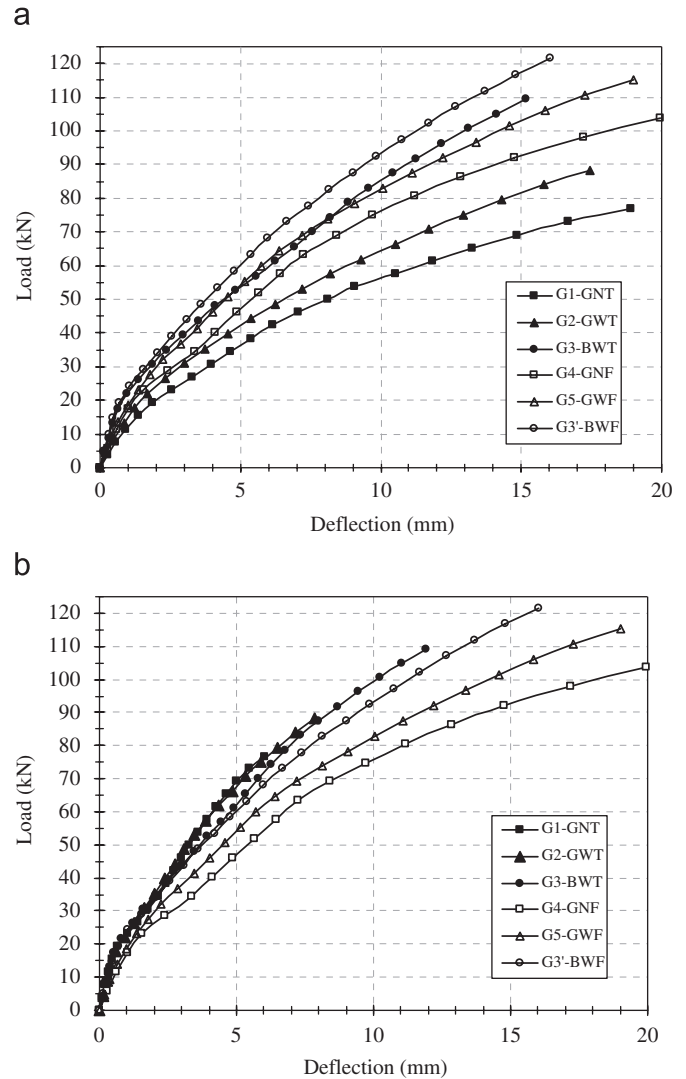


Fig. 13. Load-vertical deflection relationships at mid-span using ADINA. (a) under the loaded web and (b) under the unloaded web.

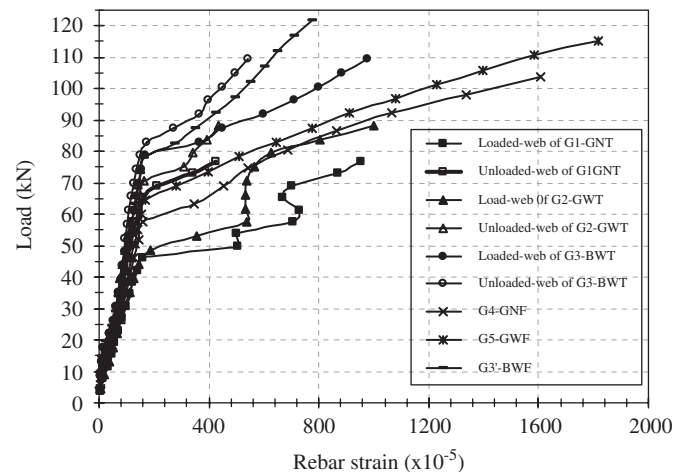


Fig. 14. Load-strain relationship of longitudinal tensile rebar at mid-span using ADINA.

whole length bottom slab is the most efficient structural type in resisting load eccentricities compared to girders with opening sections.

3.2.2. Steel strains

Fig. 14 shows the load–strain relationships of the longitudinal tensile rebars at the mid-span in loaded and unloaded webs, respectively. The loads corresponding to the yielding of the longitudinal rebars at mid-span are shown in Table 3. It can be seen that all the tensile reinforcements at mid-span already yielded before failure loads are reached. Though the tensile rebars in the unloaded webs of G1-GNT and G2-GWT have also yielded, their strain is much lower than that in loaded webs at high load levels. Furthermore, it can be seen that tensile rebars in the

loaded webs of G1-GNT and G2-GWT yielded at considerably lower load level compared to that of the unloaded webs. The other four models have relatively higher yield load. In G3-BWT, which has full-length bottom slab, the load–strain curves of the steel in loaded and unloaded webs have relatively closer response (compared to those of G1-GNT and G2-GWT) even though G3-BWT is subjected to eccentric loads. This could be attributed to the increased torsional rigidity of the box girder compared to that of double-tee girders with or without bottom slabs. From Fig. 14, it can be seen that the load–strain relationship of

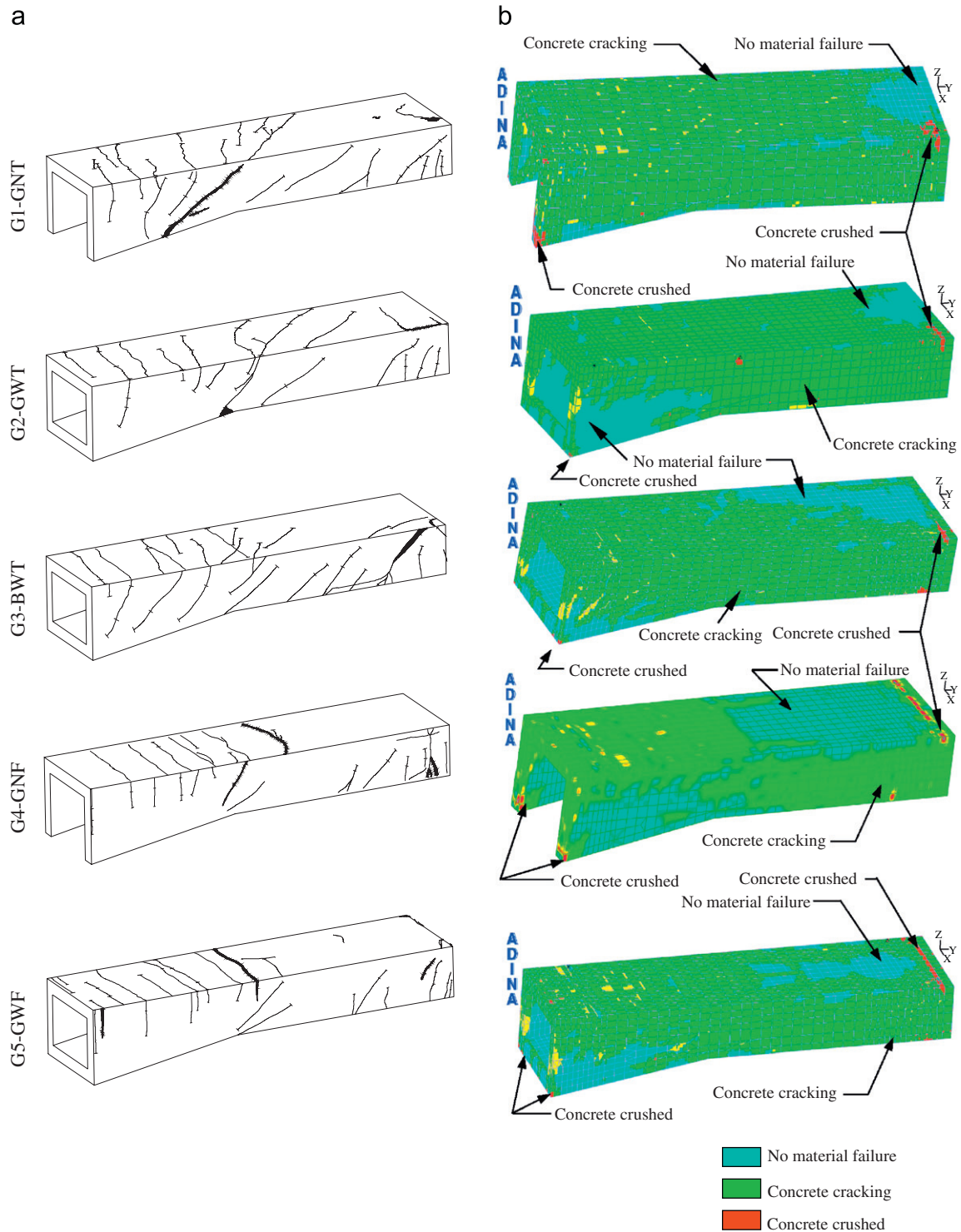


Fig. 15. Crack patterns from the FE models and tests at failure loads. (a) Experimental, (b) ADINA outputs.

longitudinal rebars at mid span of girder G3'-BWF had a response that is almost the average of those of the loaded and unloaded webs of G3-BWF.

3.2.3. Crack patterns and deformation figures

ADINA provides the capability to plot crack patterns and deformation figures. Figs. 15(b) and 16(b) show the crack patterns and deformation figures of five tested girders, respectively. Crack pattern of G3'-BWF was very close to that of G5-GWF at failure load, thus not shown for concision of the presentation.

One of the important outputs of 3D solid element on concrete material model is crack-flag that is a three-digit number (P1)(P2)(P3). Each digit represents a principal stress coordinate direction. The relationships of the three principal stresses are $\sigma_{P1} \geq \sigma_{P2} \geq \sigma_{P3}$. If the digit equals 0, there is no crack; if the digit equals 1, there is an open crack; if the crack flag equals 333, the material is crushed [10]. The crack plotting can be intuitively interpreted as follows: red color, corresponding to crack flag as 333, indicates the material is crushed; light green, corresponding to crack flag as 100, means the concrete crack; and cyan elements, corresponding to crack flag as 000, demonstrate there is neither

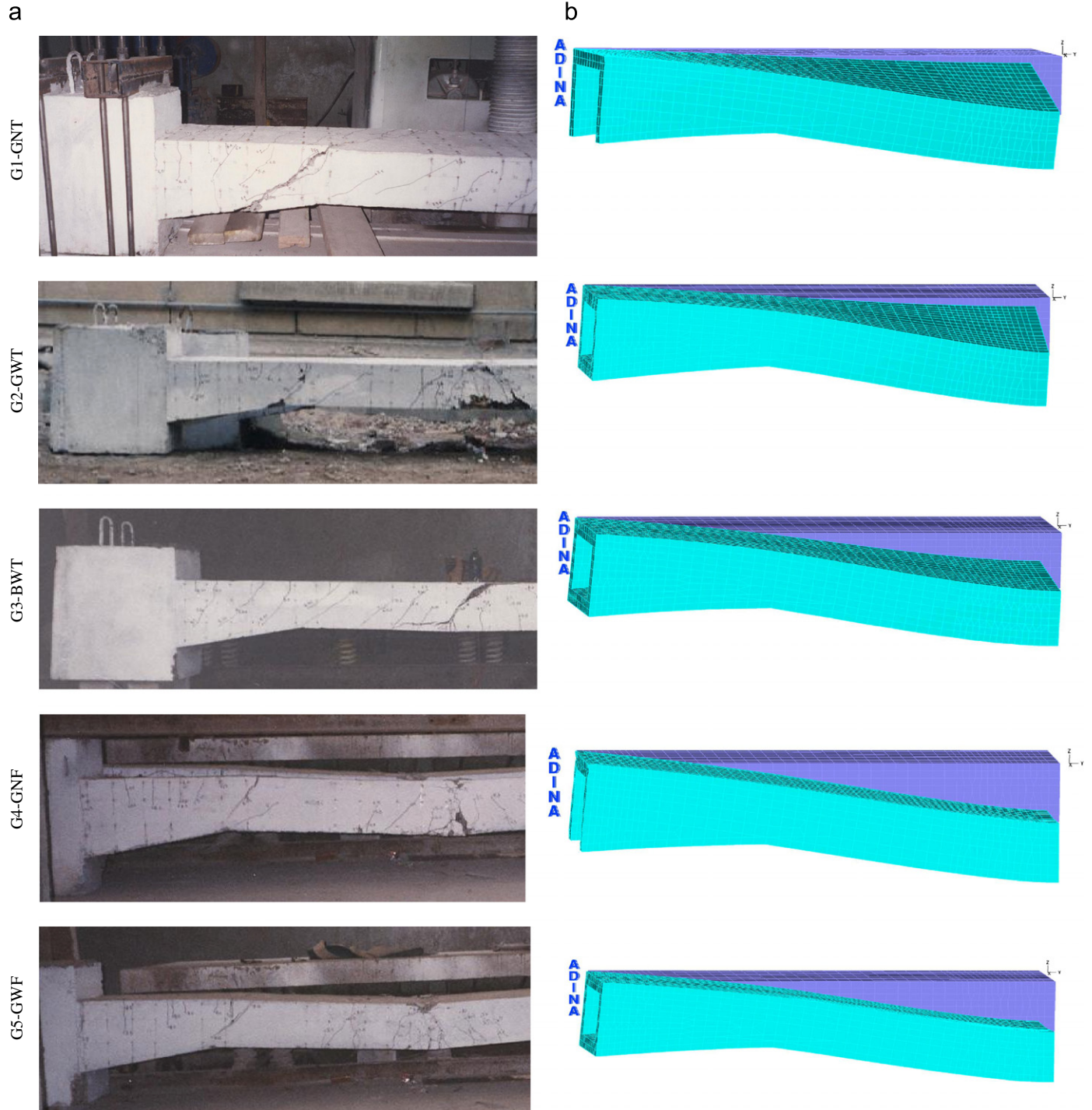


Fig. 16. Deformed shapes of the FE models and tests at failure loads. (a) Experimental and (b) ADINA outputs.

concrete cracking nor crushing. Fig. 15 shows the concrete cracking and crushing patterns at the failure stages of the models of the five tested girders.

Similar to the crack propagation observed during experimental tests, in the five FE models, for eccentric load cases, concrete cracking started from the bottom of the loaded webs, while, for centric load cases, cracking started at the bottom of two webs. The concrete crack extensions in the five FE models had a good agreement with the experimental observation. When failure loads were reached, compression concrete crushing occurred in the models. For G3'-BWF, G4-GNF and G5-GWF that were subjected to centric loads, the distribution of cracked and crushed elements, shown as green and red elements, respectively, and the extension of concrete cracks during the loaded processes were almost symmetrical with respect to the central lines of the sections. Unlike the centric loading cases, compression material crushing was only observed on the loaded sides, and the cracking patterns were asymmetric with respect to the central line of the sections. For G1-GNT, which had no bottom slab under eccentric load, few cracked elements appeared at the bottom of unloaded web even until failure loads were reached; for G2-GWT, which had bottom slab at haunched part under eccentric load, a few elements began to crack only when loads were close to the failure loads; however, for G3-BWT, which had box section along whole length also under eccentric load, the cracking load of the unloaded web was close to that of loaded web, and many cracks were developed when failure load was reached.

In addition, deformation figures, shown in Fig. 16, are also available in ADINA. Only flexural deformations were observed in the deformation responses of G3'-BWF, G4-GNF and G5-GWF. On the other hand, for the eccentric load cases, the components of deformation due to eccentric loading, i.e. flexure, torsion and distortion, can be observed from the deformation figures of the three girders, G1-GNT, G2-GWT and G3-BWT. Because of the restraint at the ends and structural symmetric, torsional warping displacements did not occur at the ends and mid-span. In the deformation of G3-BWT, higher contribution from the flexure mechanism can be seen compared to that of G1-GNT and G2-GWT.

4. Conclusions

An experimental and analytical investigation was conducted on thin-walled RC girders and box girders subjected to centric and eccentric concentrated load to evaluate the effects of load eccentricity and bottom slab length on their behavior. Five tests were conducted on medium-scale haunched RC girders and box girders with fixed end conditions, when subjected to increasing centric or eccentric loads up to failure. The tested girders were also modeled using ADINA, and a nonlinear finite element analysis was conducted. The following conclusions were drawn from the experimental and analytical investigations.

- (1) Subjecting the tested girders without bottom slabs and with bottom slabs at haunched parts to eccentric load, rather than centric load, decreased their observed initial cracking load by about 25% and 17% and decreased their load-carrying capacity by 16% and 8%, respectively.
- (2) Different failure mechanisms were observed for girders subjected to centric versus eccentric loads; centric loads resulted in flexure-dominated plastic failure, while eccentric

loads resulted in more contributions from the brittle shear and torsion mechanisms.

- (3) In general, it was observed that the presence of a bottom slab and its length significantly mitigates the impact of load eccentricity on the behavior of thin-walled RC haunched girders. The presence of bottom slab postpones the initial cracking, enhances load-carrying capacities, and reduces the difference in deformations between the loaded and unloaded webs.
- (4) On the other hand, introducing bottom slabs in the haunched part or along the full span had a relatively small effect on the behavior of girders subjected to centric loads.
- (5) Despite the complexity of the phenomenon, the models developed using ADINA were capable of capturing the nonlinear response of the tested girders. The models were able to predict the load-displacement, load-strain and crack propagation of the girders with good accuracy.

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