

Development of Fragility Curves for Reinforced-Masonry Structural Walls with Boundary Elements

Layane Hamzeh¹; Ahmed Ashour, M.ASCE²; and Khaled Galal, M.ASCE³

Abstract: Performance-based seismic design requires accurate damage/loss models for different seismic force resisting systems. Fragility functions are considered one of the most common damage/loss models that link specific demand parameter to the probability of exceedance of different damage states. Recently, reinforced masonry shear walls with boundary elements (RMSW + BEs) showed enhanced lateral performance and curvature ductility compared to that of rectangular walls. However, few experimental studies are available to date focusing on the seismic response of RMSW + BEs. FEMA guidelines provide fragility curves for reinforced masonry shear walls having only a rectangular cross section. Moreover, limited data are available to generate fragility curves for RMSW + BEs. In this paper, a numerical study is presented using the macromodeling approach embedded in the SeismoStruct software to simulate the in-plane lateral response of flexural dominated RMSW + BEs. The numerical model is validated against three RMSW + BEs walls tested under quasi-static cyclic loading reported in the literature. Subsequently, a parametric study is performed on 36 RMSW + BEs to evaluate the influence of varying the axial stress, wall aspect ratio, and vertical reinforcement ratio on the lateral response of the RMSW + BEs. It was observed that RMSW + BEs strength increases as the aspect ratio decreases, vertical reinforcement ratio increases, and level of axial stress increases. On the other hand, decreasing the wall aspect ratio, vertical reinforcement ratio, and level of axial stress enhances the RMSW + BEs displacement ductility. The drift ratios corresponding to three damage states (i.e., slight, moderate, and severe) were computed according to FEMA guidelines. Finally, fragility curves were generated for RMSW + BEs utilizing the computed numerical data of the studied walls, which can be adopted in performance-based seismic design frameworks. DOI: 10.1061/(ASCE)CF.1943-5509.0001174. © 2018 American Society of Civil Engineers.

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Introduction

In recent decades, notable developments have been achieved in understanding the seismic response of reinforced masonry rectangular shear walls (RMSW). This understanding is synchronous with the current emergence of performance-based seismic design. Performance-based design is an approach in which the structural design criteria are expressed in terms of achieving a set of performance objectives related to the level of damage [FEMA 445 (ATC 2006)]. This methodology relates the occurrence of specific damage states to the functionality or remediation costs when a structure is subjected to different levels of seismic events. For the seismic performance assessment of RMSW, FEMA 58-1 (ATC 2009a) provides a set of guidelines and procedures for evaluating the damageability of structural components using fragility functions.

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Fragility curves were introduced in the late 1980s [ATC-13 (ATC 1985); Hwang et al. 1987] in which a specific engineering demand (e.g., drift ratio) is related to the probability of exceedance of a specific damage state (Hwang and Jaw 1990). Such damage state is primarily connected to a method of repair which can be translated into repair cost.

Numerous studies have been conducted to study the nonlinear behavior of rectangular RMSW. However, few experimental and numerical studies investigated the seismic response of RMSW with end confinement, especially reinforced masonry shear walls with boundary elements (RMSW + BEs) (El-Dakhakhni and Ashour 2017). A number of early studies (e.g., Kaminosono et al. 1986; Sajjad 1990; Shing et al. 1993; Snook et al. 2005) focused on investigating the effect of confining the RMSW ends using different techniques on the lateral response of RMSW. Shedid et al. (2010) tested seven half-scale fully grouted RMSW and investigated the performance of walls with three different end configurations: rectangular, flanged, and confined boundary elements. The study showed that there is an enhancement in ductility and ultimate displacements for flanged and end confined walls in comparison to rectangular walls subjected to the same level of axial stress. Banting and El-Dakhakhni (2012, 2014) tested nine half-scale fully grouted RMSW + BEs with different design parameters such as aspect ratio, height, length, and reinforcement ratio. The study highlighted the fact that the presence of boundary elements (BE) in the walls delayed the buckling of the reinforcement as well as crushing of the grout. The enhancement of the wall response on the component level was also observed at the system level by testing a one-third scale building consisting of RMSW + BEs walls by Ezzeldin et al. (2017a). Furthermore, Ezzeldin et al. (2016) evaluated the seismic

collapse risk of numerically modeled RMSW + BEs adopting the performance factors of rectangular RMSW using the FEMA P-695 (ATC 2009b) methodology. The study concluded that RMSW + BEs experienced an enhanced performance that is enough to meet the FEMA P-695 acceptance criteria for the expected seismic collapse risk under the maximum considered earthquake (MCE) that the rectangular RMSW could not achieve. Moreover, Ezzeldin et al. (2017b) presented a methodology to generate system-level fragility curves for RMSW + BEs and applied it on four-story building. These research efforts led to the adoption of ductile RMSWs with confined boundary elements in the most recent CSA S304-14 (CSA 2014) design standard, *Design of Masonry Structures*, as a valid seismic force resisting system.

It can be inferred from the preceding literature that further experimental and analytical work is needed toward investigating the effect of different design parameters on the seismic response of RMSW + BEs. Experimental tests provide comprehensive understanding on the lateral response and damage pattern of masonry walls and systems tested under simulated seismic loading. However, experiments are usually costly and require extensive facilities and resources. Accordingly, validated and reliable numerical modeling can be one of the effective solutions to investigate the performance of RMSW + BEs having various design parameters. Predicting the inelastic lateral response of RMSW + BEs is challenging and requires accurate numerical modeling tools that incorporate relevant material and geometric properties. The two main approaches for modeling RMSW are micromodeling and macromodeling (Lourenço et al. 1998). Micromodeling is based on discretization of a structure into a finite number of small elements interconnected at a finite number of nodes. Macromodeling is based on representing the overall structure with larger elements, each of which has properties that are equivalent to the sum of its components. Although micromodeling has the ability to capture the local response of the RMSW + BEs constituent, it is a complex approach that needs a high level of computational effort. Conversely, macromodeling does not require the same level of detailed discretization used for micromodeling and has the ability to capture the overall wall response.

The objective of this paper is to develop fragility curves for RMSW + BEs having different design parameters subjected to quasi-static cyclic loading. This paper presents a two-dimensional numerical model in SeismoStruct, using fiber-based beam-column elements, to simulate the behavior of RMSW + BEs under quasi-static cyclic loading. Subsequently, data from experimental test results of three RMSW + BEs reported by Shedid et al. (2010) and Banting and El-Dakhkhni (2014) were used to validate the numerical model. Following the model validation, a parametric study was performed for 36 RMSW + BEs having different design parameters including aspect ratio, vertical reinforcement ratios in BE, and levels of axial compression stress. Results from the parametric study were then used to develop fragility curves for seismic performance assessment of RMSW + BEs according to the methodology set out by FEMA 58-1 (ATC 2009a).

Numerical Model Details

Model Overview

SeismoStruct (Seismosoft 2014), a finite-element package that uses a fiber modeling approach, is used in the current study to model the seismic response of RMSW + BEs. The walls were modeled using displacement-based (DB), inelastic beam-column elements to simulate the inelastic flexural response of the RMSW + BEs under

fully reversed cyclic loading. The DB beam-column elements are characterized by a constant axial deformation and linear curvature distribution along the element length. As such, from the nonlinear uniaxial stress-strain response of the individual fibers, the sectional moment-curvature state is obtained through the integration of the entire number of fibers (Rodrigues et al. 2012). This is to fully account for the spread of inelasticity along the section length and across the depth. However, it should be noted that the fiber-based elements in SeismoStruct do not account for the shear deformations. Therefore, all the walls considered in the current study have a shear span-to-depth ratio greater than or equal to 1.5 and are designed to fail in a flexural mode in order to minimize the shear deformation contribution to the overall RMSW + BEs response.

Wall Details

Thirty-six fully grouted RMSW + BEs were modeled using SeismoStruct software according to the details provided in Table 1. The walls were designed according to the CSA S304-14 (CSA 2014) design standards requirements. Numerical wall models are used for the assessment of different parameters on the load-displacement response of RMSW + BEs. The parameters under study are aspect ratio, axial compressive stress, and vertical reinforcement ratio in BE. All walls were detailed with the same vertical reinforcement in the web of four 20M bars [area of vertical reinforcement, $A_v = 300 \text{ mm}^2$] spaced at 800 mm and horizontal reinforcement of 10M bars [area of horizontal reinforcement, $A_h = 100 \text{ mm}^2$] spaced at 200 mm. As indicated in Table 1, different aspect ratios, H_w/L_w , varying from 1.5 to 3.75 have been considered in the current study. Fig. 1 shows the cross sections of the walls under study with different vertical reinforcements in BE. Three vertical reinforcement ratios in BE, $\rho_{v(\text{BE})}$ (0.79, 1.18, and 1.58%) were also utilized to investigate their effect on the load-displacement response of RMSW + BEs. Moreover, three axial stress levels were applied to the RMSW + BEs to represent the range of compressive stresses found in practice.

Element Size and Boundary Conditions

Fig. 2 shows a schematic diagram of the developed wall model, including the distribution of nodes and elements. The wall was divided into four elements based on recommendations of Calabrese et al. (2010) that a good approximation to the response can be obtained with a mesh discretization of at least four elements. To regularize the response of DB elements, it was also suggested by Calabrese et al. (2010) that localization of strain in the DB elements occurs in the most strained integration point, and not in a single element as typically used. Thus, to ensure the correct regularization, the weight of the most strained integration point has to be equal to the plastic hinge length, L_p . The DB elements used have two integration points per element, and each point has half the weight. Hence the regularized length of the first element should be twice L_p . Several formulas are available in the literature to estimate L_p of shear walls (Paulay and Priestley 1992; Priestley et al. 2007; Bohl and Adebar 2011). The formula proposed by Bohl and Adebar (2011), which is based on nonlinear finite-element analysis results of 22 reinforced concrete shear walls, was found to give the closest estimate of the plastic hinge length, L_p , for RMSW + BEs (Ezzeldin et al. 2016). Therefore, the L_p is calculated as proposed by Bohl and Adebar (2011) as shown in Eq. (1), whereas this equation gives a lower bound estimate of plastic hinge length.

Table 1. Test matrix and wall details

Wall identifier	Length (m)	Height (m)	H_w/L_w	Vertical reinforcement in BE		Axial compressive stress per story (MPa)	Total axial stress (MPa)
				Number and size	$\rho_{v(BE)}$ (%)		
W1	4	6	1.5	4-20M	0.79	0	0
W2	4	9	2.25	4-20M	0.79	0	0
W3	4	12	3	4-20M	0.79	0	0
W4	4	15	3.75	4-20M	0.79	0	0
W5	4	6	1.5	6-20M	1.18	0	0
W6	4	9	2.25	6-20M	1.18	0	0
W7	4	12	3	6-20M	1.18	0	0
W8	4	15	3.75	6-20M	1.18	0	0
W9	4	6	1.5	8-20M	1.58	0	0
W10	4	9	2.25	8-20M	1.58	0	0
W11	4	12	3	8-20M	1.58	0	0
W12	4	15	3.75	8-20M	1.58	0	0
W13	4	6	1.5	4-20M	0.79	0.15	0.3
W14	4	9	2.25	4-20M	0.79	0.15	0.45
W15	4	12	3	4-20M	0.79	0.15	0.6
W16	4	15	3.75	4-20M	0.79	0.15	0.75
W17	4	6	1.5	6-20M	1.18	0.15	0.3
W18	4	9	2.25	6-20M	1.18	0.15	0.45
W19	4	12	3	6-20M	1.18	0.15	0.6
W20	4	15	3.75	6-20M	1.18	0.15	0.75
W21	4	6	1.5	8-20M	1.58	0.15	0.3
W22	4	9	2.25	8-20M	1.58	0.15	0.45
W23	4	12	3	8-20M	1.58	0.15	0.6
W24	4	15	3.75	8-20M	1.58	0.15	0.75
W25	4	6	1.5	4-20M	0.79	0.3	0.6
W26	4	9	2.25	4-20M	0.79	0.3	0.9
W27	4	12	3	4-20M	0.79	0.3	1.2
W28	4	15	3.75	4-20M	0.79	0.3	1.5
W29	4	6	1.5	6-20M	1.18	0.3	0.6
W30	4	9	2.25	6-20M	1.18	0.3	0.9
W31	4	12	3	6-20M	1.18	0.3	1.2
W32	4	15	3.75	6-20M	1.18	0.3	1.5
W33	4	6	1.5	8-20M	1.58	0.3	0.6
W34	4	9	2.25	8-20M	1.58	0.3	0.9
W35	4	12	3	8-20M	1.58	0.3	1.2
W36	4	15	3.75	8-20M	1.58	0.3	1.5

$$L_p = (0.2L_w + 0.05z) \left(1 - \frac{1.5P}{f'_c A_g} \right) \leq 0.8L_w \quad (1)$$

where L_w = wall length; z = moment shear ratio; A_g = gross area of the wall cross section; f'_c = concrete compressive strength (i.e., f'_m will be used instead for masonry walls); and P = axial force on the section.

Fiber discretization is adopted to represent the behavior at the section level, where each fiber is associated with a uniaxial stress-strain law. A sufficient number, around 150–400 fibers, can lead to good estimates of the load-displacement hysteresis response according to (Boulanger et al. 2013). Consequently, based on a sensitivity analysis, the RMSW + BEs sections were divided into 300 fibers. The RMSW + BEs considered in the current study were modeled as cantilevers, and thus, this was represented in the model by perfectly fixing the base degree of freedoms. All walls were subjected to the same loading protocol shown in Fig. 3, which is similar to the one implemented by Banting and El-Dakhkhni (2014) and Shedi et al. (2010). Horizontal displacements were applied at the top of each wall at increasing multiples of yield displacement until severe flexural damage state was reached. Severe flexural damage state was defined as the point at which the lateral resistance of the wall reached 20% strength degradation from the peak resistance as recommended by FEMA 58-1 (ATC 2009a).

Material Models and Mechanical Properties

The consideration of nonlinear material behavior in the prediction of RMSW + BEs requires accurate modeling of the uniaxial material stress-strain cyclic response. Table 2 shows the mechanical material properties of steel and masonry used in the numerical modeling of the walls. The masonry was modeled using the Mander et al. (1988) nonlinear model for concrete (i.e., con_ma in SeismoStruct). This is a uniaxial nonlinear constant confinement model that follows the constitutive relationship proposed by Mander et al. (1988) and the cyclic rules proposed by Martínez-Rueda and Elnashai (1997). The confinement effects provided by the lateral transverse reinforcement are incorporated through the rules suggested by Mander et al. (1988) whereby constant confining pressure is assumed throughout the entire stress-strain range. Therefore, the core area enclosed by the vertical bars and the transverse hoops were assigned a confined material property. However, the RMSW web and the BE cover were assigned an unconfined material property. The input parameters of the model for the masonry are the compressive strength, f'_m ; the strain at peak strength, ϵ_m ; and the modulus of elasticity, E_m . The elastic modulus, E_m , was calculated according to CSA S304-14 (CSA 2014) recommendation as $850 f'_m$, in which f'_m is the masonry compressive strength.

The steel reinforcement was modeled using the Menegotto and Pinto (1973) nonlinear steel model (stl_mp in SeismoStruct). This

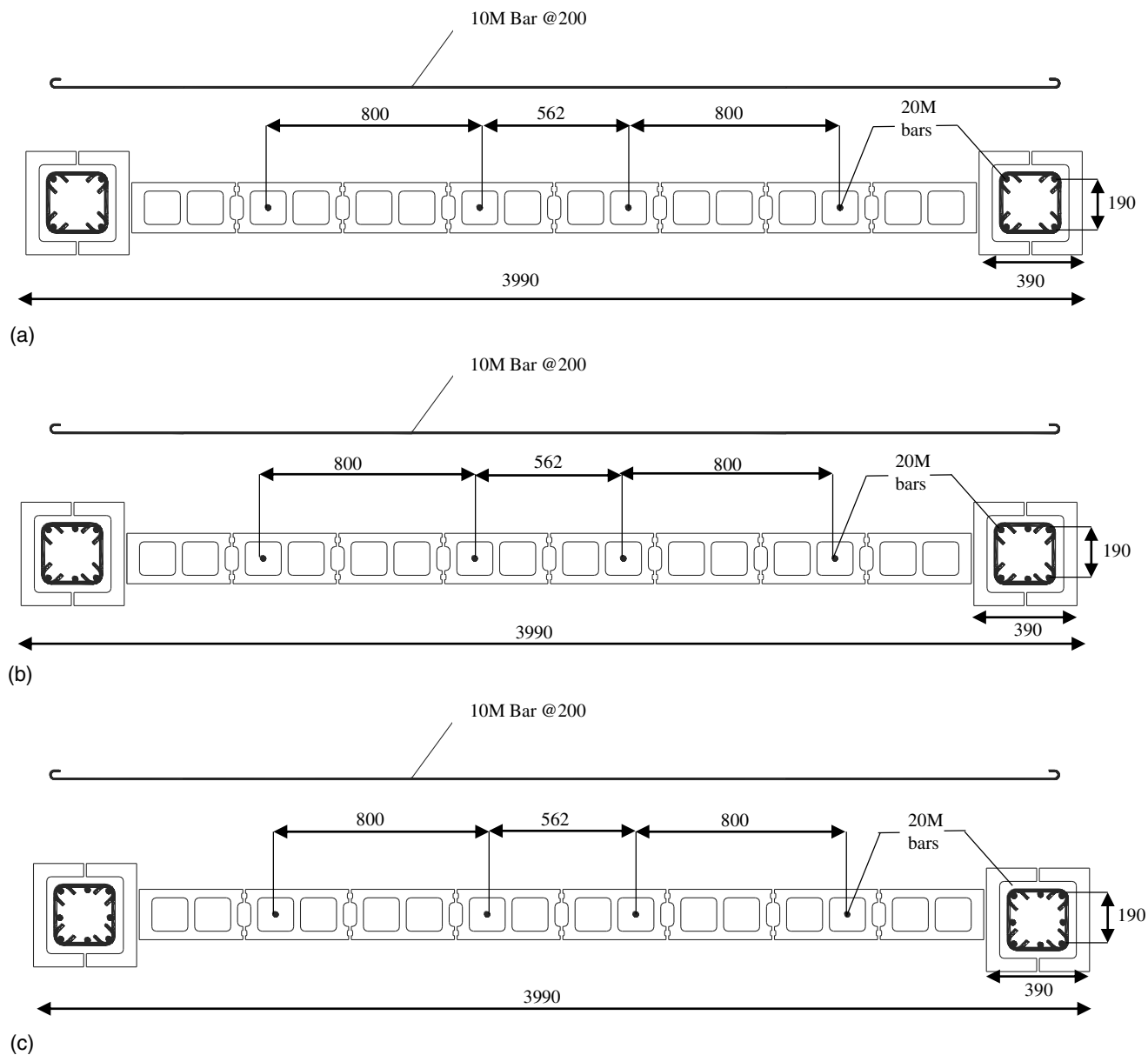


Fig. 1. Cross section of RMSW + BEs: (a) 4 bars in BE; (b) 6 bars in BE; and (c) 8 bars in BE (all dimensions in millimeters).

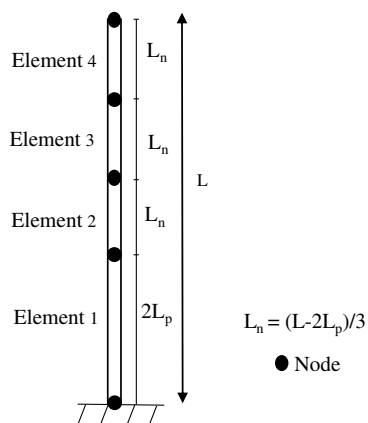


Fig. 2. Schematic diagram of wall numerical model.

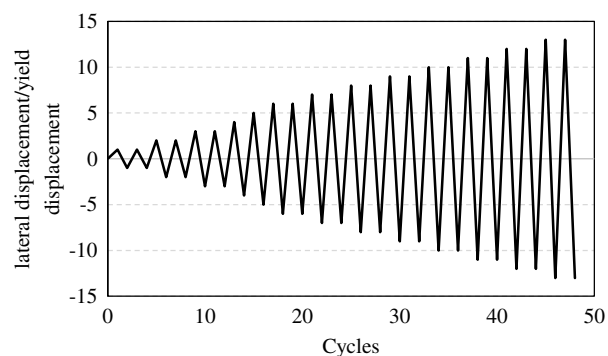


Fig. 3. Sample of the loading protocol for Wall 1.

Table 2. Material mechanical properties utilized in the numerical models

Parameter	Value
Compressive strength, f'_m (MPa)	17
Elasticity modulus of masonry, E_m (MPa)	14,850
Strain at peak strength, ϵ_c	0.0015
Elasticity modulus of steel, E_s (MPa)	200,000
Yield strength, f_y (MPa)	400
Strain hardening parameter, r (%)	0.005
Transition curve initial shape, R_0	18.8
Transition curve shape	
a_1	18.5
a_2	0.3
Isotropic hardening	
a_3	0
a_4	1

model is a uniaxial steel model proposed by Menegotto and Pinto (1973) coupled with the isotropic hardening rules proposed by Filippou et al. (1983). The input parameters are the elastic Young modulus, E_s , the yield strength, f_y , the strain hardening ratio, r , and five coefficients representing the transition from elastic to plastic zone R_0 , a_1 , a_2 , a_3 , and a_4 (Table 2).

Model Validation

Prior to the parametric study assessment, model predictions were validated using experimental data of half-scale RMSW + BEs. Fig. 4 shows the cross sections of the RMSW + BEs used in the model validation adopted from Banting and El-Dakhkhni (2014) and Shedid et al. (2010). Three RMSW + BEs specimens

experimentally tested under fully reversed displacement controlled quasi-static cyclic loading were used for the numerical model validation. The selected RMSW + BEs specimens have different aspect ratios, ranging from 1.48 to 3.23. Table 3 summarizes the various design aspects of the three RMSW + BEs used for the model validation, showing the wall dimensions, axial stress level, and vertical and horizontal reinforcement details. The vertical and horizontal reinforcement ratios (ρ_v and ρ_h , respectively) are defined as ratios of the areas of reinforcing bars to gross area of the horizontal or vertical masonry cross section, respectively. Wall VW1 adopted from Banting and El-Dakhkhni (2014) had length of 1,235 mm; height of 3,990 mm; vertical reinforcement ratio, ρ_v , of 0.69%; and horizontal reinforcement of 0.3%. Walls VW2 and VW3 adopted from Shedid et al. (2010) had length of 1,800 mm and were each detailed with ρ_v of 0.55%, but varied in horizontal reinforcement with values of 0.3 and 0.6% and heights of 3,990 and 2,660 mm, respectively.

The material mechanical properties reported by Banting and El-Dakhkhni (2014) and Shedid et al. (2010) were used in the model validation. The average compressive strength of the masonry prisms based on the experimental results was reported as 15 MPa for Wall VW1 and 16.4 MPa for Walls VW2 and VW3. Vertical steel reinforcement consisting of 10M bars (area of vertical reinforcement, $A_v = 100 \text{ mm}^2$; and nominal diameter of bar, $d_b = 11 \text{ mm}$) with a yield strength of 496 MPa and Young's modulus of 200.6 GPa was utilized for all the walls. The horizontal reinforcement and stirrups were comprised of deformed wire, D4, (area of horizontal reinforcement, $A_h = 25.4 \text{ mm}^2$; and nominal diameter of bar, $d_b = 5.7 \text{ mm}$) with a proof yield strength of 582.5 MPa for Wall VW1 and 534 MPa for Walls VW2 and VW3. As discussed previously, the steel reinforcement was modeled using the Menegotto and Pinto (1973) nonlinear steel model, and the

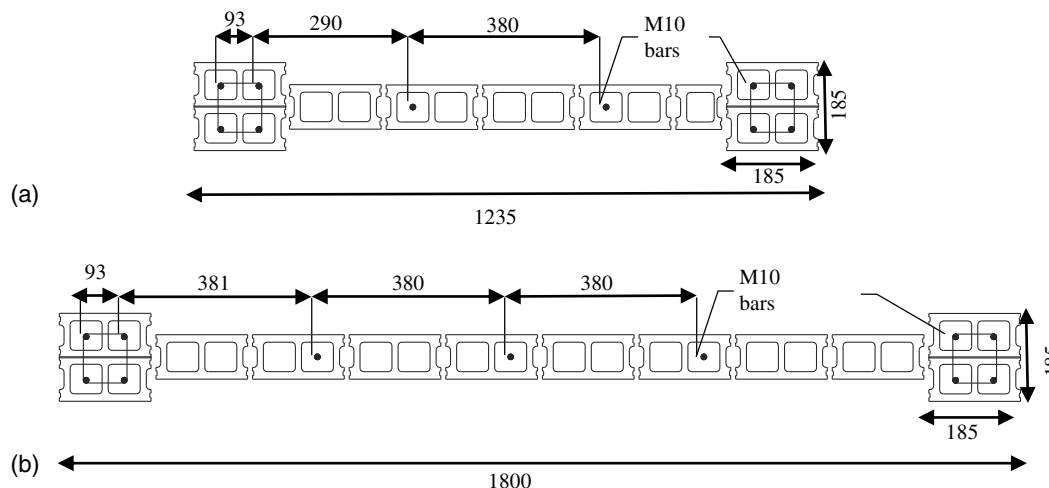


Fig. 4. Cross section of walls utilized to validate the numerical model: (a) VW1; (b) VW2; and VW3. (Data from Banting and El-Dakhkhni 2014; Shedid et al. 2010.)

Table 3. Summary of wall details used for the model validation

Wall identified	Length (mm)	Height (mm)	Vertical reinforcement		Horizontal reinforcement		Aspect ratio	Axial stress (MPa)	References
			Bar numbers and size	ρ_v (%)	Bar numbers and spacing (mm)	ρ_h (%)			
VW1	1,235	3,990	10 M10	0.69	1 @ 95	0.3	3.23	0.89	Banting and El-Dakhkhni (2014)
VW2	1,800	3,990	11 M10	0.55	1 @ 95	0.3	2.21	0.89	Shedid et al. (2010)
VW3	1,800	2,660	11 M10	0.55	2 @ 95	0.6	1.48	0.89	Shedid et al. (2010)

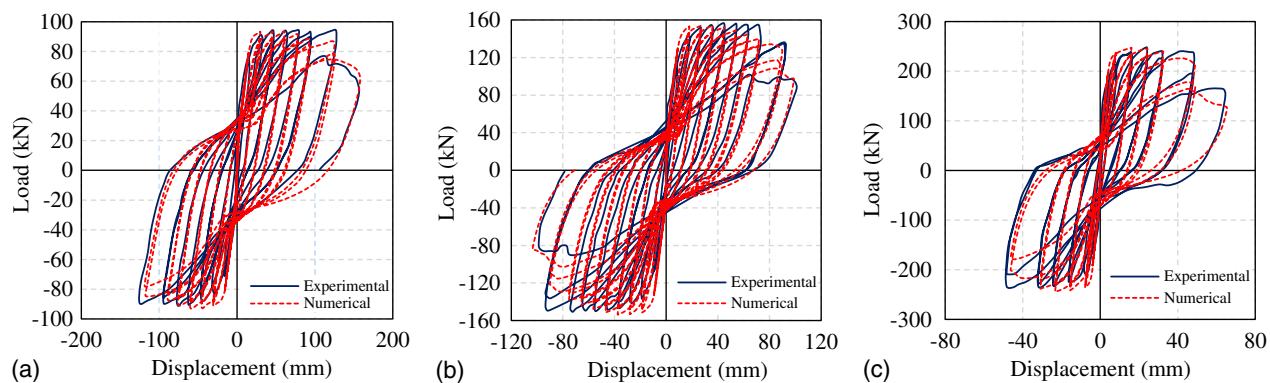


Fig. 5. Experimental versus numerical hysteresis loops: (a) VW1 (experimental data from Banting and El-Dakhkhni 2014); (b) VW2; and (c) VW3 (experimental data from Sheded et al. 2010).

Table 4. Validation of model predictions with the experimental data

Wall identifier	Load direction	Experimental				Numerical				Percentage error			
		Q_y (kN)	Q_u (kN)	$Q_{0.8u}$ (kN)	$\mu_{\Delta u}$	Q_y (kN)	Q_u (kN)	$Q_{0.8u}$ (kN)	$\mu_{\Delta u}$	Q_y (%)	Q_u (%)	$Q_{0.8u}$ (%)	$\mu_{\Delta u}$ (%)
VW1	(+)ve (push)	76.4	94.1	76	3	81.3	93.3	74.6	2.9	7.7	0.9	1.8	3.3
	(-)ve (pull)	70.9	88	70.4	2.7	78	92.1	73.7	2.9	10	4.7	4.7	7.4
VW2	(+)ve (push)	112	151	123	4.3	120.9	154	123.2	4	7.9	2	0.2	7
	(-)ve (pull)	109	147	121	4	115.5	151	120.8	3.8	5.5	2.7	0.2	5
VW3	(+)ve (push)	174	247	194	5.9	195	245.5	196.4	5.7	12.7	0.6	1.2	3.4
	(-)ve (pull)	170	232	190	6.2	190	243.9	195.1	5.9	12.4	5.1	2.7	4.8

confined and unconfined masonry was modeled using the Mander et al. (1988) stress-strain model.

The computed load-displacement hysteresis of the three RMSW + BEs was validated against the experimental results reported by Banting and El-Dakhkhni (2014) and Sheded et al. (2010). Fig. 5 shows good agreement between the experimental hysteresis loops and the corresponding computed numerical loops. The model was able to simulate the most relevant characteristics of the cyclic response, including the initial stiffness, peak load, stiffness degradation, and strength degradation at different drift levels. A summary of the strengths and displacement ductility computed from the experimental and numerical results is provided in Table 4. The key parameters of the walls include the yield load, Q_y , peak lateral load, Q_u , lateral load corresponding to 20% strength degradation, $Q_{0.8u}$, and displacement ductility at peak, $\mu_{\Delta u}$. It can be observed from Table 4 that the model was able to compute the RMSW + BEs lateral capacity with a maximum error of 13%. The displacement ductility of the three walls is represented well by the numerical model, with a maximum error of 7%. Overall, it can be concluded that a good agreement between the experimental and numerical results was achieved considering the natural variation in reinforced masonry shear wall properties.

Parametric Study

A parametric study was performed to assess the influence of different parameters on the lateral response of the RMSW + BEs. All walls were subjected to fully reversed displacement controlled quasi-static cyclic loading and were cycled up to 20% strength degradation. Fig. 6 shows the load-displacement hysteresis for Wall 1. Similar to Wall 1's envelope illustrated in Fig. 6, the envelopes of the load-displacement relationships were computed for the 36 walls

and presented in Fig. 7. The numerical yield loads $Q_{y(num)}$ and ultimate loads $Q_{u(num)}$ of the walls are shown in Table 5. The displacement ductility is a measure of the ability of the member to deform after yielding of the tension reinforcement. The idealization of the load-displacement relationship was conducted using the approach suggested by Tomazevic (1998). The idealized displacement ductility, $\mu_{\Delta 0.8u}$, was defined as the ratio between the displacement at 20% strength degradation, $\Delta_{0.8u}$, and idealized yield displacement, Δ_{yi} . The idealized displacement ductility, $\mu_{\Delta 0.8u}$, values are also presented in Table 5. It can be observed that a high level of displacement ductility was achieved. The effect of different design parameters on the displacement ductility are discussed in the following section.

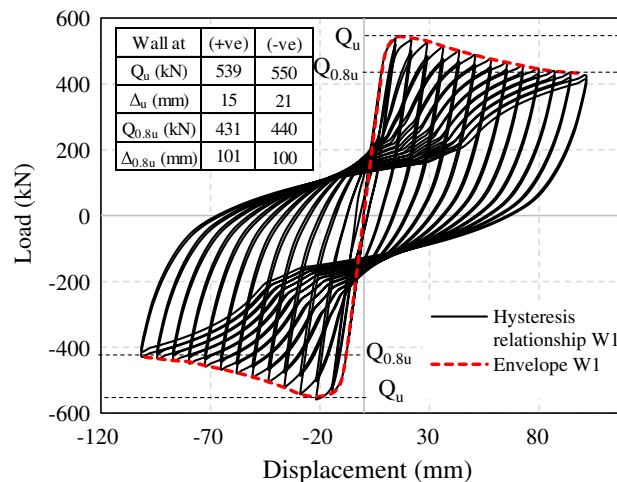


Fig. 6. Load-displacement hysteresis for Wall 1.

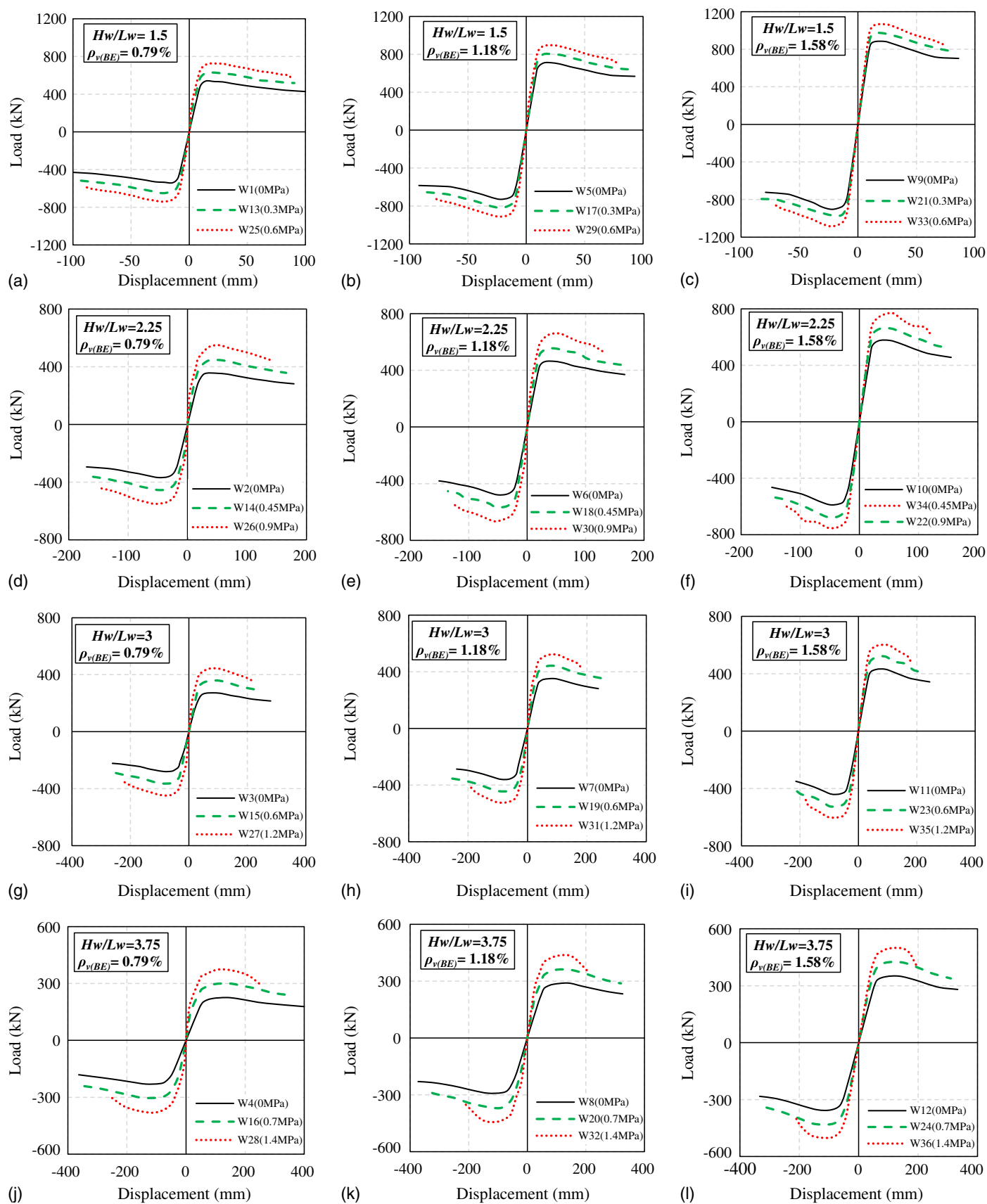


Fig. 7. Load-displacement hysteresis envelopes for RMSW + BEs with $H_w/L_w = 1.50$ and having (a) $\rho_{v(BE)} = 0.79\%$, (b) $\rho_{v(BE)} = 1.18\%$, (c) $\rho_{v(BE)} = 1.58\%$; RMSW + BEs with $H_w/L_w = 2.25$ and having (d) $\rho_{v(BE)} = 0.79\%$, (e) $\rho_{v(BE)} = 1.18\%$, (f) $\rho_{v(BE)} = 1.58\%$; RMSW + BEs with $H_w/L_w = 3.00$ and having (g) $\rho_{v(BE)} = 0.79\%$, (h) $\rho_{v(BE)} = 1.18\%$, (i) $\rho_{v(BE)} = 1.58\%$; and RMSW + BEs with $H_w/L_w = 3.75$ and having: (j) $\rho_{v(BE)} = 0.79\%$, (k) $\rho_{v(BE)} = 1.18\%$, and (l) $\rho_{v(BE)} = 1.58\%$.

Table 5. Summary of walls numerical loads, drifts, and displacement ductilities

Wall identifier	$Q_{y(num)}$ (kN)	$Q_{u(num)}$ (kN)	Top drift (%)			Displacement ductility $\mu_{\Delta 0.8u}$
			DS1	DS2	DS3	
W1	385.8	545.0	0.12	0.36	1.67	11.01
W2	266.1	360.2	0.18	0.54	1.94	8.72
W3	190.0	274.0	0.24	0.69	2.26	7.70
W4	155.0	224.9	0.30	0.88	2.54	6.99
W5	518.9	720.0	0.12	0.37	1.54	10.07
W6	354.1	469.8	0.19	0.54	1.77	7.89
W7	272.1	355.6	0.25	0.75	1.99	6.73
W8	223.3	290.7	0.31	0.92	2.33	6.33
W9	651.6	893.0	0.13	0.38	1.46	9.43
W10	441.8	575.0	0.19	0.57	1.68	7.44
W11	337.1	434.0	0.25	0.73	1.91	6.37
W12	275.0	352.1	0.32	0.92	2.22	5.98
W13	491.0	635.4	0.13	0.36	1.58	10.71
W14	355.0	450.4	0.19	0.57	1.81	8.39
W15	287.0	361.7	0.26	0.78	2.06	7.33
W16	240.0	301.0	0.33	0.95	2.29	6.49
W17	621.2	811.0	0.13	0.37	1.43	9.49
W18	454.8	557.2	0.20	0.56	1.64	7.52
W19	364.0	441.5	0.26	0.79	1.97	6.32
W20	288.0	361.3	0.33	0.98	2.15	5.94
W21	751.3	983.9	0.13	0.39	1.38	8.97
W22	540.6	665.9	0.20	0.58	1.57	6.96
W23	432.8	522.5	0.27	0.79	1.73	5.72
W24	362.0	424.8	0.34	1.02	2.07	5.57
W25	577.0	729.0	0.13	0.37	1.48	9.97
W26	440.0	551.0	0.20	0.59	1.61	7.17
W27	352.8	442.5	0.27	1.24	1.83	6.19
W28	297.2	376.0	0.35	0.68	1.67	4.37
W29	720.5	902.5	0.13	0.60	1.28	8.37
W30	548.2	667.5	0.20	0.61	1.43	6.12
W31	416.0	521.0	0.28	0.84	1.56	5.12
W32	347.2	437.0	0.35	0.69	1.43	3.67
W33	848.0	1,074.0	0.13	0.40	1.21	7.74
W34	630.6	760.0	0.21	0.61	1.36	5.87
W35	519.5	600.0	0.28	0.85	1.50	4.82
W36	443.0	499.0	0.36	0.67	1.36	3.47

Influence of Different Parameters on the Lateral Response of RMSW + BEs

Effect of Axial Compressive Stress

The effect of changing the axial compressive stress on the load-displacement behavior is assessed. As shown in Fig. 7, the RMSW + BEs ultimate strength increased with higher level of

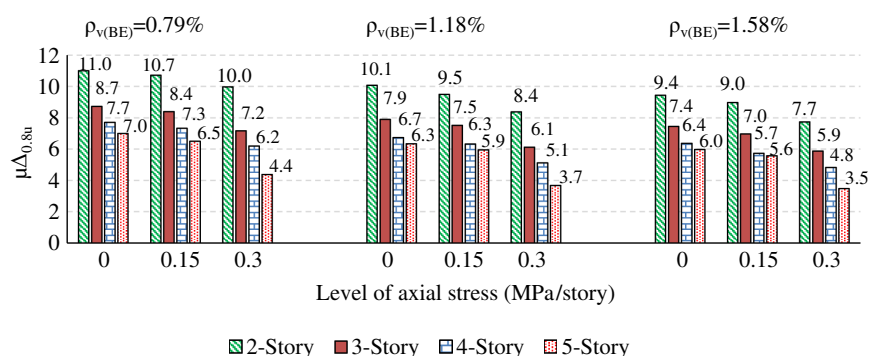
axial stresses. For walls with $\rho_{v(BE)} = 0.79\%$, as the level of axial stress increased from zero to high (0.3 MPa per story), $Q_{u(num)}$ increased by 34 and 67% for walls having aspect ratio of 1.5 and 3.75, respectively. Also, for walls with $\rho_{v(BE)} = 1.58\%$, as the level of axial stress increased from zero to high, $Q_{u(num)}$ increased by 20 and 42% for walls having aspect ratio of 1.5 and 3.75, respectively. Moreover, it can be inferred that walls with a higher level of axial stress experienced a more rapid strength and stiffness degradation. In addition, it can be observed from Fig. 8 that the walls' displacement ductility decreased with the increased axial stress, which is in agreement with the vast majority of experimental studies on RMSW (e.g., Sheded et al. 2008; Banting and El-Dakhkhni 2012). For walls with $\rho_{v(BE)} = 0.79\%$, $\mu_{\Delta 0.8u}$ decreased by 10%, and 37% for walls having aspect ratio of 1.5, and 3.75, respectively with the increase of the level of axial stress from zero to high. Moreover, for walls with $\rho_{v(BE)} = 1.58\%$, $\mu_{\Delta 0.8u}$ decreased by 18%, and 42% for walls having aspect ratio of 1.5, and 3.75, respectively, with the increase in the level of axial stress from zero to high.

Effect of Vertical Reinforcement Ratio in BE

Fig. 7 shows that the ultimate strength of walls increased as the vertical reinforcement ratio in BE increased. For walls with zero level of axial stress, as $\rho_{v(BE)}$ increased from 0.79 to 1.58%, $Q_{u(num)}$ increased by 64%, and 57% for walls having aspect ratio of 1.5, and 3.75, respectively. Likewise, for walls with a high level of axial stress, as $\rho_{v(BE)}$ increased from 0.79 to 1.58%, $Q_{u(num)}$ increased by 47%, and 33% for walls having aspect ratio of 1.5, and 3.75, respectively. The displacement ductility was dependent on the amount of vertical reinforcement. The results plotted in Fig. 8 show that walls with low vertical reinforcement ratios in BE developed higher displacement ductility than those tested with high vertical reinforcement ratios. For walls with zero level of axial stress, as $\rho_{v(BE)}$ increased from 0.79 to 1.58%, $\mu_{\Delta 0.8u}$ decreased by 14% for walls having aspect ratios of 1.5 and 3.75. Similarly, for walls with a high level of axial stress, as $\rho_{v(BE)}$ increased from 0.79 to 1.58%, $\mu_{\Delta 0.8u}$ decreased by 21% for walls having aspect ratio of 1.5 and 3.75.

Effect of Aspect Ratio

The relationship between the wall aspect ratio and load-displacement behavior is evaluated for four different aspect ratios (1.5, 2.25, 3, and 3.75). Fig. 7 shows that the lateral resistance of masonry walls increases as the height-to-length ratio decreases. For walls having $\rho_{v(BE)} = 0.79\%$, as aspect ratio increased from 1.5 to 3.75, $Q_{u(num)}$ decreased by 59 and 48% for walls with zero and high level of axial stress, respectively. For walls having $\rho_{v(BE)} = 1.58\%$, as aspect ratio increased from 1.5 to 3.75, $Q_{u(num)}$ decreased by

**Fig. 8.** Displacement ductility.

60 and 54% for walls with zero and high level of axial stress, respectively. Similar trends were reported by Sheded et al. (2010) and Banting and El-Dakhkhni (2014). It is also seen in Fig. 8 that $\mu_{\Delta 0.8u}$ decreased as aspect ratio increased, given that the walls had the same length, vertical reinforcement ratio, and axial stress. As aspect ratio increased from 1.5 to 3.75, $\mu_{\Delta 0.8u}$ decreased by 37 and 56% for walls with zero and high level of axial stress, respectively. The inversely proportional aspect ratio-ductility relationship is mainly attributed to the increase in the yield displacement value, as the aspect ratio increases, which was not offset by the increase in the peak displacement.

Fragility Assessment

Fragility curves are developed to contribute toward a growing seismic performance database regarding the behavior of RMSW + BEs. The fragility curves of RMSW + BEs are generated based on the guidelines provided in FEMA 58-1 (ATC 2009a). To develop fragility curves, distinct damage states are associated with the failure modes of the walls. In this study, the fragility functions are limited to flexure failure mode. Following the FEMA 58-1 (ATC 2009a) recommendations, flexure failure damage states can be categorized into: slight (DS1), moderate (DS2), and severe damage state (DS3). Damage state DS1 represents slight flexural damage and corresponds to a state at which the wall has been loaded to 80% of its peak resistance. Damage state DS2 represents moderate flexural damage and corresponds to a state at which the wall has reached its maximum flexural strength. Damage state DS3 represents severe flexural damage and corresponds to a state at which the wall has been loaded beyond its peak strength and exhibited a load drop of 20% with respect to the peak.

Fragility functions relate the probability of exceedance of a damage state to a demand parameter such as story drift ratio or floor acceleration. Component fragility functions provide a conditional probability that a particular damage state will occur in a component for a given demand value. The story drift ratio, Δ_i , of each wall is selected as the demand parameter. A lognormal probability distribution function is used to generate fragility curves. The lognormal probability distribution, F , for any level of top drift, Δ_i , is given in Eq. (2) (FEMA P-58)

$$F(\Delta_i) = \varphi\left(\frac{\ln(\Delta/\theta_i)}{\beta_i}\right) \quad (2)$$

$$\theta_i = e^{\left(\frac{1}{M} \sum_{i=1}^M \ln \Delta_i\right)} \quad (3)$$

$$\beta = \sqrt{\beta_r^2 + \beta_u^2} \quad (4)$$

$$\beta_r = \sqrt{\left(\frac{1}{M-1}\right) \left(\sum_{i=1}^M \left(\ln\left(\frac{\Delta_i}{\theta_i}\right)\right)^2\right)} \quad (5)$$

where θ_i = median drift for each damage state given in Eq. (3); β = logarithmic standard deviation (dispersion) given in Eq. (4); and φ = standard normal (Gaussian) cumulative distribution function. The dispersion parameter β_r given in Eq. (5) accounts for random variability. The dispersion parameter β_u accounts for uncertainty and is assumed to be 0.1 according to FEMA 58-1 (ATC 2009a). For each damage state, the limiting level of top drift associated with the first occurrence of any damage state for a particular wall is also presented in Table 5. The total number of wall specimens is given as M and the calculated values of θ_i and β are listed in Table 6. The fragility functions are represented by a smoothed curve fit to the

Table 6. Lilliefors test results for total set of walls

Damage state	Logarithmic distribution		Lilliefors test		
	θ (%)	β	D	D_{crit}	H_0
DS1	0.22	0.38	0.144	0.146	Accept
DS2	0.63	0.36	0.116	0.146	Accept
DS3	1.71	0.23	0.086	0.146	Accept

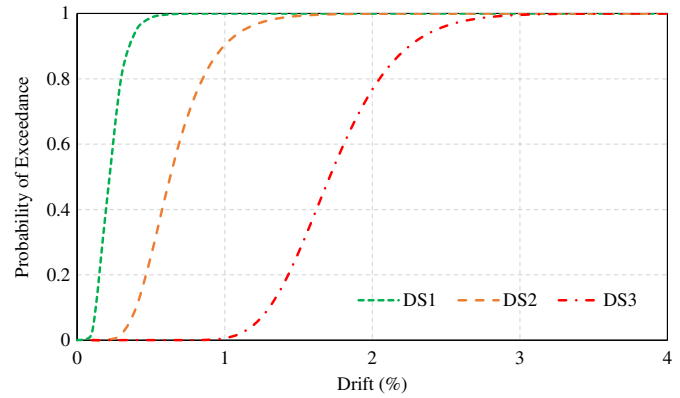


Fig. 9. Fragility curves for all sets of walls.

occurrence of each damage state in each wall. Fig. 9 shows the fragility curves for each damage state (DS1, DS2, and DS3) for the studied 36 walls. It can be seen in Table 6 that the median drift θ decreased from 0.22% at DS1 to 1.71% at DS3.

Also, a set of fragility functions for each damage state (DS1, DS2, and DS3) depending on the design parameters is presented in Fig. 10. As shown in Fig. 10(a), the reinforcement ratio, $\rho_{v(BE)}$, had minimal influence on the fragility curves for damage states DS1 and DS2. However, at DS3, θ decreased by 18% for walls having high reinforcement ratio in the BE (i.e., $\rho_{v(BE)} = 1.58\%$) when compared to walls having low vertical reinforcement ratio in BE (i.e., $\rho_{v(BE)} = 0.79\%$). In addition, the drifts for 80% probability of achieving DS3 are 2.25, 2.05, 1.95% for walls having $\rho_{v(BE)}$ of 0.79, 1.18, and 1.58%, respectively. Fig. 10(b) shows that the axial load had a noticeable effect on the developed fragility curves for DS3, but a slight effect on damage states DS1 and DS2. For DS3, θ decreased by 23% for walls under a high level of axial stress when compared to walls under zero level of axial stress. Furthermore, the drifts corresponding to 80% probability decreased from 2.3 to 1.7% for walls with a high level of axial stress when compared to walls with zero level of axial stress. It can also be seen in Fig. 10(c) that the median drift θ increased as aspect ratio increased for all damage states (DS1, DS2, and DS3). For DS1, θ increased from 0.13 to 0.33% for walls having $H_w/L_w = 1.5$ and $H_w/L_w = 3.75$, respectively. For DS2, θ increased from 0.38 to 0.86% for walls having $H_w/L_w = 1.5$ and $H_w/L_w = 3.75$, respectively. For DS3, θ increased from 1.44 to 1.96% for walls having $H_w/L_w = 1.5$ and $H_w/L_w = 3.75$, respectively. It can also be inferred from Fig. 10(c) that the drifts associated with 80% probability for all damage states are lower for walls having a low aspect ratio (i.e., $H_w/L_w = 1.5$) in comparison to walls with higher aspect ratios (i.e., $H_w/L_w = 3.75$). For walls with $H_w/L_w = 3.75$, the drifts corresponding to 80% probability of exceedance varied with values of 0.14, 0.42, and 1.65% for DS1, DS2, and DS3, respectively. For walls with $H_w/L_w = 1.5$, the drifts at 80%

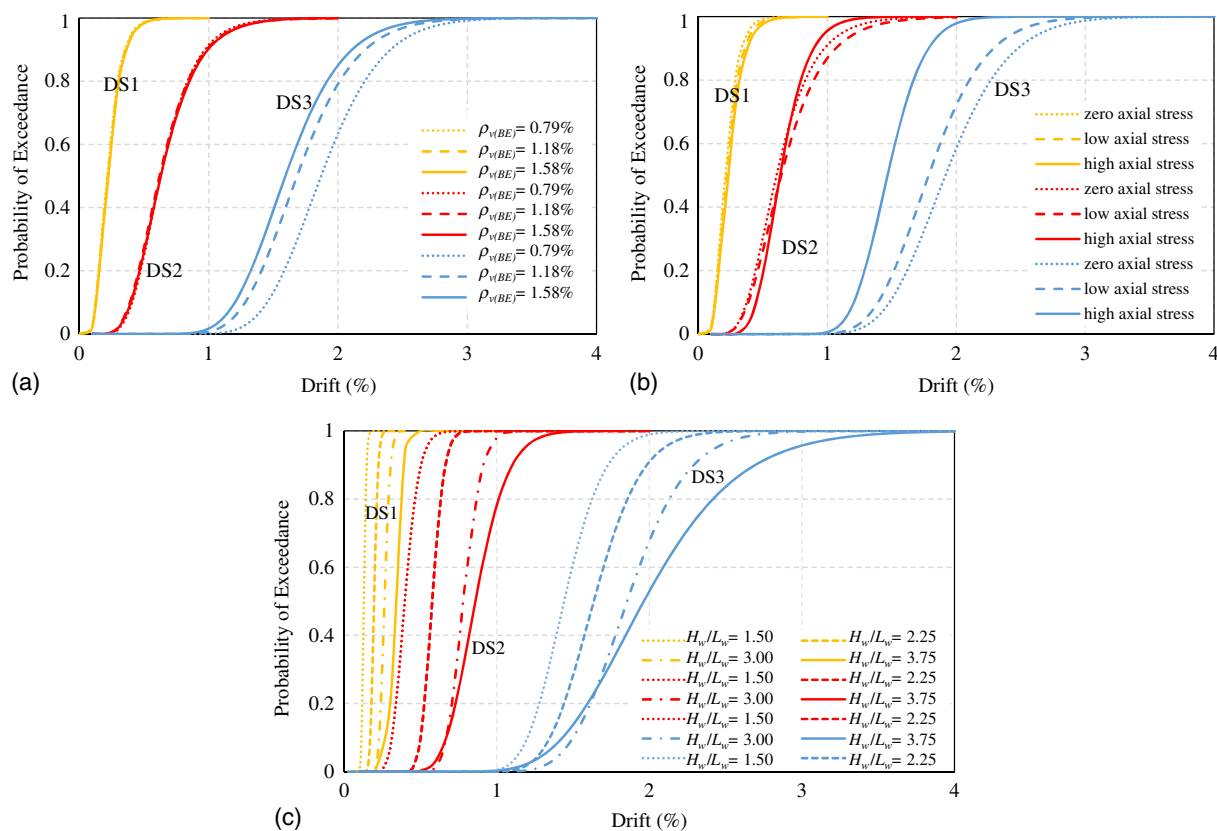


Fig. 10. Fragility curves based on design parameters: (a) $\rho_{v(BE)}$; (b) axial stress; and (c) aspect ratio.

Table 7. Lilliefors test results based on design parameters

Damage state	Design parameters	Logarithmic distribution		Lilliefors test		
		θ	β	D	D_{crit}	H_0
DS1	$\rho_{v(BE)} = 0.79\%$	0.212	0.39	0.146	0.243	Accept
	$\rho_{v(BE)} = 1.18\%$	0.216	0.39	0.149	0.243	Accept
	$\rho_{v(BE)} = 1.58\%$	0.220	0.39	0.148	0.243	Accept
DS2	$\rho_{v(BE)} = 0.79\%$	0.607	0.365	0.139	0.243	Accept
	$\rho_{v(BE)} = 1.18\%$	0.619	0.360	0.142	0.243	Accept
	$\rho_{v(BE)} = 1.58\%$	0.630	0.357	0.146	0.243	Accept
DS3	$\rho_{v(BE)} = 0.79\%$	1.87	0.197	0.137	0.243	Accept
	$\rho_{v(BE)} = 1.18\%$	1.68	0.213	0.138	0.243	Accept
	$\rho_{v(BE)} = 1.58\%$	1.59	0.219	0.150	0.243	Accept
DS1	$H_w/L_w = 1.5$	0.128	0.105	0.264	0.274	Accept
	$H_w/L_w = 2.25$	0.195	0.109	0.256	0.274	Accept
	$H_w/L_w = 3$	0.263	0.113	0.261	0.274	Accept
	$H_w/L_w = 3.75$	0.332	0.116	0.251	0.274	Accept
DS2	$H_w/L_w = 1.5$	0.378	0.111	0.270	0.274	Accept
	$H_w/L_w = 2.25$	0.576	0.110	0.266	0.274	Accept
	$H_w/L_w = 3$	0.781	0.121	0.258	0.274	Accept
	$H_w/L_w = 3.75$	0.860	0.194	0.190	0.274	Accept
DS3	$H_w/L_w = 1.5$	1.441	0.142	0.154	0.274	Accept
	$H_w/L_w = 2.25$	1.635	0.151	0.168	0.274	Accept
	$H_w/L_w = 3$	1.854	0.165	0.180	0.274	Accept
	$H_w/L_w = 3.75$	1.964	0.247	0.252	0.274	Accept
DS1	Zero axial stress	0.205	0.372	0.173	0.243	Accept
	Low axial stress	0.217	0.386	0.177	0.243	Accept
	High axial stress	0.227	0.398	0.184	0.243	Accept
DS2	Zero axial stress	0.604	0.375	0.155	0.243	Accept
	Low axial stress	0.641	0.396	0.187	0.243	Accept
	High axial stress	0.633	0.269	0.227	0.243	Accept
DS3	Zero axial stress	1.915	0.201	0.107	0.243	Accept
	Low axial stress	1.785	0.194	0.119	0.243	Accept
	High axial stress	1.467	0.153	0.131	0.243	Accept

probability of exceedance varied with values of 0.37, 1.02, and 2.40% for DS1, DS2, and DS3, respectively.

Lilliefors test is used to quantify the goodness of fit of the fragility functions for all 36 walls (Lilliefors 1967). This test is carried out to assess the acceptability of the lognormal distribution for a 95% confidence level as required by FEMA 58-1 (ATC 2009a). The results for the Lilliefors test are presented in Tables 6 and 7. The D statistic is the Kolmogorov-Smirnov test parameter and corresponds to the maximum of the absolute value of the differences between the empirical and theoretical cumulative distribution function. The null hypothesis, H_0 , for the test is the decision on whether to accept or reject the hypothesis (A = Accept, R = Reject). The null hypothesis is accepted if D is less than or equal to Lilliefors test parameter D_{crit} at 5% significance level. The developed fragility functions passed the Lilliefors test for damage states (DS1, DS2, and DS3) of the 36 walls. FEMA 58-1 (ATC 2009a) quantifies the fragility functions as high quality if the fragility functions pass the Lilliefors test and have logarithmic standard deviations, β , less than 0.6. The results indicate that developed set of fragility functions for RMSW + BES corresponding to flexure-dominated damage states (DS1, DS2, and DS3) are considered high-quality fragility functions.

Conclusions

This paper presented a numerical simulation of the in-plane response of 36 reinforced masonry shear walls with boundary elements (RMSW + BEs) loaded under displacement controlled quasi-static cyclic loading. The RMSW + BEs macro model, utilizing SeismoStruct, was validated using experimental results of three RMSW + BEs tested under cyclic loading adopted from literature. The model predictions showed a good agreement

between the numerical and experimental load-displacement responses. A numerical investigation study was performed to quantify the influence of key parameters, namely, the vertical reinforcement ratio in the BE, the level of axial compressive stress, and the aspect ratio on the inelastic behavior and ductility of RMSW + BEs. The wall's design and detailing represented a range of parameters that would be anticipated to vary within low-rise reinforced masonry buildings. Based on the need to develop the next generation of performance-based seismic design codes, fragility curves were generated to assess the damageability of RMSW + BEs subjected to simulated seismic loads. In this study, fragility functions for RMSW + BEs were developed for three damage states (DS1, DS2, and DS3) that correspond to flexure-dominated failure modes according to FEMA P-58 (ATC 2009a) guidelines. Based on the results of numerical modeling, the following conclusions can be drawn:

1. The variation of vertical reinforcement in BE had a noticeable effect on the RMSW + BEs peak strength. Results shows that walls with higher vertical reinforcement ratio in the BE ($\rho_{v(BE)} = 1.58\%$) achieved a higher peak strength than those with lower vertical reinforcement ratios in BE ($\rho_{v(BE)} = 0.79\%$). In this respect, as $\rho_{v(BE)}$ increased from 0.79 to 1.58%; $Q_{u(num)}$ increased by 60, 47, and 38% for walls with zero, low, and high axial stress, respectively. However, it caused a drop in $\mu_{\Delta 0.8u}$ by 15, 17, and 20% for walls having zero, low, and high axial stress, respectively. This was explained based on the fact that the yield displacements of walls were highly dependent on the amount of vertical reinforcement, whereas the displacement at maximum loads for all walls were almost similar.
2. Similar to increasing vertical reinforcement in BE, increasing the level of axial stress from zero to high (0.3 MPa per story) resulted in increasing $Q_{u(num)}$ by 26% for walls with ($H_w/L_w = 1.5$), 42% for walls with ($H_w/L_w = 2.25$), 48% for walls with ($H_w/L_w = 3$), and 53% for walls with ($H_w/L_w = 3.75$). However, the displacement ductility tended to decrease with increasing axial stress, which is mainly attributed to an increase in yield displacement with increased axial stress. It was found that there was a decrease in $\mu_{\Delta 0.8u}$ by 15% for walls with ($H_w/L_w = 1.5$), 20% for walls with ($H_w/L_w = 2.25$), 23% for walls with ($H_w/L_w = 3$), and 40% for walls with ($H_w/L_w = 3.75$).
3. Results on the effect of varying the aspect ratio ($H_w/L_w = 1.5, 2.25, 3$, and 3.75) showed that the maximum lateral resistance of the walls increased as the height-to-length ratio decreased. Results indicated that walls with lower aspect ratio developed higher displacement ductility than those with higher aspect ratios. As H_w/L_w increased from 1.5 to 3.75, $\mu_{\Delta 0.8u}$ dropped by 37, 39, and 55% for walls with zero, low, and high axial stress, respectively. The inversely proportional aspect ratio–ductility relationship is due to the increase in the yield displacement as the aspect ratio increases, which was not offset by the increase in the displacement at maximum loads.
4. A series of fragility curves were developed using FEMA P58-1 guidelines (ATC 2009a) and checked using Lilliefors test (Lilliefors 1967) for confidence. Fragility functions relate the probability of exceedance of a damage state to the story drift ratio, Δ_i . Three damage states associated with flexure failure modes were considered: slight (DS1), moderate (DS2), and severe (DS3). Fragility curves for distinct key design parameters ($\rho_{v(BE)}$, H_w/L_w , and axial stress) were investigated. Within the design parameters considered in this study, DS1 was mostly influenced by altering the aspect ratio rather than altering the vertical reinforcement ratio in BE or axial stress. Moreover,

DS2 was more sensitive to the axial stress and aspect ratio rather than vertical reinforcement in the BE. This was not the case for DS3, which had a noticeable effect on all design parameters considered in this study.

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Notation

The following symbols are used in this paper:

- A_g = gross area of the wall cross section (mm^2);
- A_h = area of horizontal reinforcement (mm^2);
- A_v = area of vertical reinforcement (mm^2);
- D = Kolmogorov-Smirnov test parameter;
- D_{crit} = Lilliefors test parameter at 5% significance level;
- d_b = diameter of steel reinforcement;
- E_m = Young's modulus of masonry (MPa);
- E_s = Young's modulus of steel (MPa);
- F = lognormal probability distribution;
- f'_c = concrete compressive strength (MPa);
- f'_m = masonry compressive strength (MPa);
- f_y = yield stress of vertical reinforcement (MPa);
- H_w/L_w = height to length aspect ratio of the walls;
- L_p = length of plastic hinge (mm);
- L_w = wall length (mm);
- M = total number of wall specimens;
- P = axial force on the section (kN);
- $Q_{0.8u}$ = lateral strength corresponding to 20% strength degradation (kN);
- Q_u = ultimate strength (kN);
- Q_y = yield strength (kN);
- $Q_{u(num)}$ = numerical ultimate flexural strength (kN);
- $Q_{y(num)}$ = numerical yield strength (kN);
- R_0, a_1, a_2, a_3 , and a_4 = coefficients representing the transition from elastic to plastic zone;
- r = strain hardening ratio;
- z = moment shear ratio;
- β = logarithmic standard deviation (dispersion);
- β_r = dispersion parameter accounts for random variability;
- β_u = dispersion parameter accounts for uncertainty;
- $\Delta_{0.8u}$ = displacement at 20% strength degradation (mm);
- Δ_i = story drift ratio;
- Δy_i = idealized yield displacement (mm);
- ε_m = strain at peak strength;
- θ_i = median drift for each damage state;
- $\mu_{\Delta 0.8u}$ = idealized displacement ductility;
- $\mu_{\Delta u}$ = displacement ductility at peak;
- ρ_h = horizontal reinforcement ratio (%);
- ρ_v = vertical reinforcement ratio (%);
- $\rho_{v(BE)}$ = vertical reinforcement ratio in boundary element (%);
- and
- φ = standard normal (Gaussian) cumulative distribution function.

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