

Developing a plastic hinge model for reinforced concrete beams prone to progressive collapse

Farzad Rouhani, Lan Lin, and Khaled Galal

Abstract: It is known that building structures would undergo nonlinearity during progressive collapse. Given this, modelling the nonlinear behaviour of structural members is critical for assessing their resistance. The objective of this study is to develop the nonlinear modelling parameters of reinforced concrete (RC) beams for the progressive collapse analysis. To achieve this, three types of RC moment-resisting buildings located in high, moderate, and low seismic zones in Canada are designed. Nonlinear pushdown analyses are then conducted on 27 three-dimensional finite element models using ABAQUS to examine the case that one column on the ground level is removed. Based on the analysis results, an idealized moment-rotation curve for modelling the plastic hinge in beams with different ductility is proposed. In comparison with the 2013 GSA modelling parameters, smaller chord rotations are observed from the detailed finite element analysis.

Key words: progressive collapse, GSA, nonlinear, plastic hinge, modelling, reinforced concrete buildings, moment-rotation, seismic, ductility.

Résumé : On sait que les charpentes de bâtiments subiraient la non-linéarité pendant un effondrement progressif. Étant donné ceci, la modélisation du comportement non linéaire des éléments de structure est critique afin d'évaluer leur résistance. L'objectif de cette étude consiste à développer les paramètres de modélisation non linéaires de poutres en béton armé aux fins de l'analyse d'effondrement progressif. Dans le but de réaliser ceci, trois types de bâtiments en béton armé résistants aux moments et situés dans des zones à haut, modéré et faible risque sismique au Canada sont conçus. Des analyses de poussée non linéaire sont effectuées sur 27 modèles d'éléments finis tridimensionnels au moyen d'ABAQUS dans le cas où une colonne au niveau du sol est enlevée. En se fondant sur les résultats des analyses, on propose une courbe de moment-rotation idéalisée afin de modéliser la rotule plastique des poutres de différents niveaux de ductilité. Comparativement aux paramètres de modélisation de la GSA 2013, des rotations de membrure plus petites sont observées en effectuant une analyse par éléments finis détaillée. [Traduit par la Rédaction]

Mots-clés : effondrement progressif, « US General Services Administration » (GSA), non linéaire, rotule plastique, modélisation, bâtiments en béton armé, moment-rotation, sismique, ductilité.

Past studies

Since the partial collapse of the Ronan Point Apartment Building in London in 1968, design of building structures against progressive collapse have brought attention to researchers around the world. Furthermore, comprehensive research work has been conducted after the collapse of the World Trade Center Twin Towers in 2001. It includes both experimental and numerical studies; some of the most recent ones are described below along with their findings.

Analytical studies

Progressive collapse analysis was advanced significantly from 1996 in terms of the analysis method, simulation of the blast loads, etc. For example, [Mlakar et al. \(2003\)](#) prepared a technical report on the investigation of the performance of the Pentagon Building under the 2001 9/11 attack. A beam element formulation and solution was introduced for the dynamic method to improve the accuracy of the results from the progressive collapse analysis. The proposed formulation would predict the capacity of the concrete beams associated with the level of ductility during an event of progressive collapse. It can also be used to develop moment-

rotation curves for reinforced concrete (RC) beams. In addition, the responses from the finite element analysis were compared with peak values recommended by GSA (U.S. General Service Administration) Guideline. [Kaewkulchai and Williamson \(2004\)](#) proposed a beam formulation and solution for the purpose of progressive collapse of analysis of planar frame buildings. More specifically, they considered a beam-column element of frame buildings developed by [Kim \(1995\)](#) in the study. They introduced a lumped plasticity approach, in which plastic hinges were assumed at the ends of the element, to take into account the effect of the nonlinearity of the elements. This approach was then applied to a two-bay two-storey frame to investigate the dynamic effects for the progressive collapse analysis. Their results showed that the vertical displacement and the plastic rotations could be underestimated if the dynamic effects were not considered. [Grierson et al. \(2005\)](#) proposed a methodology for using linear-static method for progressive collapse analysis. The step-by-step procedure of this method to follow was well explained in the article. The advantage of the proposed method is that, it is best suited for use by the practicing engineers in design offices to avoid running a complicated nonlinear analysis. It was reported in [Grierson et al. \(2005\)](#) that the results from the simplified method

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are compatible with those from dynamic analysis. In addition, the analysis can be conducted by using any structural analysis software, such as, SAP2000. Kim and Park (2006) proposed a concept called Energy Balance to simplify the analysis. Furthermore, a comprehensive study was conducted by Kim et al. (2008) on three steel seismic moment-resisting frame buildings, namely, 3, 6, and 15 storeys to investigate the contribution of the seismic design to the building progressive collapse resistance. Lin et al. (2011) conducted a study on RC frame buildings designed according to the seismic provisions of the 2005 edition of the National Building Code of Canada (NBCC 2005). In total, six buildings located in Ottawa and Vancouver were considered in the study. The progressive collapse resistance of the buildings was evaluated according to the 2003 GSA Guideline (GSA 2003), which was available at that time. The results from the study showed that the vulnerability of the progressive collapse of seismically designed buildings depended greatly on the differences between the spans of the longitudinal and the transverse frames, i.e., larger differences between the spans led to higher vulnerability. Qian and Li (2012) examined the performance of a RC frame due to the removal of a corner and edge column by comparing the test results with those from the finite element analysis. They also evaluated the dynamic impacts on the elements due to the sudden removal of a column. They concluded that seismically detailed elements would have a satisfactory performance in resisting progressive collapse. Livingston et al. (2015) examined the behaviour of RC beams by conducting a 2D push down numerical analysis. Sensitivity analyses were conducted in terms of lateral load, yield stress of reinforcement, span length, etc. Their study also confirmed the contribution of the slab to the progressive collapse resistance of a frame. The numerical model for the study was validated with a 3/8 scale experimental test.

Experimental tests

Due to safety reasons, it is hard to conduct tests on the progressive collapse on real buildings. However, several researchers did conduct experimental tests that provide very valuable data and observations of the performance of buildings against progressive collapse. For example, Astaneh-Asl (2001) examined a typical steel building by removing a middle column on the building's perimeter. Their test results showed that the loads were redistributed very well due to the catenary action of the steel deck and girders. Catenary action is defined as an ability of a beam to transfer vertical loads to the bearing elements (i.e., columns) when its maximum flexural capacity is reached. This will develop significantly large deformation where the vertical elements of beams resist the acting load on the beam. Sadek et al. (2011) performed two full-scale tests on a RC beam-column assembly comprised of a beam with two spans supported by three columns. A downward displacement was applied at the center column during the test, and was increased gradually until the assembly failed. In their study, finite element models were also developed to investigate the characteristics of the primary response and the failure modes. Tran and Li (2015) studied five half-scale RC columns with light transverse reinforcement and tested the columns up to the axial failure. Backbone curves of the column displacement were developed in the study. They reported that the initial stiffness of the column is overestimated while the ultimate displacement is underestimated by both FEMA 356 (FEMA 2000) and ASCE 41-13 (2014). Xiao et al. (2015) conducted a collapse test of a half-scale three-storey RC frame. Failure mechanisms, load-transfer path, and the dynamic response were discussed in the study. They concluded that the slabs and beams directly connected to the failed columns have significant effects on disproportionate collapse resistance. They also suggested that the requirement of providing sufficient anchorage capacity to the joints should be provided in the guidelines for the progressive collapse analysis to achieve the catenary action, which is beneficial for the collapse resistance.

Existing guidelines

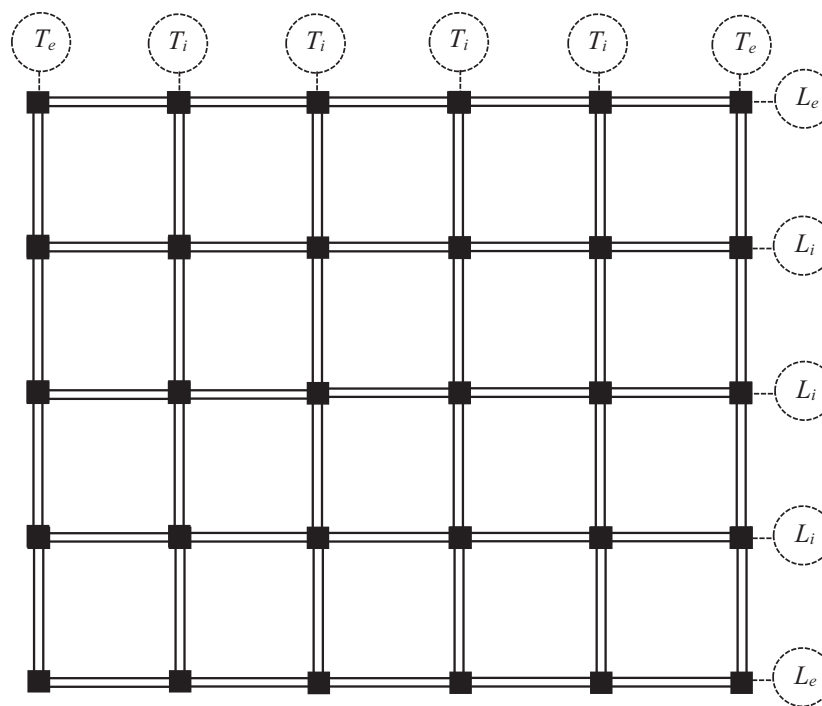
GSA published guidelines for the progressive collapse resistance of buildings in 2003 and 2013, respectively. It should be noted that no guidelines on progressive collapse analysis are available in Canada. According to GSA, progressive collapse is defined as an event triggered by the local failure of the primary structural members due to the column removal, which might, in turn, cause the collapse of the adjacent members. The latest 2013 Guidelines were developed based on the seismic provisions of ASCE-41.13 (2014) by considering the structural integrity, ductility, and nonlinear behaviour due to the sudden removal of a column. Requirements for redundancy, overall structural integrity and resilience specified by the American Concrete Institution (ACI) were also considered in GSA Guidelines. In addition, United States Department of Defense (DoD) issued guidance on protection of facilities in case of abnormal loading and progressive collapse in 2001 and 2005, respectively. Currently, GSA guidelines are adopted more often than DoD for the analysis. The typical approach considered in GSA is designated as Alternative Path Method (APM) in which, first, instantaneous loss of a vertical load-bearing element is assumed; then the capability of the beam elements supported by the column is evaluated. To comply with the requirements of APM, several analysis procedures are specified in GSA, namely, linear static, nonlinear static, and nonlinear dynamic. The acceptance criteria for progressive collapse analysis are based on the response of beams. Although the results from the experimental tests and numerical analyses have made significant contribution to the latest GSA Guidelines published in 2013, there is still a lack of detailed implementation rules for the numerical modelling of progressive collapse analysis.

It is well known that significant nonlinear deformation (e.g., plastic hinge) will develop in building elements when the progressive collapse occurs. More specifically, the analytical results depend greatly on how the plastic hinges are modeled in the finite element analysis. Given this, the objective of this study is to evaluate the plastic hinge model for RC beams of moment-resisting frame buildings defined in 2013 GSA.

Design of the buildings

According to a survey conducted by Natural Resources Canada (NRCan 2013), high-rise commercial buildings (10 or more floors) only accounted for approximately 0.4% of the total number of buildings in Canada. Therefore, low-rise buildings were selected for the analysis in this study. As mentioned in the previous section, progressive collapse analysis is intended to protect the new and very important office buildings from failure due to any abnormal loading. Given this, office buildings with four storeys were chosen for the study. It should be noted the progressive collapse resistance of a building did not change significantly with the number of storeys as reported in Lin et al. (2011). The buildings were designed for the following locations: Toronto, Montreal, and Vancouver. These locations were selected to represent the low, moderate, and high seismic hazard zone in Canada, respectively. In each location, three span lengths were considered, namely, 4.0 m, 6.0 m, and 8.0 m. The storey heights of all the buildings are 4.0 m. The lateral load resisting system consists of moment-resisting RC frames in both the longitudinal and the transverse directions. There are five frames in the longitudinal direction (designated L_e and L_i in Fig. 1; L_e is exterior frame and L_i is interior frame) and six frames in transverse direction (T_e and T_i). The floor system consists of a two-way slab supported by the beams of the longitudinal and transverse frames. The slab is cast integrally with the beams.

The buildings were designed in accordance with the 2010 edition of the National Building Code of Canada (NBCC 2010) and Canadian standard CSA A23.3-14 (CSA 2014). Regarding the loads, the superimposed dead load considered in the design was 2.0 kPa, which included the loads due to floor finishing, mechanical

Fig. 1. Typical floor layout of structure (span = 4.0, 6.0, and 8.0 m).**Table 1.** Dimensions and reinforcement ratios of the studied beams.

	<i>l</i> (m)	<i>b</i> × <i>h</i> (mm)	Beam	Longitudinal rebar percentages in beams (%)			
				At support		At mid-span	
				Top	Bottom	Top	Bottom
Conventional* (Toronto)	8.0	400 × 600	C1 (Minimum)	0.33%	0.17%	0.04%	0.32%
			C2 (Design)	0.50%	0.17%	0.08%	0.38%
			C3 (Maximum)	2.00%	0.58%	0.29%	0.88%
	6.0	300 × 500	C4 (Minimum)	0.32%	0.27%	0.07%	0.47%
			C5 (Design)	0.36%	0.13%	0.07%	0.27%
			C6 (Maximum)	2.00%	0.67%	0.20%	1.33%
	4.0	300 × 400	C7 (Minimum)	0.33%	0.17%	0.33%	0.30%
			C8 (Design)	0.42%	0.17%	0.42%	0.33%
			C9 (Maximum)	2.00%	0.33%	1.67%	0.92%
Moderately ductile† (Montréal)	8.0	400 × 600	M1 (Minimum)	0.32%	0.17%	0.08%	0.31%
			M2 (Design)	0.67%	0.42%	0.42%	0.42%
			M3 (Maximum)	2.00%	1.25%	0.63%	1.25%
	6.0	300 × 500	M4 (Minimum)	0.33%	0.13%	0.07%	0.30%
			M5 (Design)	0.87%	0.40%	0.20%	0.47%
			M6 (Maximum)	2.00%	0.67%	0.47%	1.13%
	4.0	300 × 400	M7 (Minimum)	0.32%	0.17%	0.08%	0.32%
			M8 (Design)	0.58%	0.50%	0.58%	0.50%
			M9 (Maximum)	2.00%	0.83%	0.42%	1.08%
Ductile† (Vancouver)	8.0	400 × 600	D1 (Minimum)	0.32%	0.17%	0.08%	0.31%
			D2 (Design)	0.71%	0.42%	0.29%	0.42%
			D3 (Maximum)	2.00%	0.83%	0.63%	1.13%
	6.0	300 × 500	D4 (Minimum)	0.32%	0.13%	0.13%	0.31%
			D5 (Design)	0.73%	0.40%	0.33%	0.47%
			D6 (Maximum)	2.00%	0.93%	0.47%	1.07%
	4.0	300 × 400	D7 (Minimum)	0.31%	0.17%	0.08%	0.31%
			D8 (Design)	0.75%	0.33%	0.42%	0.33%
			D9 (Maximum)	2.00%	1.17%	0.42%	1.17%

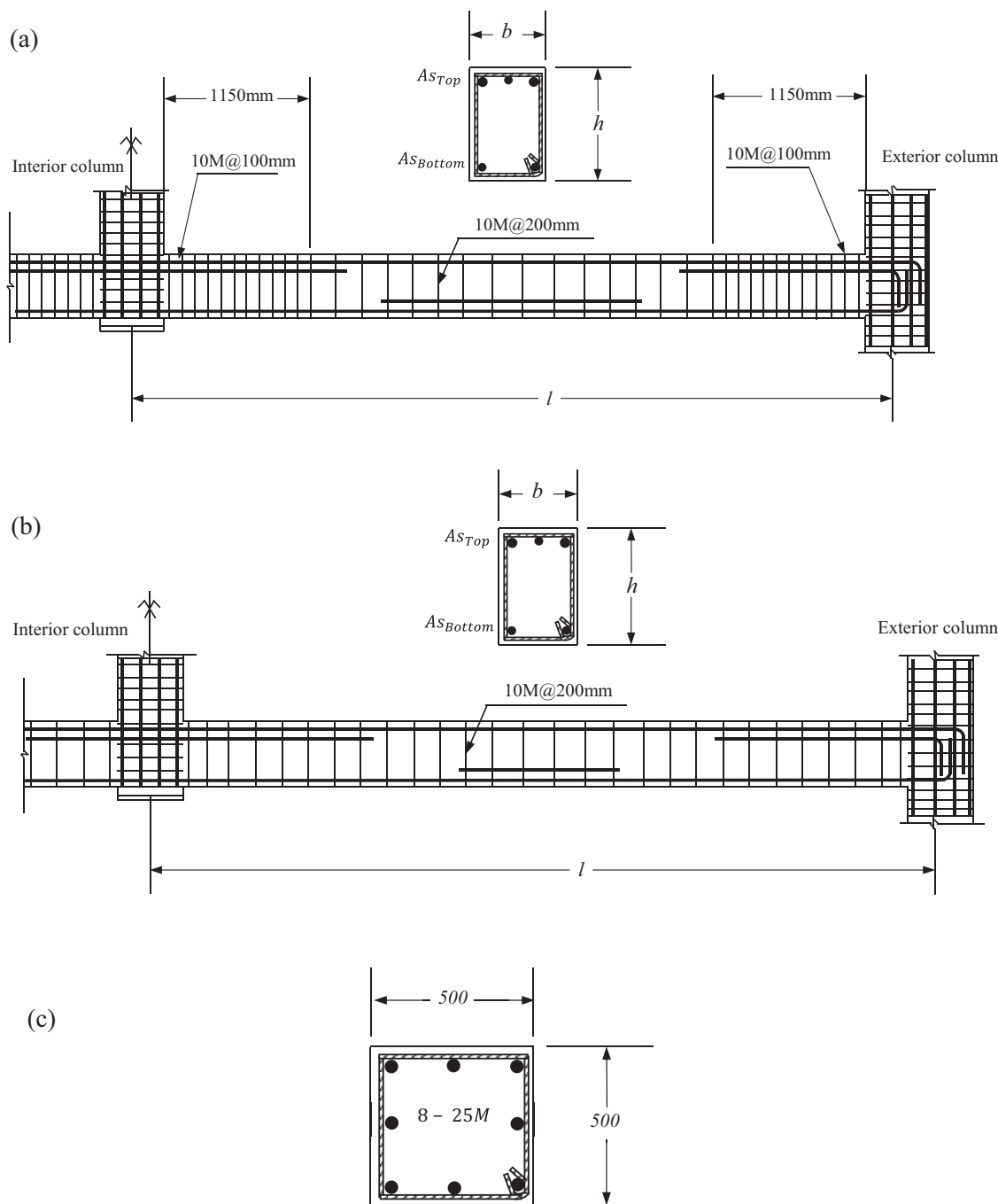
*Transvers reinforcement in All regions: 10M@200 mm.

†Transvers reinforcement in P.H. region: 10M@100 mm; non P.H. region 10M@200 mm.

services, partitions, and suspended ceiling. The design live loads were 1.0 kPa and 2.4 kPa for the roof and floors, respectively. The design base shears for earthquake loads were determined using the seismic design spectrum for the location of the building. The

foundations were assumed to be on the soil represented by site class C in NBCC (shear wave velocity between 360 m/s and 750 m/s). The fundamental period of the frames was about 0.6 s based on a formula to estimate the building period on NBCC. The other pa-

Fig. 2. Typical reinforcement layout of beams: (a) ductile (Vancouver buildings) and moderately-ductile (Montreal buildings), (b) conventional (Toronto buildings), and (c) column section.



parameters used in the calculation of the base shears were the ductility-related force modification factor R_d , the overstrength-related force modification factor R_o , the higher mode factor $M_v = 1$, and the importance factor $I_E = 1$. Given the seismicity of the building location, the frames in Toronto were designed as conventional frames (i.e., $R_d = 1.5$, $R_o = 1.3$); in Montreal they were designed as moderately ductile frames (i.e., $R_d = 2.5$, $R_o = 1.4$), and in Vancouver, they were designed as ductile frames (i.e., $R_d = 4.0$, $R_o = 1.7$) in accordance with NBCC.

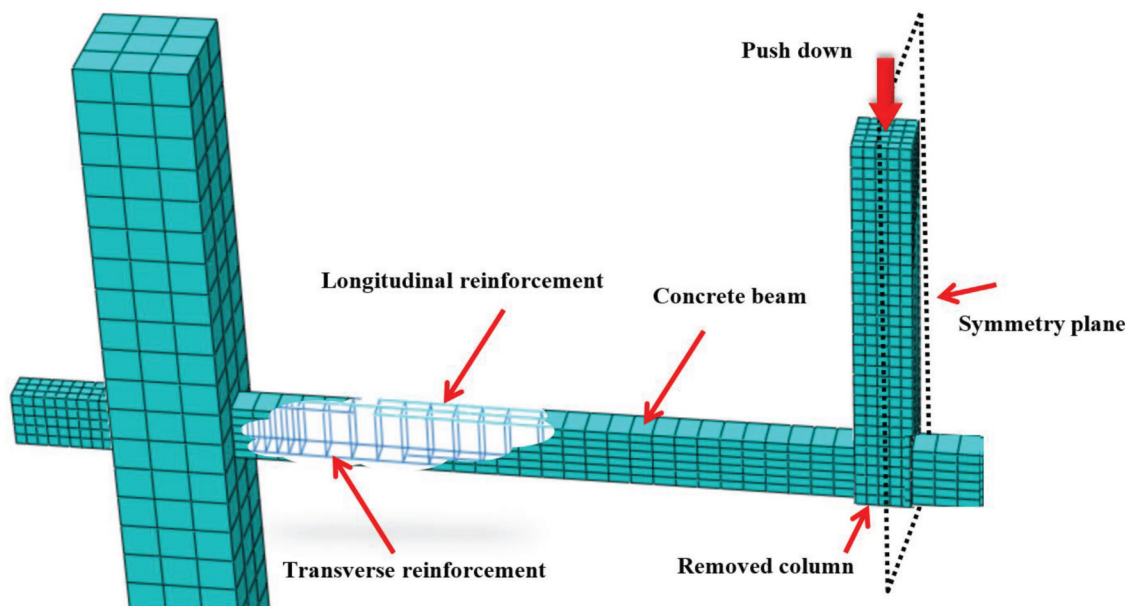
Compressive strength of concrete $f'_c = 40$ MPa, and yield strength of reinforcement $f_y = 400$ MPa were used in the design. The dimensions of beams and reinforcement from the design of the buildings are given in Table 1. It is necessary to mention that

two additional design cases are also provided in Table 1 that are labelled as Maximum and Minimum. They were based on the maximum and the minimum allowable reinforcement specified in CSA A23.3-14 to cover a wide range of the reinforcement provided in the design. For illustration, Fig. 2 shows the layout of the beam flexure and shear reinforcement for a typical span.

Modelling techniques

Figure 3 schematically shows the 3D model developed using ABAQUS for the beam-column assembly, in which one of the interior columns is removed as highlighted in Fig. 2. Please note that beam-column assembly is a typical element considered in the

Fig. 3. Schematic 3D ABAQUS model of the studied beam-column assembly. [Colour online.]



current progressive collapse analysis (Mlakar et al. 2003; Sadek et al. 2011; etc.). Beams and columns were modelled using 3D deformable homogeneous solid element C3D8R (i.e., continuum, 3-D, 8-node, reduced integration). All the columns are modelled as fixed-fixed in axial, flexural, and shear reactions. Beam-column joints are assumed to be rigid, i.e., the joint undergoes the equal rotation of the corresponding beam. Equal sizes of the meshes are defined in beam-column faces to provide the connection between mesh edges. It is necessary to mention that failure of side columns and beam-column joints is not considered in modelling because of the strong column – weak beam criteria for the seismic design, i.e., the plastic hinges are expected to form in the beams, not in the columns.

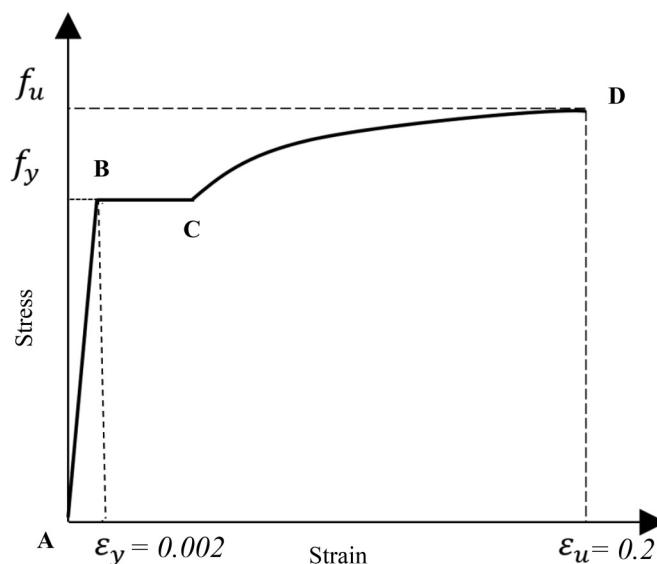
Modelling of steel bars

In this study, steel bars are modeled using 3D truss element T3D2 defined in ABAQUS to take the axial forces for tension and compression. More specifically, T3D2 is a 3D spar element having two nodes with three degrees of freedom at each node (i.e., translation in the x , y , and z direction). A perfect bond is assumed between concrete and steel bars. In our ABAQUS modelling, an “Embedded region” was used to embed the steel bars that were (3D truss element T3D2) inside the concrete medium (solid elements C3D8R). This option will assure a perfect bond between steel and concrete is achieved. More specifically, this option will constrain the translational degrees of freedom of the embedded nodes. The longitudinal and transverse reinforcement developed in ABAQUS model is illustrated in Fig. 3. The stress-strain relationship of the steel was represented by an isotropic elastic-plastic model (Fig. 4) defined in ABAQUS which consists of three parts to present the behaviour of the steel at three stages, i.e., elastic, plastic, and strain hardening for both compression and tension. Rupture of steel bars is assumed to occur when a strain of 0.2 is reached.

Modelling of concrete

The typical stress-strain relationship used for concrete under tension and compression is shown in Fig. 5. The curve for compressive concrete was adopted from the model developed by Kent and Park (1971), and it is applicable to both unconfined and confined concrete. The tensile curve was based on the model proposed in Fib (2010) as defined in eq. (1), which can be used to

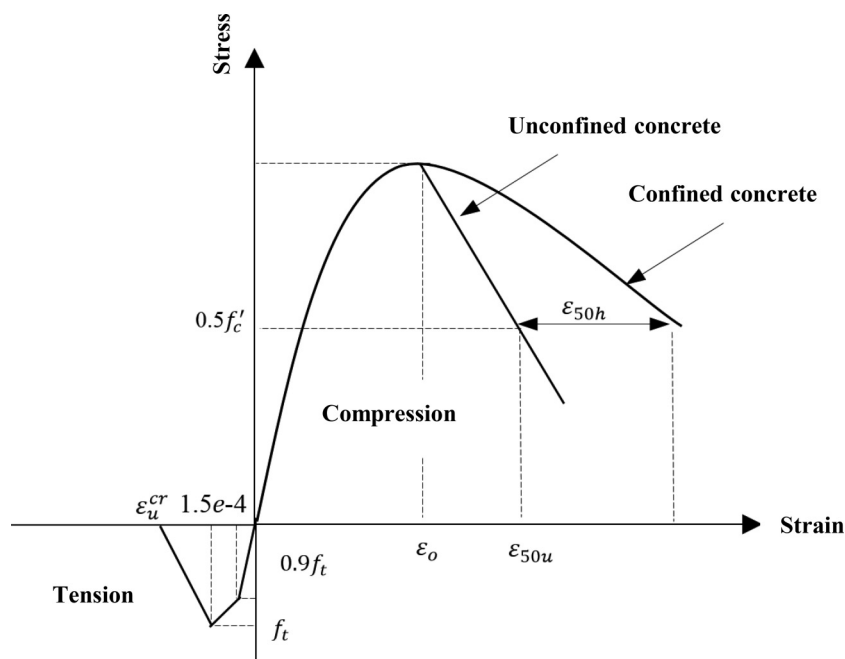
Fig. 4. Steel stress-strain curve (Holzer et al. 1975).



represent the ascending branch of the tensile curve. The cracked concrete follows a straight descending line to approximate its tensile behaviour.

It is known that RC members could crack at very small loads due to the extremely low tensile strength of concrete. To take into account the effects of cracking on the performance of structures, three models are defined in ABAQUS, which are Concrete Smeared Crack model (CSC), Brittle Cracking model for Concrete (BCC), and Concrete Damaged Plasticity model (CDP). In this study, CDP model was selected to simulate cracking and post failure of concrete due to its advantages over the other two models. One of them is that damage to concrete in compression and tension in CDP model can be assigned with different values while a single number is used in CSC model. It should be noted that CDP model has also been used by other researchers, such as Xue and Le (2016); Jeyarajan et al. (2015); Li et al. (2015); etc. in the modelling of RC structures.

Fig. 5. Concrete stress–strain curve (Reprinted from Kent and Park 1971 with permission from ASCE).



The CDP model follows the concepts of isotropic damage elasticity with isotropic tensile and compressive plasticity. The two major failure mechanisms considered in the model are concrete compressive crushing and tensile cracking. Figures 6a and 6b illustrate the response of concrete subjected to uniaxial loading in tension and compression defined in ABAQUS. The parameters ϵ_t^{pl} and ϵ_c^{pl} are used to define the post failure behaviour of concrete in tension and compression, respectively, and they can be calculated using the formula given in ABAQUS Manual. Table 2 lists the values for the parameters used to define CDP model in this study.

$$(1) \quad \sigma_{ct} = \begin{cases} E_{ci}\epsilon_{ct} & 0 < \epsilon_{ct} < 0.9f_t/E_{ci} \\ f_t - 0.1f_t \frac{0.00015 - \epsilon_{ct}}{0.00015 - 0.9f_t/E_{ci}} & 0.9f_t/E_{ci} < \epsilon_{ct} < 0.00015 \end{cases}$$

Meshing

As reported by Bao et al. (2013) the mesh size and its configuration could affect ABAQUS results significantly. Given this, sensitivity analyses on meshing were conducted in this study. In total, five cases were considered. The selected mesh sizes of the beam elements ranged from 50 mm to 200 mm, which gives aspect ratios of the five cases between 1.0 and 3.0. It is necessary to mention that the maximum aspect ratio recommended by ABAQUS is 5.0. With respect to the truss elements for modelling rebars, the maximum mesh size is 50 mm, and the meshes are distributed along the axial direction of the elements. Figure 7 shows the five cases of meshing on the beam of a two-span frame in a building located in Vancouver where the middle column is removed. In particular, the mesh of the concrete beam contains 56 elements in the beam cross-section, i.e., 7 elements along the height and 8 elements along the width. As illustrated in Fig. 7, relatively smaller mesh is used in the plastic hinge zone located at the end of beams. More specifically, the numbers of elements considered in the plastic zone (Fig. 7) are 5, 8, 11, 22, and 33 for Cases 1 to 5, respectively. The number of beam mesh elements at the side of the missing column was selected to be the same as that in the plastic hinge zone to achieve symmetrical meshing about the middle span of the beam. Accordingly, the divisions for the remaining parts of the beam are 13, 16, 26, 52, and 26 for Cases 1 to 5, respectively. It is necessary to mention that the preliminary

results showed that the meshing of the region outside the plastic hinge zone did not have significant effects on the beam response.

During the analysis, a vertical (downward) load was applied at the location of the removed column. This load was gradually increased; in the meantime the displacement at the location of the missing column was monitored until the beam(s) failed. The bending moment at the left face of the column and the vertical displacement of the beam at the place where the load was applied were recorded, and the results are illustrated in Fig. 8. Note that the displacement in the figure was normalized with respect to the beam span length. The results in the Fig. 8 clearly show that the responses provided by Cases 3, 4, and 5 are almost the same while the response given by Case 1 is the smallest among the five cases considered, and the response provided by Case 2 might be considered as an average of the five cases. Given this, the size of meshing defined in Case 3 was selected for further analyses due to its least computation time compared to Cases 4 and 5.

Analysis results

Following the techniques described in the previous sections, comprehensive 3D ABAQUS models were developed for the designed beams listed in Table 1. In each model, only a beam-column assembly was considered and the interior column was removed. Nonlinear pushdown analysis was conducted on each model following the loading procedure for the meshing sensitivity analysis as described above. The yield (M_y) and ultimate (M_u) moment capacities of the beams from the analysis are presented in Table 3. For the purpose of comparison, the nominal moment resistance (M_n) of each beam based on hand calculation is also provided in the table. It was found that M_u of each case from ABAQUS is about 10% larger than that given by hand calculation for the ductile and moderately ductile beams, while the hand calculation and the ABAQUS results match each other for conventional beams. The larger difference observed in the ductile and moderately ductile beams might be due to the detailed modelling of the confinement in ABAQUS. It is necessary to mention herein that relatively higher longitudinal reinforcement ratio is used in the design of ductile and moderately ductile beams compared to the conventional beams (see Table 1), which leads to larger M_y for all the cases. On the other hand, M_u significantly depends on the level of

Fig. 6. ABAQUS response curve of concrete due to uniaxial loading: (a) tension and (b) compression (ABAQUS 2011).

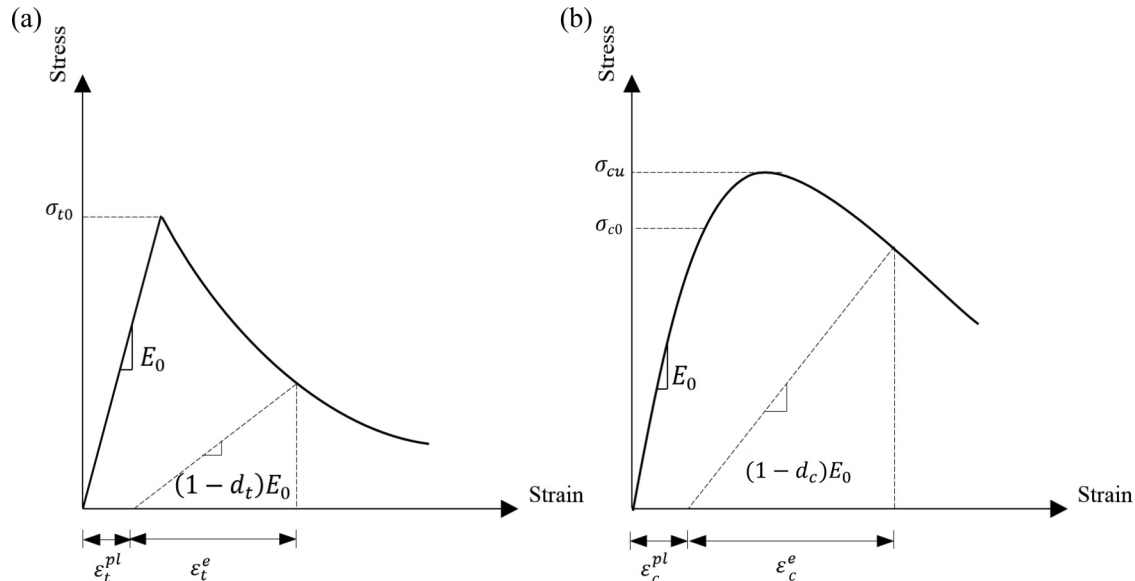


Table 2. Values for d_c and d_t .

Compression (d_c)		Tension (d_t)	
Damage	Inelastic strain	Damage	Inelastic strain
0	0.0000000	0	0
0	0.0000294	0.2	3.08505E-05
0	0.0000495	0.55	0.000365064
0	0.0000764	0.9	0.001018065
0	0.0001103		
0	0.0001514		
0	0.0002551		
0	0.0003855		
0	0.0005392		
0	0.0007120		
0	0.0009000		
0	0.0010997		
0.008337335	0.0013081		
0.022903215	0.0015227		
0.041802466	0.0017416		
0.063582535	0.0019634		
0.08714985	0.0021870		
0.111693643	0.0024115		
0.136621512	0.0026365		
0.599559465	0.0035991		

the ductility that delays the crushing of the concrete. Furthermore, the higher transverse reinforcement ratio of the ductile and moderately ductile beams prevents the buckling of the compressive longitudinal steel bars before the tensile bars yield. It, in turn, results in larger moment resistance as demonstrated in the results for the beams D1 to D9, M1 to M9 in Table 3. These observations are consistent with those from the experimental tests reported in Qian and Li (2012).

Figures 9a to 9c show the moment at the fixed end of the beam versus the vertical displacement at the place where the column was removed for ductile, moderately ductile, and conventional beams, respectively. In each figure, each dashed line represents the response of one of the nine beams in a given location while the solid line presents the mean response curve. For the purpose of comparison, the bending moment is normalized to the nominal moment resistance, and the displacement is normalized to the span length. The results in Fig. 9 show that the response curve for ductile and moderately ductile beams is very similar, and they are

different from that for conventional beams due to the different requirements for the seismic design (i.e., different seismic hazard). Moreover, the slope of the response curve for conventional beams for the displacement ratio Δ/L between 0.02 and 0.04 is relatively larger than that for ductile and moderately ductile beams for the same reason discussed above. It was also found that the slope of the curve beyond a Δ/L of 0.04 for the three types of beams is quite close. This is because they all lost their capacity when the displacement ratio Δ/L reached 0.04.

Development of plastic hinge model

Figure 10 illustrates the response curves proposed in this study to model the plastic hinges in beams for the progressive collapse analysis, in which Fig. 10a is for ductile and moderately ductile beams since the behaviour of these two types of beams is very similar, while Fig. 10b is for conventional beams. It is necessary to mention herein that the mean response curve in Fig. 10 for each type of the beam (i.e., ductile, moderately ductile, and conventional) is adapted from that shown in Fig. 9 where the horizontal axis is converted to the rotation from the normalized displacement ratio Δ/L . More specifically, the rotation was calculated by using the vertical displacement of each beam at the location of 12% of span length divided by the corresponding horizontal displacement of the point in the beam assessed. It has been observed that the points located in the distance between the column face and 12% of span length rotate equally, which is equivalent to about 1.5 times the beam depth. The computed mean response curve is then idealized by a multilinear (i.e., quadri-linear) curve for ductile and moderately ductile beams (Fig. 10a), a tri-linear curve for conventional beam (Fig. 10b). The idealized curve was chosen in such a way that it stays as close as possible to the computed curve obtained from ABAQUS. The detailed description of each curve is given below, and the modelling parameters to define the curves proposed are given in Table 4.

- For ductile and moderately ductile beams (Fig. 10a): point A is used to represent the first yield of the longitudinal steel bars, which corresponds to about 50% of the nominal capacity of the section; point B stands for the nominal bending moment capacity of the beam; point C represents the ultimate capacity, which is about 10% higher than the nominal capacity; and point D represents the failure of the section.
- For conventional beams (Fig. 10b): point A is used to represent the first yield of the longitudinal steel bars, which corresponds

Fig. 7. Discretization of mesh size sensitivity analyses. [Colour online.]

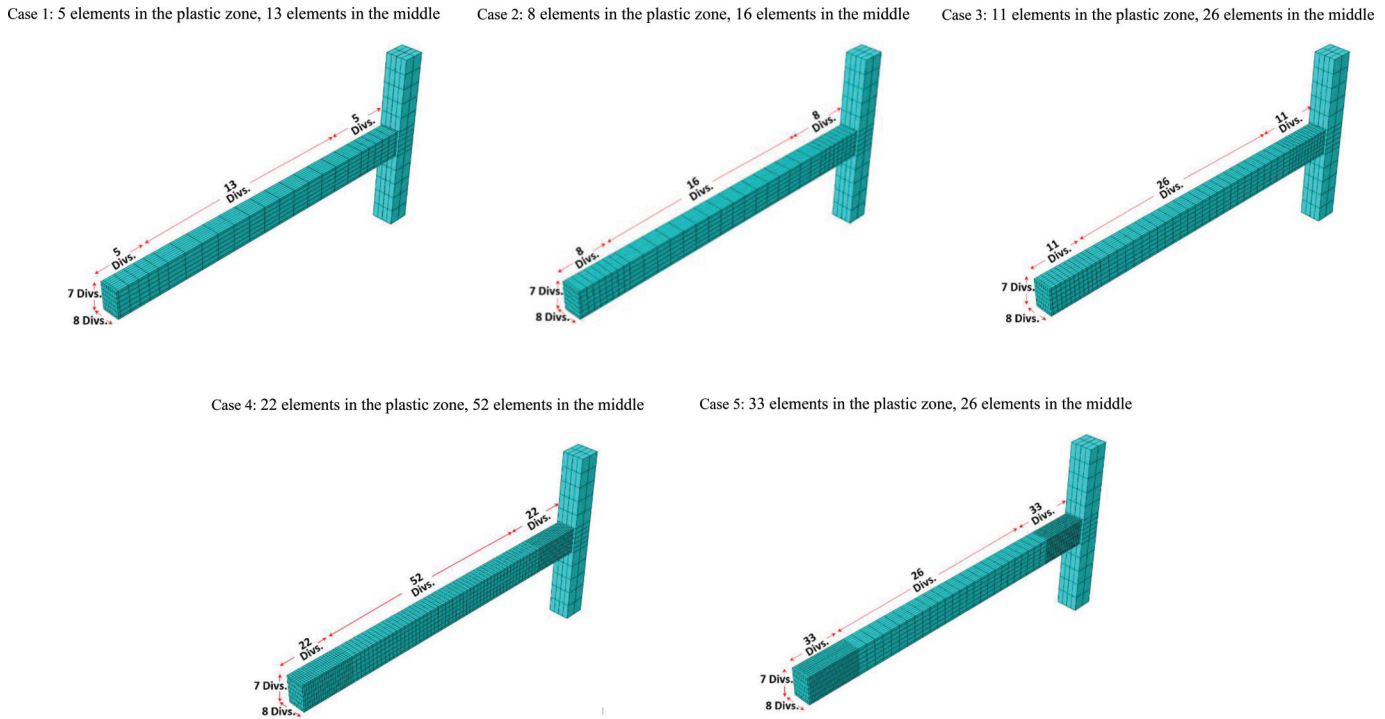


Fig. 8. Results of mesh sensitivity analyses. [Colour online.]

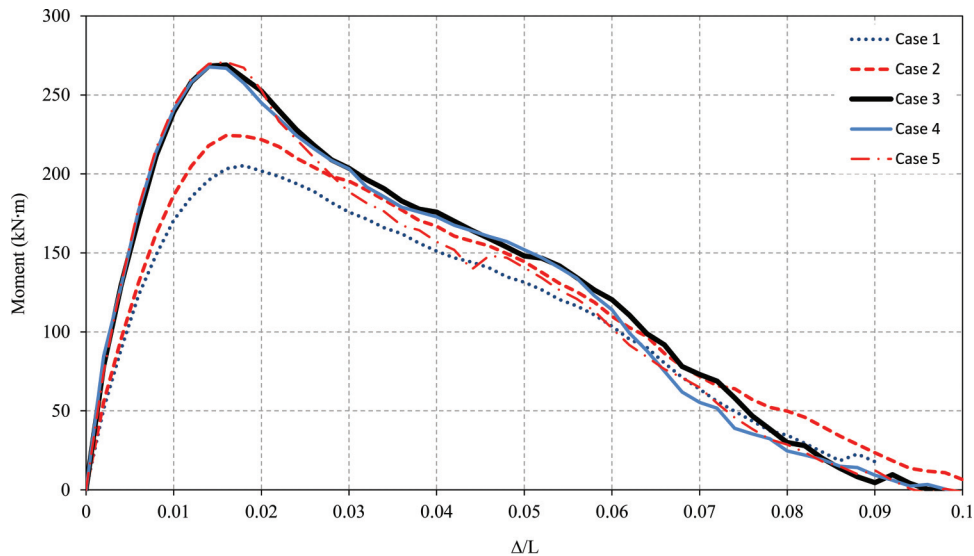
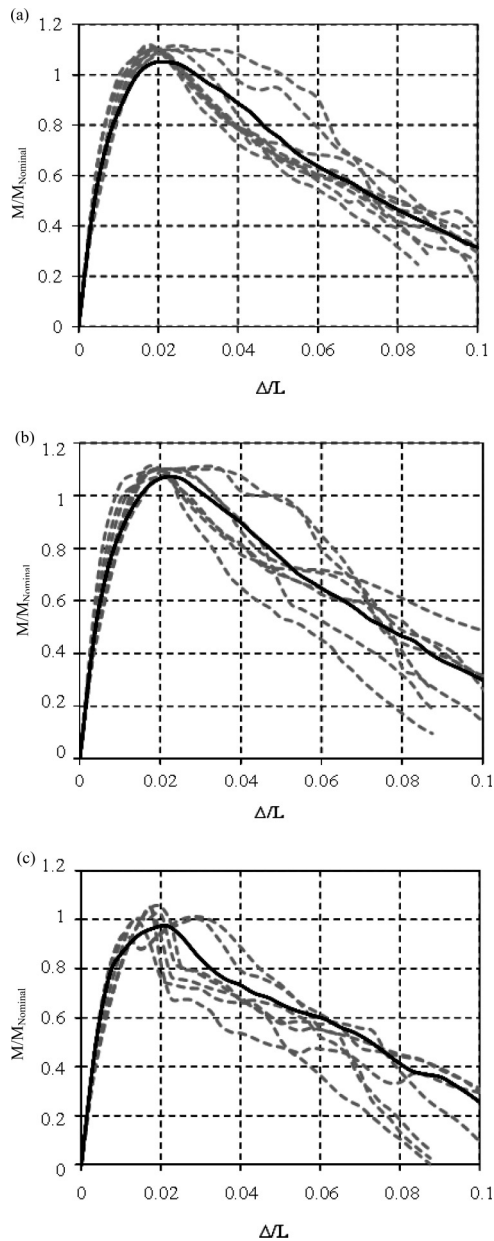


Table 3. Beam flexural capacity.

Beam (Toronto)	M_y (kN-m)	M_u FEM (kN-m)	M_u Theoretical (kN-m)	Beam (Montreal)	M_y (kN-m)	M_u FEM (kN-m)	M_u Theoretical (kN-m)	Beam (Vancouver)	M_y (kN-m)	M_u FEM (kN-m)	M_u Theoretical (kN-m)
C1	474.3	539	550	M1	281.1	530.4	473.6	D1	302.7	550.3	487
C2	465.5	541.3	492.1	M2	292.5	573.6	516.8	D2	286.1	572.1	515.4
C3	497	546.2	530.3	M3	314.3	604.4	521	D3	339.5	640.6	552.2
C4	231.9	252.1	242.4	M4	88.2	180.1	159.4	D4	135.2	281.6	258.3
C5	250.4	287.8	282.2	M5	142.1	284.2	251.5	D5	139.7	297.2	265.4
C6	223.6	294.2	300.2	M6	161.3	336	297.3	D6	166.8	320.8	267.3
C7	120	141.2	128.4	M7	78.2	150.4	129.7	D7	69.1	125.6	112.1
C8	125.9	159.4	144.9	M8	81.7	157.2	131	D8	85.6	152.9	141.6
C9	134.5	166	152.3	M9	105.8	179.4	148.3	D9	121.7	206.2	171.8

Note: C, conventional; M, moderately-ductile; and D, ductile.

Fig. 9. Bending moment in the beam at the fixed support vs. displacement: (a) ductile, (b) moderately ductile, and (c) conventional beam.



to about 80% of the nominal capacity of the section; point B stands for the nominal bending moment capacity of the beam; and point C represents the failure of the section.

- The slope of every two adjacent points on the proposed curve is defined with respect to the initial elastic stiffness K_e using parameters α , β , and γ .
- The parameter a represents the rotation corresponding to the peak value of M/M_{nominal} .
- The parameter e represents the rotation when the ratio of M/M_{nominal} is equal to 1.0, i.e., the moment reaches the nominal moment.

Instead of using the mean values of the typical points given in Table 4 to define the plastic hinge model, formulations of the typical model parameters a' and e' for the ductile and moderately ductile, and conventional beams were developed in this study, and they are given in eqs. (2) and (3), respectively. These two

Fig. 10. Multilinear backbone curves proposed: (a) ductile and moderately ductile beams and (b) conventional beams.

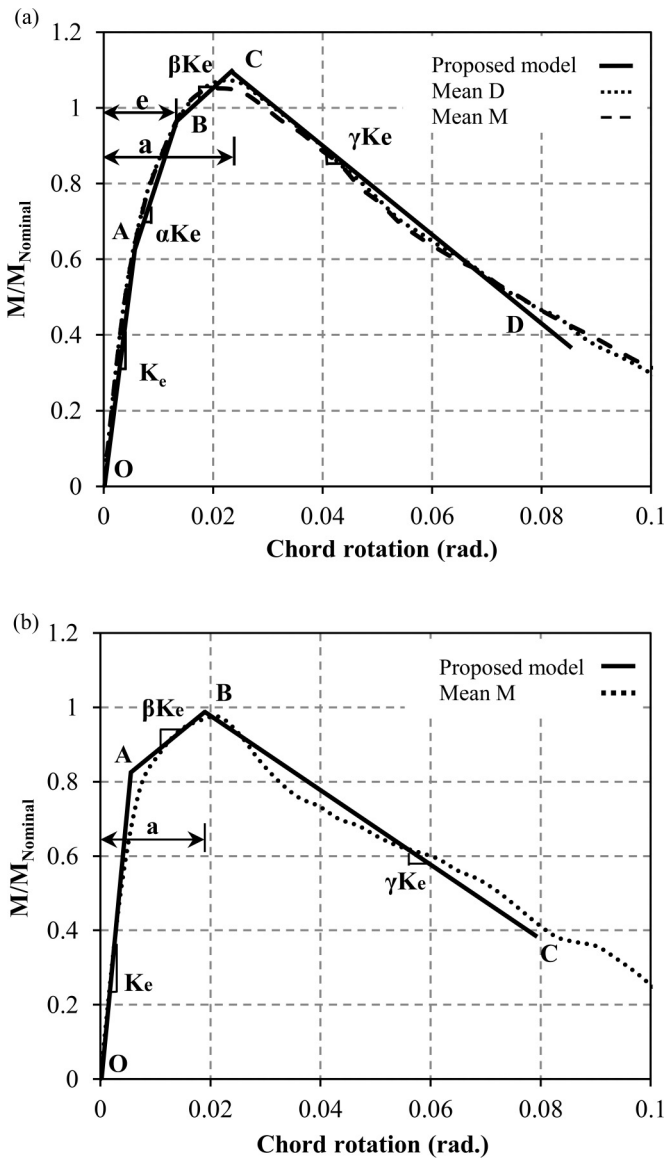
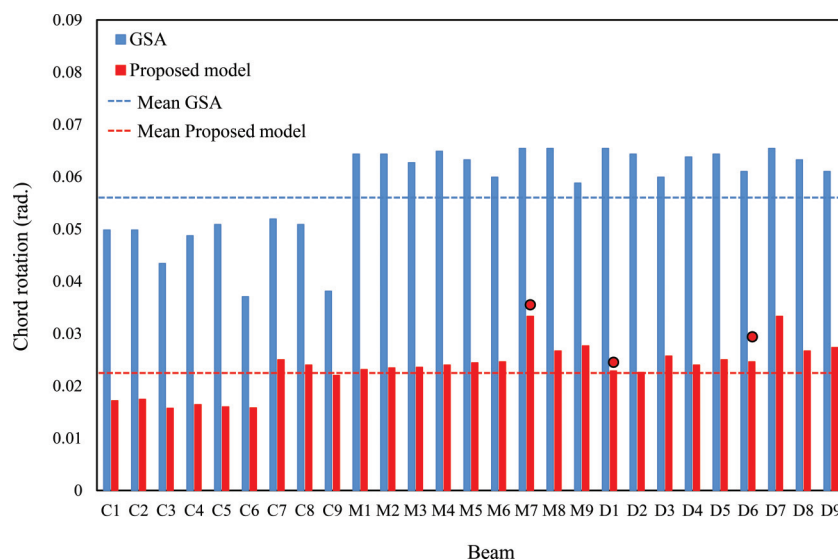


Table 4. Proposed mean values for the modelling parameters.

		Ductile and moderately ductile beam	Conventional beam
Tangents	Parameters		
	α	0.26	—
	β	0.45	0.14
Coordinates	γ	−0.075	−0.08
	A	(0.003, 0.5)	(0.006, 0.8)
	B	(0.0145, 1)	(0.02, 0.99)
	C	(0.025, 1.1)	(0.08, 0.32)
	D	(0.08, 0.4)	—

parameters have the same definition as that for parameters a and e described above. However, they were proposed to model the plastic hinges of a specific beam with respect to its span length (L), ductility (R_d), and the longitudinal reinforcement factor (η) in which $\eta = (\rho - \rho'/\rho_b)$, ρ and ρ' are reinforcement ratios for tension reinforcement and compression reinforcement, respectively; and ρ_b is the reinforcement ratio for a balanced section. It is expected

Fig. 11. Comparison of the value for parameter a' based on the proposed model with 2013 GSA. [Colour online.]

that the parameters given by the proposed equations is more appropriate for modelling beam-specific plastic hinges than those shown in Table 4.

$$(2) \quad a' = \frac{LR_d}{81 - L(0.06 + 7R_d - 69\eta)}$$

$$(3) \quad e' = \frac{LR_d}{316 - L(40 + 10R_d - 21\eta)}$$

It is worth mentioning that eqs. (2) and (3) were developed based on the results from all the 27 beams under investigation as shown in Table 1. In fact, the response curve of each beam shown in Fig. 9 was idealized as a multilinear curve (for ductile and moderately ductile beams) or tri-linear curve (for conventional beams). In total, 27 and 18 data points for parameters a' and e' were defined respectively, in which each point is associated with three variables, i.e., L , R_d , and η . Gauss–Newton algorithm was chosen to fit each of the two parameters a' and e' with the variables mentioned above. The mean squared errors of the proposed functions for a' and e' were about 0.0014 and 0.0057, respectively. Furthermore, it was found that the maximum residual, which represents the difference between the values of the parameter (a' and e') predicted using the proposed equation and the actual value, was about 0.07. For comparison, the value of the parameter a' for each of the 20 beams examined in ABAQUS was calculated using the proposed eq. (2). The results were presented in Fig. 11 in which the labels in the horizontal axis are consistent with those provided in Table 3. It was found that the average value (about 0.023) of a' was about 40% less than the GSA value (0.057) (Rouhani 2015). It is worth mentioning herein that very close response to the proposed model was observed in the most recent experimental study conducted by Qian and Li (2012). The circles shown in Fig. 11 represent the chord rotation of three similar beams considered in Qian and Li (2012).

Comparison with the modelling parameters proposed in 2013 GSA

Figures 12a and 12b present the response curves proposed in this study with the superposition of those specified in 2013 GSA for ductile and moderately ductile, and convention RC beams, respectively. The shaded areas in the figures represent the range of the

response curve enclosed by the lower and upper bounds of the parameters defined in 2013 GSA. The thicker solid line represents the response curve for each type of the beams proposed in this study while the thinner solid lines show the lower and upper bounds of the proposed model. The major observations of the results shown in Fig. 12 are summarized as follows,

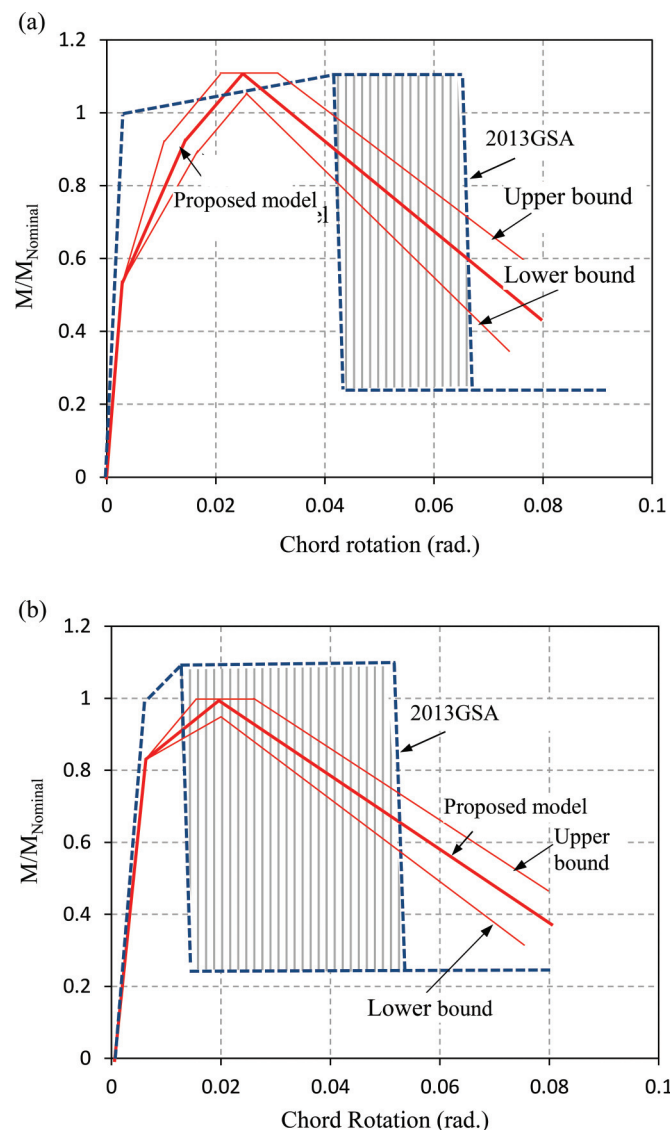
- According to the 2013 GSA definition, the maximum bending moment capacity of beam section is equal to its nominal capacity regardless the level of the ductility. By comparison with the curves proposed in this study, it is noticed that the bending moment capacity of the ductile and moderately ductile beams is underestimated in the 2013 GSA.
- The stiffness of the post peak (ultimate) response curve remains constant in 2013 GSA. However, the results from the detailed finite element analyses in this study show that the change of the stiffness depends on the level of the plasticity in the beam.
- The post yield and failure mechanism of the beams are observed to follow a relatively small descending slope (i.e., 0.075 for ductile and moderately ductile, and 0.08 for conventional). On the contrary, 2013 GSA defines a sudden drop on the response once the full capacity is reached.

Conclusions

The main conclusions of this study are summarized as follows:

- The level of seismic design ductility of the RC building significantly affects its progressive collapse resistance. This is attributed to two reasons: (i) the special detailing of longitudinal and transverse reinforcements in beams required for the seismic design could result in a higher capacity of plastic rotation; (ii) the confinement of concrete provided by the transverse reinforcement could increase the compression capacity of concrete, which will, in turn, increase the bending moment capacity of the beam.
- To model the nonlinear behaviour (moment–rotation curve) of RC beams for progressive collapse analysis, multilinear curve (i.e., quadri-linear) should be used for the ductile and moderately ductile beams while trilinear curve should be used for conventional beams. In addition, recommended values (i.e., mean value) of the modelling parameters for the above mentioned curves are given based on the analyses of the 27 beams in this study.

Fig. 12. Comparison of the proposed model with 2013 GSA model: (a) ductile and moderately ductile beams and (b) conventional beams. [Colour online.]



- Two equations are proposed for nonlinear modelling of plastic hinges in RC beams associated with the beam span length, ductility ratio, and the reinforcement ratio.
- In comparison with the 2013 GSA modelling parameters, smaller chord rotations (about 50% less) were observed from the detailed finite element analysis.
- The allowable rotation for RC beams to assess the progressive collapse resistance depends on the rotation with respect to the ultimate bending moment. Detailed characterization of the beam behaviour after its peak capacity may not be needed due to the sudden failure of the element.
- The length of the beam plastic hinge was found to be approximately about 1.5 times the overall thickness of the beam, which is consistent with that specified in the seismic design provisions of CSA A23.3-14.

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List of symbols

ε_c^{pl}	equivalent plastic strains in compression
ε_t^{pl}	equivalent plastic strains in tension
σ_{c0}	initial yield in compression
σ_{t0}	failure stress in tension

σ_{cu}	ultimate compression stress
f_t	tensile strength
σ_t	tensile stress
d_c	degradation of elastic stiffness in compression
d_t	degradation of elastic stiffness in tension
E_c	initial (undamaged) module of concrete
ε_c^{cr}	cracking strain in compression
ε_t^{cr}	cracking strain in tension
ε_{c0}^{el}	elastic strain in compression
ε_{t0}^{el}	elastic strain in tension
$\varepsilon_{50c(u)}$	strain corresponding to the stress equal to 50% of the maximum strength of confined or un-confined concrete
Δ	displacement at the bottom of the removed column
h	height of the concrete beam section