



Design of rectangular industrial duct plates subjected to out-of-plane pressure considering nonlinear large deformations



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ARTICLE INFO

Article history:

Received 28 July 2013

Received in revised form

5 November 2013

Accepted 22 November 2013

Available online 14 December 2013

Keywords:

Rectangular industrial duct

Stiffened plate

Finite element analysis

Bezier curves

Large deflection

Normalized Load Factor

ABSTRACT

Large industrial steel ducts are often rectangular and are built-up of stiffened plates. The plates along with stiffeners act to resist the pressure loads and transfer these loads to the supports. Parallel wide flange steel profiles, usually beams, are used as transverse stiffeners that are spaced perpendicular to the longitudinal axis of duct. The design process involves determining the load carrying capacity and deflection of the plate based on the plate thickness and the spacing between stiffeners. The current analysis and design method for industrial ducts is based on the elastic large deflection plate theory using standard tables.

A nonlinear finite element parametric study was conducted on dimensionless parameters to investigate the behavior of laterally loaded long plates. Through-thickness yielding of the plate and formation of partial plastic hinges at the ends is allowed. The results are presented in terms of a proposed dimensionless Normalized Load Factor (NLF) representing the applied pressure, the Normalized Deflection Factor (NDF) representing the out-of-plane deflection and the Normalized Maximum Stress Factor (NMSF) representing the maximum stresses induced in the plates. Design equations for deflection and stresses of plates are established with the aid of Bezier curves. A simple design procedure allowing for large deflection and partial yielding of edges is proposed. A limiting value for pressure has been found where it becomes irrelevant to check deflection. Results show that the proposed design procedure is simple and can lead to economic plate thicknesses and spacing of stiffeners in industrial ducts in ambient temperatures.

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1. Introduction

A series of large rectangular duct systems is required to transport large amount of air and flue-gases in many industrial processes such as coal power stations, industrial boiler applications and furnace off-gas systems. Duct systems are air tight conduits needed to transport air or flue gases under positive or negative pressure. They are also exposed to high temperatures due to the fact that the air or flue gases are transported at elevated temperatures. The large industrial ducts are often rectangular having dimensions that may range from 5 m to 15 m, sometimes even larger. Fig. 1 shows a schematic of typical components of a large rectangular industrial duct.

The design of large industrial ducts is not yet covered by any design standard; however, most practitioners follow the *guidelines* of the ASCE [1]. One of the difficulties facing engineers in the analysis and design procedures of this particular structure is the uncertainty of how the components should be designed, especially that the plates exhibit large deformations which make the analysis process completely different than using the regular small deflection theory. Very few technical information and design guidelines have been published describing analysis and design techniques for

such structures such as ASCE [1]. These guidelines require that the duct system should be designed to resist the following loads: (a) internal positive or negative pressure (operating or transient), (b) weight of the duct including insulation and lugging, (c) live loads such as ash weight, wind, snow and seismic (d) other loads such as thermal expansion or support reactions. Usually, the stiffener design is generally governed by the combination of transient internal pressure and wind load [2].

The behavior of the components of large industrial ducts is significantly different from those of regular plate girders. The design process of the large rectangular ducts generally involves local structural analysis of stiffeners and plates, and a global analysis of the whole duct structure. Although the plate is the most important structural element, it cannot be designed in isolation of the stiffeners supporting it. Typically, the design process involves determining the duct plate thickness, stiffener spacing, and proportioning of the stiffener section by considering one structural element at a time. In reality, the steel plate and the stiffeners act as one composite section resisting the lateral pressure. As such, the duct cross section, plate and stiffeners, act together to provide the necessary flexural stiffness for resisting transverse bending stresses. On the other hand, the side panels of the duct segment transfer the gravity loads to the supports by shear.

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Nomenclature

b	span of the plate between stiffeners [mm]
F_y	yield stress of the steel plate [MPa]
E	elasticity modulus of steel [MPa]
NDF	Normalized Deflection Factor [dimensionless]
NLF	Normalized Load Factor [dimensionless]
NMSF	Normalized Maximum Stress Factor [dimensionless]
t	thickness of the plate [mm]
p	lateral uniform pressure on the plate [MPa]
w	width of the plate strip [mm]
X_{pn}	X coordinate of Bezier curve control point n [dimensionless]

Y_{pn}	Y coordinate of Bezier curve control point n [dimensionless]
β	dimensionless factor representing plate slenderness [dimensionless]
Δ	deflection of plate [dimensionless]
λ	width-to-thickness ratio (b/t) [dimensionless]
σ	general term representing normal stresses in the plate [MPa]
σ_{max}	maximum stress induced in the plate (bending+membrane) [MPa]
ψ	dummy variable used to determine deflection of the plate [dimensionless]
ζ	dummy variable used to determine the maximum stress in the plate [dimensionless]

There is limited reported research work on large industrial ductworks, which resulted in the delay in development of updated guidelines and up-to-date design codes. The solutions for large deflections theory in plate design are complicated. The analysis of rectangular plates subjected to uniform lateral pressure and undergoing large out of plane deflection received increasing attention in 1940s and several successful attempts were made by Levy [3,4]. However, the outcome was a series of lengthy mathematical formulas that were difficult to use in regular design procedures. Young [5] has provided tabulated numerical values for dimensionless coefficients for the relations among pressure load, deflection and stresses for rectangular plates under uniform load producing large deflections. Unfortunately, these tables are limited to relatively low values of pressure. More recently, Wang et al. [6] proposed a simpler, yet still lengthy, mathematical procedure for an exact large-deflection mathematical analysis of rectangular plates with both fixed and simply supported edges. Such tables or solutions for the rectangular plate analysis are useful for design of the large industrial ductworks. Thanga et al. [2] carried out a comprehensive work on large industrial ductworks and proposed several dimensionless factors governing the behavior and strength of stiffened panels.

The objective of this study is to establish relations between loads, plate thickness and stiffener spacing for large industrial

ducts. In order to achieve this goal, a nonlinear finite element model was developed to accurately simulate the plate behavior in both the elastic and plastic ranges. The model was validated using results from theoretical solution and the values from “Roark’s Formulas” by Young [5] and the work done by Thanga et al. [2]. Another objective is to identify the fundamental parameters that dictate the behavior of laterally loaded plates for the numerical study. This necessitates a method of analysis that can trace the behavior of plates after undergoing large deflection. This study considers only plates subjected to static pressure loading and under ambient temperature.

2. Large versus small deflection theories

The classic theory of laterally loaded plate bending is often classified as *small* or *large* deflection plate theory, based on the ratio of the out-of-plane deflection, Δ , to the plate thickness, t . The small deflection theory does not consider the membrane stresses that develop in the plate when deflection to plate thickness ratio becomes large and when the plate edges are restrained from pulling-in. Hence, the relationship between pressure and deflection will be linear during small deflection [7].

Large deflection response occurs when the magnitude of the out-of-plane deflection becomes equal to or greater than half the thickness of the plates subjected lateral pressure [7]. The large out-of-plane deflection causes stretching of the plate resulting in membrane stresses in addition to bending stresses. If the edges are prevented from pulling in, membrane action becomes more significant and the overall behavior becomes nonlinear; this is what is referred to as nonlinear large deflection plate theory.

Deformation predictions based on the small deflection theory lead to excessive thickness or smaller spacing between the stiffeners. The plate between the stiffeners in large industrial ducts can undergo large deflections while performing safely and satisfying the serviceability limit. Such larger deflections would cause higher membrane stresses and, consequently, a higher proportion of the acting pressure will be supported through the development of membrane stresses rather than by bending stress. This illustrates that membrane action requires relatively higher deflections to be triggered. Therefore, it is anticipated that allowing partial yielding in the plate edges near supports will increase the deflection, hence increasing the membrane stresses.

In the past, the analysis and design of the plate thickness, i.e., the stiffener spacing, was done in accordance with pure plate bending behavior known as the elastic small deflection theory. Currently, many engineering firms determine the spacing of stiffeners based on elastic large deflection plate theory. Based on

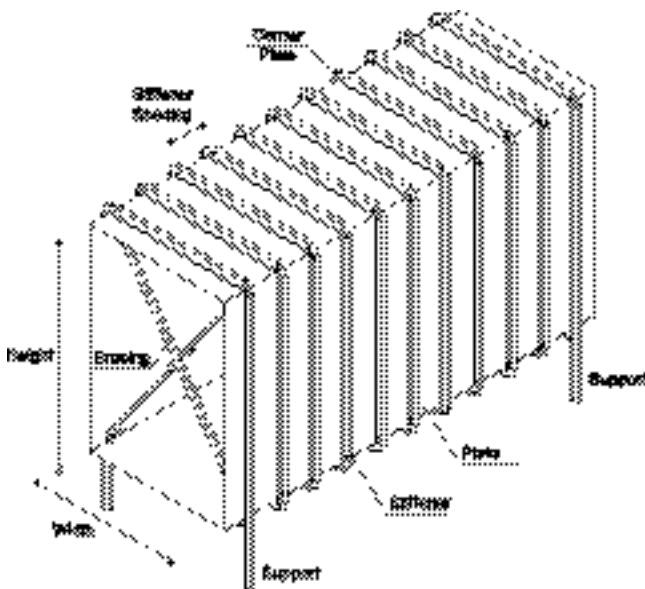


Fig. 1. Schematic of large rectangular ducts and its main structural components.

previous analytical studies for uniformly loaded rectangular plates with large deflections, various publications, such as Roark's formulas for stress and strain by Young [5], provide simplified design tables which are widely used in the industry. On the other hand, these tables are limited to relatively low values of lateral pressure.

3. Finite element parametric study

In this section, a nonlinear finite element (FE) model is developed using the commercial multi-purpose FE program ABAQUS [8] in order to investigate the nonlinear large deformation behavior of long steel duct plates subjected lateral pressure. The aim of this model is to investigate the best modeling techniques to trace the developed stresses and lateral deflections under uniform lateral pressure.

In order to capture the large deformations and the material nonlinearity, a general static analysis using nonlinear geometry option was used. The long plate was modeled using 4-noded "S4" shell elements from ABAQUS elements library. This general-purpose shell element can be used to model thick and thin shell structures and can be used in large displacement problems. All materials were modeled using an elastic-perfectly plastic material model. The FE model will be verified using theoretical closed form solutions and previous validation models from literature.

3.1. Numerical integration through thickness

Since the nonlinear FE is supposed to capture the onset and spread of material yielding, it is important to have the integration points (at which material yielding is established) distributed through the plate's thickness, including its top and bottom surfaces. This cannot be obtained using the Gauss quadrature rule as its sampling points lie within the thickness of the plate. On the other hand, this can be easily obtained using the Simpson rule. ABAQUS provides up to 99 through-thickness integration points;

for simplicity without sacrificing accuracy, only 11 points were considered in this study dividing the thickness into 10 layers. Fig. 2 illustrates the distribution of the integration points through the thickness of the shell element.

3.2. FE model and boundary conditions

In order for this study to be generic and independent of the shape and cross section of the stiffeners, only the portion between the stiffeners will be modeled. It will be assumed that the stiffeners are stiff enough to provide clamping of the plate and can be considered as a total fixation. Several sensitivity analyses were performed to know the effects of increasing the number of through thickness integration points and for the meshing of the plate strip. It was found that 11 points are sufficient to detect the onset and propagation of yielding through the thickness. Moreover, element length of $span/30$ was chosen to accurately model the deflection of the plate strip. The two edges representing the connection with the stiffeners were totally clamped. The other two edges were simply supported. The FE model and the boundary conditions with corresponding DOFs are shown in Fig. 3. The strip width "w" was taken equal to twice the strip span "b" to insure that the behavior of the strip is representing the plates in industrial duct which usually have width-to-span ratio varying from 2 to 10. It is worth mentioning that imposing an initial imperfection in the plates is not expected to significantly affect the plate's response at such high lateral pressures.

3.3. Verification model

Before proceeding into a detailed parametric study, a verification finite element (FE) model is developed in order to verify the accuracy of the proposed model in the linear range. In this model, the strip span $b=1000$ mm, strip width $w=200$ mm, plate

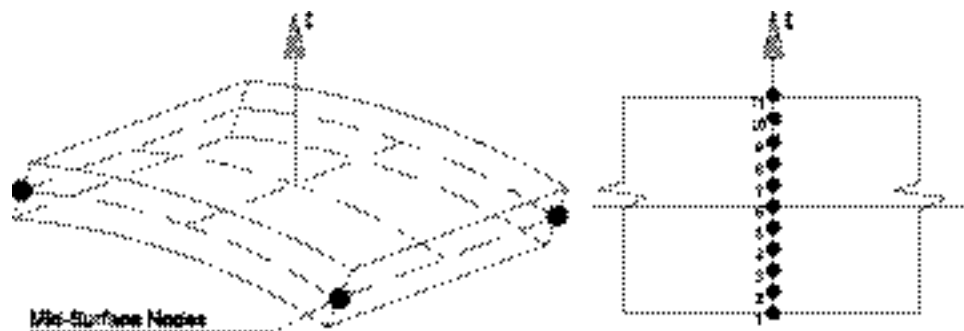


Fig. 2. Simpson (Newton-Cotes) integration points through the thickness.

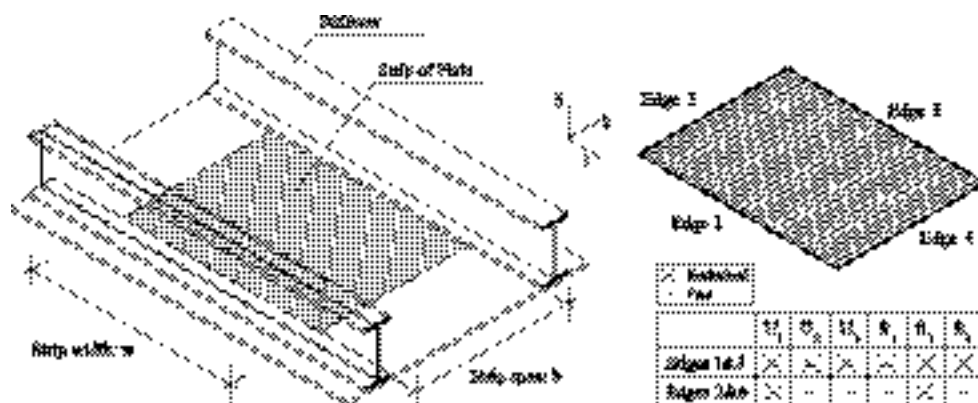


Fig. 3. Stiffener modeling and finite element model.

Table 1
Results of the verification study.

Clamped edge elemental strip of plate			
Reference	Membrane stress (MPa)	Maximum stress (MPa)	Maximum deflection (mm)
Roark's formula [3]	17.45	129.05	5.52
Timoshenko [5]	17.45	131.18	5.78
Thanga et al. [2]	16.70	132.00	5.84
Present study	17.89	131.04	5.78

thickness $t=5$ mm, the modulus of elasticity for steel is taken as 200 GPa and Poisson's ratio as 0.3. A uniform pressure $p=10$ kPa is applied to the plate. Table 1 lists the results obtained from the FE model versus the corresponding results obtained by the methods used in Roark's formula [5], Timoshenko [7] and Thanga et al. [2]. The obtained results from the model are in good agreement with other methods. The model will be then extended to study the full pressure-deformation nonlinear relationship while considering the effects of large deformation and membrane actions.

3.4. Parametric study

The design of industrial ducts is usually concerned with determining the optimum stiffener spacing and the plate thickness which is generally governed by serviceability rather than ultimate limit states. Due to the fact that spacing between stiffeners almost remains unchanged after deformation, and assuming that there is no relative deflection of the stiffeners in the U3 direction, thus the boundary condition of these stiffeners can be considered as clamped and restraining the pull in of stiffeners in case of the lateral loading of plates. These assumptions are seen to sufficiently represent the plate and stiffeners away from the duct ends.

As the plate is laterally loaded, yielding would first occur near the clamped edges. It is important to clearly estimate the level of lateral pressure that can be applied beyond the elastic range while satisfying the serviceability imposed requirements. The purpose of this study is to determine the relationship between the lateral pressure loads along with the plate characteristics and the induced stresses and deflections. Two steel materials which are commonly used in steel ductworks were chosen for this study with yield stresses F_y of 250 MPa and 350 MPa, respectively. The modulus of elasticity for steel E is taken as 200 GPa and Poisson's ratio as 0.3.

3.5. Results

Thanga et al. [2] proposed utilizing the dimensionless factor Δ/t to represent the deflection (which will be called the "Normalized Deflection Factor" (NDF) in this paper) and the dimensionless factor pE/F_y^2 to represent the loading factor considering the yield stress F_y of the plate steel. Thanga et al. [2] also showed and proved that all specimens having the same factor $\beta = b/t\sqrt{F_y/E}$ have the same load–deflection behavior. In these relationships, Δ is the deflection at the plate mid-point, b is the clear width of the plate strip between the stiffeners, t is the plate thickness; p is the applied lateral pressure; E is the modulus of elasticity and F_y is the yield stress. Results for the deflection factor versus load factor for different b/t ratios for two different practical cases, namely (a) $t=5$ mm, $F_y=250$ MPa and (b) $t=8$ mm, $F_y=350$ MPa are shown in Fig. 4. The two values of F_y represent the cases of a plate made of carbon steel Grade A36 and Gr50, respectively, which are common steel material used in the ductwork.

From Fig. 4, it can be seen that the deflection–load factors' relationships follow several curves. In this study, several deflection–load factors' relationships were examined in order to

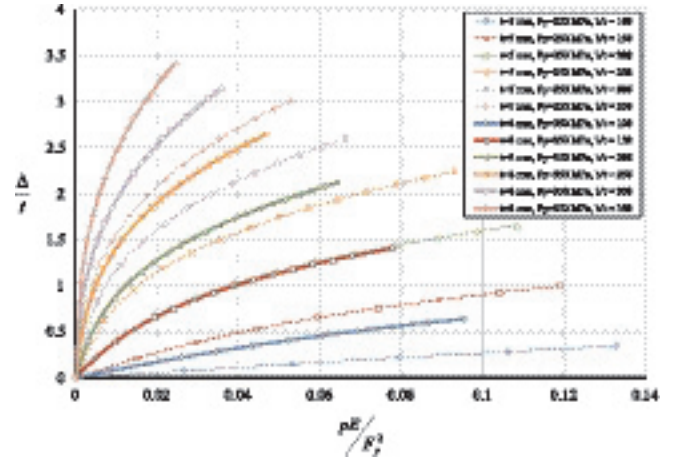


Fig. 4. Deflection factor versus load factor for different b/t ratios for two cases: (a) $t=5$ mm, $F_y=250$ MPa and (b) $t=8$ mm, $F_y=350$ MPa.

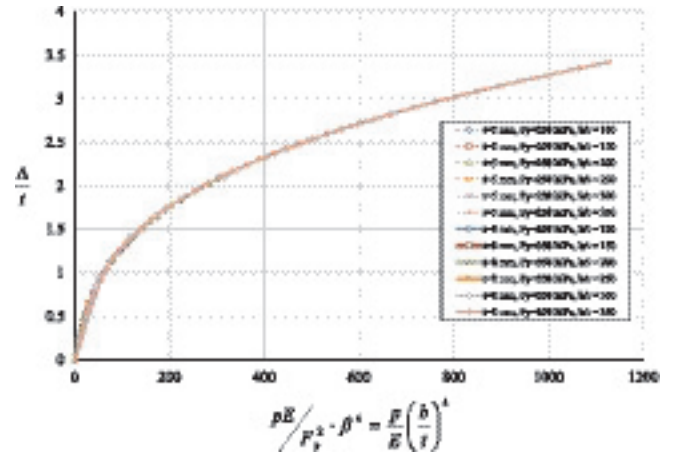


Fig. 5. Normalized Deflection Factor (NDF) versus the Normalized Load Factor (NLF) for a wide range of plate dimensions and yield strength.

normalize the relationship. It was found that by multiplying the x-coordinate of all points by the dimensionless factor β^4 which can be simplified as shown in Eq. (1), all the curves shown in Fig. 4 converge to one single, unified, curve as shown in Fig. 5. Therefore, the dimensionless factor representing the pressure will be here-with called the "Normalized Load Factor" (NLF) and will be represented by Eq. (2).

$$\therefore \beta^4 = \left(\frac{b}{t} \sqrt{\frac{F_y}{E}} \right)^4 = \left(\frac{b}{t} \right)^4 \frac{F_y^2}{E^2} \quad (1)$$

$$\therefore \text{Normalized Load Factor (NLF)} = \frac{pE}{F_y^2} \beta^4 = \frac{pE}{F_y^2} \left(\frac{b}{t} \right)^4 \frac{F_y^2}{E^2} = \frac{p}{E} \left(\frac{b}{t} \right)^4 \quad (2)$$

It is worth noting that the "Normalized Load Factor" shown in Eq. (2) was found to be the same as pressure coefficient used in "Roark's formula" [5]. But on the other hand, the available dimensionless coefficients in "Roark's formula" [5] are reported to be limited to low pressure loads and are valid within the elastic range before the onset of yielding. Herein, the use of this NLF factor for different values of F_y proved to be very efficient and representing the behavior of a wide range of clamped steel plates under uniform lateral pressure even after the partial yielding of the edges. It has also been noticed that the strain at the edges after yielding in all different specimens did not exceed 2.2% which is

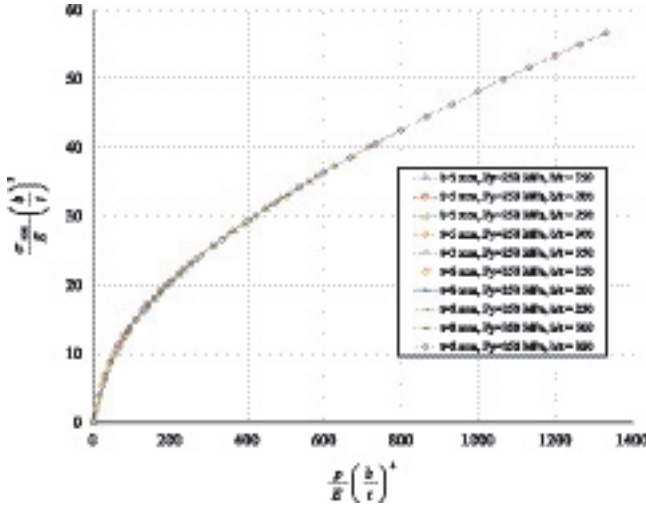


Fig. 6. Normalized Maximum Stress Factor (NMSF) versus the Normalized Load Factor (NLF) for a wide range of plate dimensions and yield strength.

less than the strain-hardening strain for the studied steel materials. This indicates that using the elastic-perfectly plastic material model is valid. It has also been noticed that the through-thickness yielding decreases as the slenderness of the plate increases and as the value of F_y increases. For example, none of the specimens with $F_y=350$ MPa had any yielding through their fibers near the edges under the studied range of the NLF from 0 to 1200.

Another aspect of the design process of industrial ducts is to check the maximum stresses in the mid-plate. According to “Roark’s formula” [5], both membrane and bending stresses are expressed by the dimensionless factor $\sigma/E \cdot (b/t)^2$. This dimensionless factor representing stresses in the plate will be herewith called the “Normalized Stress Factor” (NSF). The maximum stress σ_{max} induced in the plate for several thicknesses, widths and yield stresses will be plotted. Similar to the NSF, the maximum stress will be normalized and presented in the form of $\sigma_{max}/E \cdot (b/t)^2$ and will be called the “Normalized Maximum Stress Factor” (NMSF) and plotted versus the “Normalized Load Factor” as shown in Fig. 6.

4. Proposed design equations

4.1. Bezier curves for load–deflection

By plotting the Normalized Deflection Factor (NDF) versus the Normalized Load Factor (NLF), we can easily obtain a unified design chart for determining the nonlinear large deformation deflections of any plate subjected to uniform lateral pressure. This curve can be fitted using a sixth order polynomial, but this is seen to be impractical to codify for the use of practicing engineers. In this research, the use of Bezier curves [9] is proposed. Bezier curves are parametric functions which are very powerful in drawing curves and is highly utilized in computer graphics and drawing software. The NDF–NLF relationship in Fig. 5 can be represented by a 3-points Bezier curve as shown in Fig. 7. Moving the points P1, P2 and P3 in Fig. 7 adjusts the trend curve to fit the original one. The 3-points Bezier curve is the simplest possible curve as its order is quadratic. Using more points increases the accuracy but results in more complicated equations. It has been found that in this simple application, the 3-points Bezier curve yielded very accurate results. By observation, P1 is the start point and is located at the origin having coordinates of (0, 0). Point P3 was chosen as another point on the curve and was found after

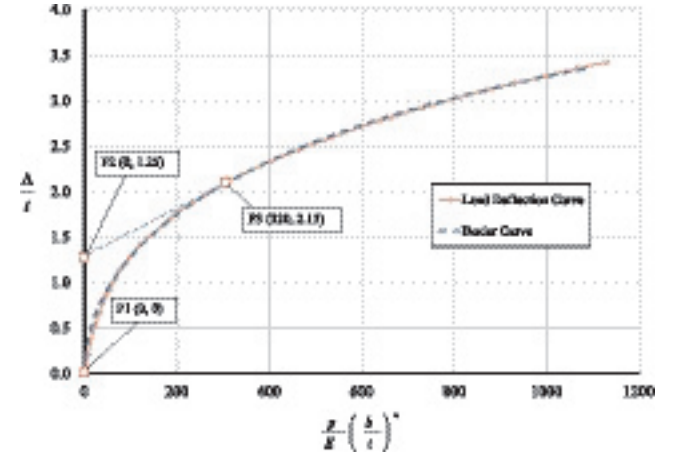


Fig. 7. Using 3-points Bezier curve and its fitting with the NDF–NLF relationship.

several trials and errors to be (320, 2.13). Point P2 was chosen to be located on the Y-axis in order to eliminate the x-coordinate and simplify the equations, thus P2 was found to be (0, 1.25).

The equation of Bezier curves—similar to any parametric function—can be represented using a dummy factor, ψ , where each coordinate x or y can be represented as a function in this factor as follows:

$$\begin{aligned} x &= X_{P1} \cdot (1-\psi)^2 + 2X_{P2} \cdot (1-\psi) \cdot \psi + X_{P3} \cdot \psi^2 \\ &= 0 \cdot (1-\psi)^2 + 2 \cdot 0 \cdot (1-\psi) \cdot \psi + 320 \cdot \psi^2 \\ &= 320 \cdot \psi^2 \end{aligned} \quad (3)$$

$$\begin{aligned} y &= Y_{P1} \cdot (1-\psi)^2 + 2Y_{P2} \cdot (1-\psi) \cdot \psi + Y_{P3} \cdot \psi^2 \\ &= 0 \cdot (1-\psi)^2 + 2 \cdot 1.25 \cdot (1-\psi) \cdot \psi + 2.13 \cdot \psi^2 \\ &= 2.5 \cdot (1-\psi) \cdot \psi + 2.13 \cdot \psi^2 \\ &= 2.5\psi - 0.37 \cdot \psi^2 \end{aligned} \quad (4)$$

where X_{Pn} , Y_{Pn} are the x and y coordinates, respectively, of each point n along the NDF–NLF curve. Therefore, in order to determine the deflection due to any lateral pressure p , the following systematic procedure should be followed:

- (1) Get the Normalized Load Factor NLF, the variable λ will represent the plate strip slenderness b/t

$$\text{NLF} = \frac{p}{E} \left(\frac{b}{t} \right)^4 = \frac{p}{E} \lambda^4 \quad (5)$$

- (2) Calculate the dummy variable ψ from

$$\psi = \sqrt{\frac{\text{NLF}}{320}} \quad (6)$$

- (3) Calculate the deflection from the following equation:

$$\Delta = t \cdot (2.5\psi - 0.37 \cdot \psi^2) \quad (7)$$

Note that the deflection, Δ , obtained from Eq. (6) should be less than $b/100$ in order to meet the current serviceability criteria set by ASCE [1]. Otherwise, the plate thickness need to be increased. Alternatively, the above set of design equations could be re-arranged in order to determine the smallest plate thickness that satisfies the ASCE’s limit of $b/100$. Setting Eq. (6) to be equal to $b/100$, we have

$$100 \cdot (2.5\psi - 0.37 \cdot \psi^2) = \frac{b}{t} \quad (8)$$

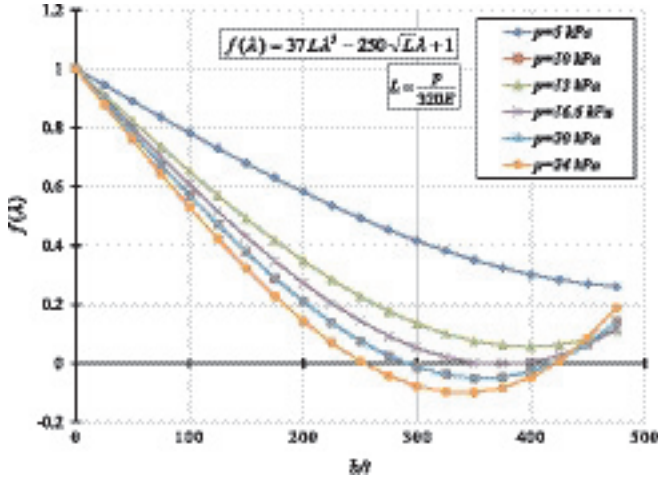


Fig. 8. Graphically finding the roots of Eq. (11) for different values of pressure p .

By rearranging and equating in order to obtain the limiting $\lambda = b/t$ ratio

$$\lambda = \frac{b}{t} = 250\psi - 37\psi^2 \quad (9)$$

From Eq. (6) and using $L = p/320E$

$$\psi = \sqrt{L\lambda^4} = \lambda^2 \sqrt{L} \quad (10)$$

By substituting Eq. (10) in Eq. (9), rearranging and simplifying we get

$$\lambda(37L\lambda^3 - 250\sqrt{L}\lambda + 1) = 0 \quad (11)$$

In Eq. (11), λ cannot be equal to 0, therefore the equation can be further simplified to

$$37L\lambda^3 - 250\sqrt{L}\lambda + 1 = 0 \quad (12)$$

By plotting Eq. (12) for several values the lateral pressure p as shown in Fig. 8, it has been found that Eq. (12) does not have real roots if the value of the lateral pressure p is less than 16.6 kPa. Thus, it could be said that it is not important to check the ASCE's deflection limit of $b/100$ when the pressure is less than 16.6 kPa. It should be noted that the pressure p refer to the resultant of various pressures acting on the plate, i.e., its self-weight, internal pressure, external pressure, wind, etc.

Thus the proposed procedure for finding the maximum allowable spacing from the serviceability limit can be summarized as follows:

- (1) Given a design pressure “ p ” greater than 16.6 kPa, calculate $L = p/320E$
- (2) Solve Eq. (12) for λ and take the smaller root value.
- (3) By assuming a certain thickness, we can easily obtain the spacing $b = \lambda t$

4.2. Load–maximum stress relationship

Similar to the NDF–NLF relationship developed in Section 4.1, the relationship between the “Normalized Maximum Stress Factor” (NMSF) versus the “Normalized Load Factor” (NLF) will be fitted using Bezier curves. By plotting the NMSF–NDF relationship, we can easily obtain a unified design chart for determining the maximum stresses induced in any plate subjected to uniform lateral pressure taking into consideration the large-deformations. The NMSF–NLF relationship in Fig. 6 can be represented by a 3-points Bezier curve as shown in Fig. 9. Moving the points P1, P2

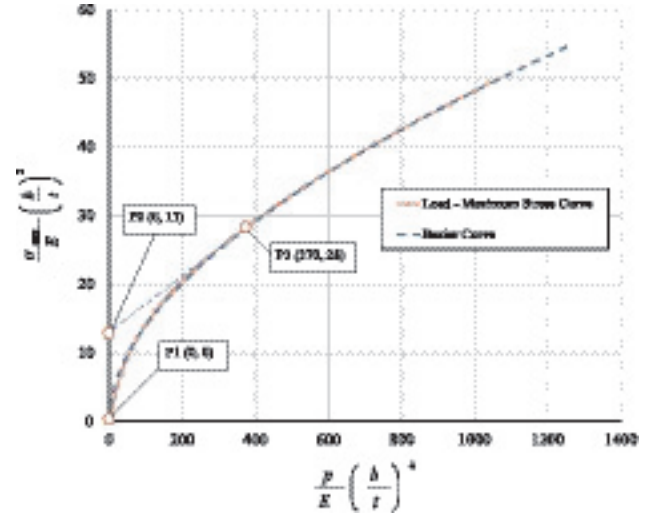


Fig. 9. Using 3-points Bezier curve and its fitting with the NMSF–NLF relationship.

and P3 in Fig. 9 adjusts the trend curve to fit the original one. By observation, P1 is the start point and is located at the origin having coordinates of (0, 0). Point P3 was chosen as another point on the curve and was found after several trials and errors to be (370, 28). Point P2 was chosen to be located on the Y-axis in order to eliminate the x-coordinate and simplify the equations, thus P2 was found to be (0, 13).

Similar to Section 4.1 and using a dummy variable, ζ , each coordinate x or y can be represented as a function in this factor as follows:

$$\begin{aligned} x &= X_{P1} \cdot (1-\zeta)^2 + 2X_{P2} \cdot (1-\zeta) \cdot \zeta + X_{P3} \cdot \zeta^2 \\ &= 0 \cdot (1-\zeta)^2 + 2 \cdot 0 \cdot (1-\zeta) \cdot \zeta + 370 \cdot \zeta^2 \\ &= 370 \cdot \zeta^2 \end{aligned} \quad (13)$$

$$\begin{aligned} y &= Y_{P1} \cdot (1-\zeta)^2 + 2Y_{P2} \cdot (1-\zeta) \cdot \zeta + Y_{P3} \cdot \zeta^2 \\ &= 0 \cdot (1-\zeta)^2 + 2 \cdot 13 \cdot (1-\zeta) \cdot \zeta + 28 \cdot \zeta^2 \\ &= 26 \cdot (1-\zeta) \cdot \zeta + 28 \cdot \zeta^2 \\ &= 26\zeta - 2 \cdot \zeta^2 \end{aligned} \quad (14)$$

Therefore, in order to determine the maximum stress due to any lateral pressure p , the following systematic procedure should be followed:

Calculate the Normalized Load Factor NLF from

$$\text{NLF} = \frac{p}{E} \left(\frac{b}{t} \right)^4 = \frac{p}{E} \lambda^4 \quad (15)$$

Calculate the dummy variable d from

$$\zeta = \sqrt{\frac{\text{NLF}}{370}} \quad \zeta = 0.93\psi \quad (16)$$

Calculate the maximum stress from the following equation:

$$\sigma_{\max} = E \cdot \left(\frac{t}{b} \right)^2 \cdot (26\zeta - 2 \cdot \zeta^2) \quad (17)$$

Or in terms of ψ

$$\sigma_{\max} = E \cdot \left(\frac{t}{b} \right)^2 \cdot (24.18\psi - 1.7298 \cdot \psi^2) \quad (18)$$

Note that the maximum stress, $\sigma_{\max}$, obtained from Eqs. (17) or (18) should usually be less than $0.6F_y$. Critical values for lateral pressure “ p ” can be easily determined by solving Eq. (18) for different yield stresses. It has been found that for $F_y = 350$ MPa, the limiting pressure = 121.5 kPa and for $F_y = 250$ MPa, the limiting pressure = 61.5 MPa.

5. Applications of the proposed procedures

5.1. Analysis example

In this section, a simple analysis example will be illustrated. The example will use a certain uniform lateral pressure load which is a result of the combination of several primary load cases. Given $b=1500$ mm, $t=5$ mm, $b/t=300$, $p=15$ kPa, $F_y=250$ MPa. Since $p < 16.6$ kPa, thus there is no need to check deflection as it will satisfy the $b/100$ limit set by the ASCE (1995). Yet for demonstration, the full procedure will be shown.

$$NLF = \frac{p}{E} \left(\frac{b}{t} \right)^4 = \frac{15/1000}{200000} \cdot 300^4 = 607.5$$

$$\psi = \sqrt{\frac{NLF}{320}} = 1.377$$

$$\zeta = \sqrt{\frac{NLF}{370}} = 1.281 \quad (\zeta = 0.93\psi)$$

$$\Delta = t \cdot (2.5\psi - 0.37 \cdot \psi^2) = 5 \cdot (3.44 - 0.7016) = 13.69 \text{ mm} \leq \frac{b}{100} = 15 \text{ mm}$$

$$\begin{aligned} \sigma_{max} &= E \cdot \left(\frac{t}{b} \right)^2 \cdot (26\zeta - 2 \cdot \zeta^2) \\ &= 200,000 \cdot \left(\frac{5}{1500} \right)^2 \cdot (33.306 - 3.282) \\ &= 66.72 \text{ MPa} \leq 0.6F_y = 150 \text{ MPa} \end{aligned}$$

5.2. Design example

This example illustrates the systematic procedures for determining the stiffener spacing for a pressure $p=18$ kPa (which is greater than 16.6 kPa). In this case, it is anticipated that we need to calculate the limiting slenderness.

$$L = \frac{p}{320E} = 2.8125 \times 10^{-10}$$

$$37L\lambda^3 - 250\sqrt{L}\lambda + 1 = 0$$

By solving, we get two roots, the least of them is 319.5.

By taking $t=6$ mm, therefore the maximum spacing $= 6 \times 319.5 = 1917$ mm.

We will take $b=1900$ mm, therefore $b/t=1900/6=316.67$.

$$NLF = \frac{p}{E} \left(\frac{b}{t} \right)^4 = \frac{18/1000}{200,000} \cdot 316.67^4 = 905.05$$

$$\psi = \sqrt{\frac{NLF}{320}} = 1.68 \quad \therefore s = 0.93\psi = 1.562$$

$$\Delta = t \cdot (2.5\psi - 0.37 \cdot \psi^2) = 6 \cdot (4.2 - 1.044)$$

$$= 18.93 \text{ mm} \leq \frac{b}{100} = 19 \text{ mm (safe and economic)}$$

$$\begin{aligned} \sigma_{max} &= E \cdot \left(\frac{t}{b} \right)^2 \cdot (26\zeta - 2 \cdot \zeta^2) = 200,000 \cdot \left(\frac{6}{1900} \right)^2 \\ &\cdot (40.612 - 4.879) = 71.27 \text{ MPa} \leq 0.6F_y = 150 \text{ MPa} \end{aligned}$$

6. Conclusions

The current practice in the design of large industrial ducts is to use design charts utilizing the elastic large deflection theory as this leads to a more economical design than using the small deflection theory. In this research, partial yielding of the plate edges clamped to the stiffeners is allowed and taken into consideration in the design. A parametric study was performed covering a wide range of plate spans, thicknesses and material properties. A “Normalized Load Factor” (NLF) was proposed and proved to be an effective dimensionless factor for plotting a unified load–deflection curve for all possible rectangular plates subjected to uniform lateral pressure. Two relationships were proposed, the Normalized Deflection Factor–Normalized Load Factor (NDF–NLF) and the Normalized Maximum Stress Factor–Normalized Load Factor (NMSF–NLF) relationships were proposed using Bezier curves. A simple analysis and design procedure and several equations were proposed to check the deflection and maximum induced stresses of such plates subjected to uniform lateral pressure.

Acknowledgments

The financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC) is greatly appreciated.

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