

A numerical element for vehicle–bridge interaction analysis of vehicles experiencing sudden deceleration



Hossein Azimi, Khaled Galal ^{*}, Oscar A. Pekau

Dept. of Building, Civil & Environmental Engineering, Concordia University, Montréal, Québec, Canada H3G 1M8

ARTICLE INFO

Article history:

Received 15 May 2012

Revised 14 September 2012

Accepted 27 December 2012

Available online 1 February 2013

Keywords:

Vehicle–bridge interaction

Bridge dynamics

Sudden braking

Numerical time-history analysis

Finite element method

ABSTRACT

Train sudden deceleration (braking) that causes sliding in wheels could impact the design of rail components, supports, bearings, and bridge substructure. Despite the availability of research on the vehicle–bridge interaction (VBI) analysis of vehicles with constant velocity, few studies have investigated the effect of vehicle longitudinal acceleration on VBI and more research is required. In this study, the effect of vehicle horizontal acceleration is incorporated into an available numerical procedure developed for VBI analysis of vehicles with constant velocity, and a modified numerical element is proposed. The effect of acceleration is considered as external loads acting on each part of the vehicle masses. Using suitable transformation matrices and defining a new parameter called acceleration parameter, the vertical contact forces are reformulated. Subsequently, a new formulation for a VBI element containing the effect of vehicle acceleration is obtained. VBI elements are those bridge elements in contact with wheels of moving vehicle. Numerical results are presented to evaluate the importance of the proposed VBI element, and to investigate the vehicle and the bridge responses. Considerable effect was observed on vehicle vertical contact forces, vehicle vertical acceleration, and somewhat on midspan deflection. Minor effect was observed on bridge midspan acceleration, for which the effect of vehicle initial velocity is prominent, not the vehicle acceleration.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

There has been a growing interest to model and analyze vehicle–bridge interaction (VBI) of intricate vehicles on bridges. VBI analysis is used if the dynamic response of the vehicle is required in addition to that of the bridge [1]. This is particularly found in case of high-speed trains where the vehicle acceleration is a design criterion for passenger comfort and needs to be well predicted. Following the growth in the use of high speed trains in Japan, China, and Europe, the past two decades witnessed more research activities in this area. Even though there is only one high-speed route currently in North America (Amtrak's Acela Express that runs on the Northeast Corridor from Boston to Washington, DC), there are several projected high-speed constructions in USA commencing 2012. For Canada, implementing new routes has been under discussion for a while in order to connect Edmonton to Calgary, and also to have another route from Windsor to Quebec City.

Majority of the research on the VBI analysis addresses vehicles moving with constant speed [2–4]. For VBI analysis, two sets of coupled equations of motion, one for the vehicle and one for the bridge should be solved. In very basic and simple cases, where

the moving load or moving mass models are considered together with basic beam models, closed-form solutions are available in literature [5,6]. For more complex vehicle models, numerical direct integration methods were adopted such as Newmark- β method [7], and α method [8]. In addition, modal superposition method has been extensively used [2,9,10]. Moreover, the Fourier transformation method has also been used [11]. In the vehicle–bridge interaction analysis that are based on the time history analysis, most of the methods used are iterative incremental analyses [2–4,11] where a convergence criterion is verified in each time increment.

Several researchers studied the response of bridges subjected to vehicles moving with constant speed. Yang and Wu [12] introduced a procedure to study the vehicle–bridge interaction which requires no iteration in each time increment using the Newmark- β unconditionally stable method [13]. They utilized the concept of dynamic condensation to modify the mass, damping, and stiffness matrices of those bridge elements in contact at each time increment. The bridge elements in contact are called VBI elements. The vertical contact force at the end of the time step was formulated based on the vehicle equations of motion. Building on the formulation for contact force developed by Yang and Wu [12], Azimi et al. [14] applied a relationship between bridge and wheel displacement vectors and their derivatives to obtain a new VBI

^{*} Corresponding author. Tel.: +1 514 8482424x3196; fax: +1 514 8487965.

E-mail addresses: h_azim@encs.concordia.ca (H. Azimi), galal@bcee.concordia.ca (K. Galal), oaapekau@civil.concordia.ca (O.A. Pekau).

element that considers the effects of shear deformation. The latter two methods will be used in the current study to develop a new VBI element that accounts for the effect of vehicle horizontal acceleration.

Unlike the wide range of available methods for the analysis of vehicle–bridge dynamic interaction for vehicles with constant velocity, few studies have investigated the effect of vehicle horizontal acceleration on the response of the vehicle and its interaction with the bridge. Among those, more studies dealt with highway bridges [15,16], and few were concerned with railway bridges [12]. The interaction of a three-axle truck model with the bridge having a non-uniform cross-section was studied [15], comprising the vehicle acceleration and the effect of road roughness. Ju and Lin [16] presented a finite element model of vehicle–bridge interaction considering braking and acceleration. Their method needs iteration in each time increment and they assumed constant horizontal acceleration for their formulation. Yang and Wu [12] modeled horizontal contact forces using friction coefficients multiplied by vertical contact forces. However, they used the same vertical contact forces formulated based on vehicles with constant velocity.

Nevertheless, the case of a vehicle with sudden braking with sliding has not been explicitly investigated. The vehicle acceleration during traction (when vehicle starts moving) is usually small since the driver slowly increases the speed in most cases. On the other hand, for sudden braking, the vehicle deceleration can be considerable and may affect the vehicle and bridge responses significantly. Although, sudden deceleration is to be avoided during the train movement, it is inevitable and can be applied by the driver or the passengers. Extreme braking situations can alter the contact forces significantly, that may affect the design of fasteners, supports, bearings, and bridge substructure such as piers.

In this paper, a modified two-dimensional VBI element is developed for vehicles experiencing sudden deceleration with sliding which is an extension to the models developed by Yang and Wu [12] and Azimi et al. [14]. In the current study, the analysis type used for the VBI problem is the non-iterative numerical time history analysis with suitable time increments. The developed VBI element incorporates the effect of the horizontal inertial forces on the response of the vehicle and the bridge. These forces will change the vertical contact forces due to a pitching moment. Vertical contact forces are reformulated by defining transformation matrices, a geometry matrix and a dimensionless equilibrium matrix. Then, a new parameter named as *acceleration parameter* is defined assisting the formulation in a practical and concise form. This parameter is calculated at the beginning of the analysis and is constant throughout the analysis, which makes it convenient for numerical programming. The new formulation for the vertical contact forces, and also the effect of horizontal contact forces, will result in a new VBI element capable of modeling and analyzing intricate vehicle models and bridges.

Vehicle models excluding horizontal DOFs are used for developing the VBI element. This type of vehicle modeling is more common and was used by majority of researchers [2,3,13,17,18]. Nevertheless, models including horizontal DOFs were also used by few researchers [16]. Consequently, the current paper will focus only on models excluding horizontal spring-dampers; nonetheless, it can be simply adjusted for vehicle models including horizontal DOFs.

A numerical model is used to validate the proposed VBI element and also to verify a module written in MATLAB® [19] developed for executing the time-history dynamic analysis. A 4-DOF vehicle model is applied to compare the results of the new VBI element with published ones in the literature. In addition, a high-speed train model of 10-DOF is considered to study the effect of horizontal deceleration on the train and also bridge responses. Various

important bridge and vehicle responses are considered in the analysis. The first response is the midspan deflection as the main parameter used in the bridge design manuals (e.g. AREMA [20]) to calculate the dynamic amplification factor. Another important response is the bridge midspan acceleration essential in the design of railway bridges for high-speed trains. The other important responses are the vehicle vertical acceleration governing the passenger comfort in high-speed trains, and vertical contact forces affecting wheel and road surface wearing. In this paper, the suspension system is modeled with linear properties. Despite the application of nonlinear suspension [9], linear suspension is commonly applied in most VBI analysis studies, for train models in particular.

2. Formulation of acceleration forces

In the development of the VBI element, the effect of vehicle horizontal deceleration should be included in contact forces. Hence, inertial forces generated by vehicle horizontal acceleration need to be formulated appropriately. The term *acceleration forces* is used here to distinguish those inertial forces generated by horizontal vehicle acceleration in the considered DOFs of the vehicle model. These forces are treated as external forces and can be added to the regular vector of external forces. A procedure is described here to formulate such forces based on vertical contact forces. This is a key step in the formulation of contact forces for the new VBI element. The procedure is first described for an example vehicle and will be later generalized for a general vehicle model.

An example vehicle model is shown in Fig. 1 formed of two wheels, and one car body. r_1 to r_4 denote vehicle DOFs. They all represent vertical DOFs except r_2 denoting pitching DOF of the car body in the direction shown. The suspension system consists of linear spring-dampers (k_{vi} and c_{vi} , $i = 1, 2$) with the vehicle body mass m_c , the rotary inertia I_c , and the mass of wheels m_{w1} and m_{w2} . The vehicle velocity is denoted as v and the vehicle acceleration as a in the positive direction. They will have negative values in the reverse direction. Moreover, d_1 and d_2 are the spacing of the center of gravity of wheels with respect to that for the car body.

The free body diagrams of dynamic forces for the car body and one wheel are shown in Fig. 2 where SP_w are the spring-damper forces, F_w are the internal horizontal forces, V_i are the total vertical contact forces, V'_i are the dynamic vertical contact forces, and μ is the kinetic friction coefficient in case of braking with sliding (or friction coefficient in case of traction from rest with sliding). T_E is the torque applied by the engine exerted on the wheel axle.

In this study, matrices and vectors will be represented by quantities enclosed by $[]$ and $\{ \}$, respectively. The symbol $\langle \rangle$ stands for row vectors, i.e. $\{ \} = \langle \rangle^T$. In the formulation of VBI element, it is assumed that the vehicle has reached its static equilibrium at rest before starting the movement which is valid for all conventional vehicles. Therefore, vehicle self-weight and associated static deflections are not considered in the dynamic analysis of the vehicle. The vertical contact force V'_i is used excluding the effect of the

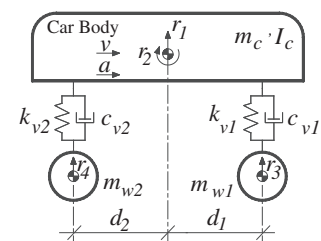


Fig. 1. A generic vehicle model of 4-DOF.

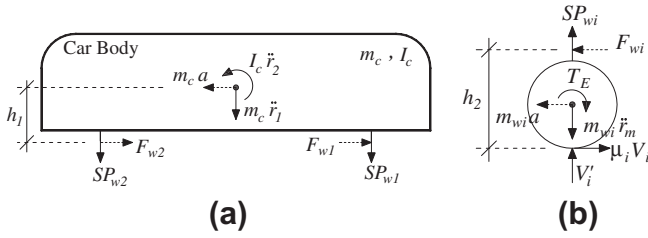


Fig. 2. Free body diagrams of 4-DOF model of: (a) the car body, (b) wheels.

vehicle self-weight. However, for the bridge analysis and also for calculation of the horizontal contact forces generated by friction, the total vertical contact force denoted as V_i has to be used. The effect of the vehicle self-weight is treated as external loads acting at the contact points remaining constant throughout the analysis. Equilibrium for wheels implies that:

$$\mu_i V_i - m_{wi} a + F_{wi} = 0 \quad i = 1, 2 \quad (1a)$$

$$V'_1 + SP_{w1} - m_{w1} \ddot{r}_3 = 0; \quad V'_2 + SP_{w2} - m_{w2} \ddot{r}_4 = 0 \quad (1b)$$

and for the car body,

$$F_{w1} + F_{w2} - m_c a = 0 \quad (2a)$$

$$SP_{w1} + SP_{w2} + m_c \ddot{r}_1 = 0 \quad (2b)$$

$$I_c \ddot{r}_2 + SP_{w2} d_2 - SP_{w1} d_1 = -(F_{w1} + F_{w2}) h_1 \quad (2c)$$

where a is the vehicle acceleration. Knowing that Eqs. (1b), (2b), and (2c) are those for the assumed DOFs, they comprise the equations of motion for the vehicle. Comparing those three equations with the case with no horizontal acceleration, only one new term is present which is $-(F_{w1} + F_{w2}) h_1$ in Eq. (2c) representing an extra pitching moment on the car body. Hence, the vector of acceleration forces $\mathbf{f}_a$ defined as forces exerted on vehicle dynamic DOFs due to horizontal vehicle acceleration is written,

$$\mathbf{f}_a = \langle 0, -(F_{w1} + F_{w2}) h_1, 0, 0 \rangle^T \quad (3)$$

Then, the vector of acceleration forces $\mathbf{f}_a$ can be decomposed into two parts as:

$$\mathbf{f}_a = \Phi \mathbf{F} \quad (4)$$

where Φ is named as geometry matrix and for this example is obtained as:

$$\Phi = \begin{bmatrix} 0 & 0 \\ -h_1 & -h_1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (5)$$

and $\mathbf{F}$ is the vector of internal horizontal forces, $\mathbf{F} = \langle F_{w1}, F_{w2} \rangle^T$. To determine $\mathbf{F}$, Eq. (1a) is used where F_{wi} ($i = 1, 2$) are based on vertical contact forces V and vehicle acceleration a . Since there are no horizontal dynamic DOFs, all parts of the vehicle move equally in the horizontal (longitudinal) direction with the same velocity and acceleration due to vehicle sliding. Hence, the vehicle horizontal acceleration a can be calculated based on the horizontal contact forces and total vehicle masses as:

$$a = \frac{\sum_{i=1}^n \mu_i V_i}{M_t} \quad (6)$$

where M_t is the vehicle total mass and n is the number of wheels. Assuming that μ_i are known (sudden braking with sliding), horizontal internal forces F_{wi} are determined only as function of vertical contact forces according to Eq. (1a). In the case of vehicle braking, μ_i are substituted with negative values in the developed numerical

element. Using Eq. (1a) together with Eq. (6), vector $\mathbf{F}$ is formulated as $\mathbf{F} = \Omega \mu \mathbf{f}_c$ where Ω is defined as dimensionless matrix of equilibrium,

$$\Omega = \begin{bmatrix} 1 - \frac{m_{w1}}{M_t} & -\frac{m_{w1}}{M_t} \\ -\frac{m_{w2}}{M_t} & 1 - \frac{m_{w2}}{M_t} \end{bmatrix} \quad (7)$$

and $\mathbf{f}_c$ is the vector of vertical contact forces and also μ is a 2×2 diagonal matrix of the corresponding kinematic friction coefficients between each wheel and the bridge surface. Finally, the vector of acceleration forces $\mathbf{f}_a$ is obtained as a function of vertical contact forces $\mathbf{f}_c$,

$$\mathbf{f}_a = \Phi \Omega \mu \mathbf{f}_c \quad (8)$$

where the matrices Φ , Ω , and μ are calculated for any vehicle model and are known throughout the analysis. The vector of acceleration forces can be written as $\mathbf{f}_a = \langle \mathbf{f}_{ua}, \mathbf{f}_{wa} \rangle^T$, where $\mathbf{f}_{wa}$ is due to wheel DOFs and $\mathbf{f}_{ua}$ is pertinent to the upper (or vehicle body) DOFs. Similarly, the geometry matrix Φ can be decomposed into the counterparts Φ_w and Φ_u corresponding to wheel and upper DOFs, respectively. The procedure described for the example vehicle can be further generalized. Fig. 3 shows a flowchart for the formulation of acceleration forces $\mathbf{f}_{wa}$ and $\mathbf{f}_{ua}$ of a general vehicle. In Fig. 3, subscript n denotes the number of wheels, m represents the number of DOFs of upper part of the vehicle model, s is the total number of vehicle DOFs (i.e. $s = n + m$), and k is the number of internal horizontal forces.

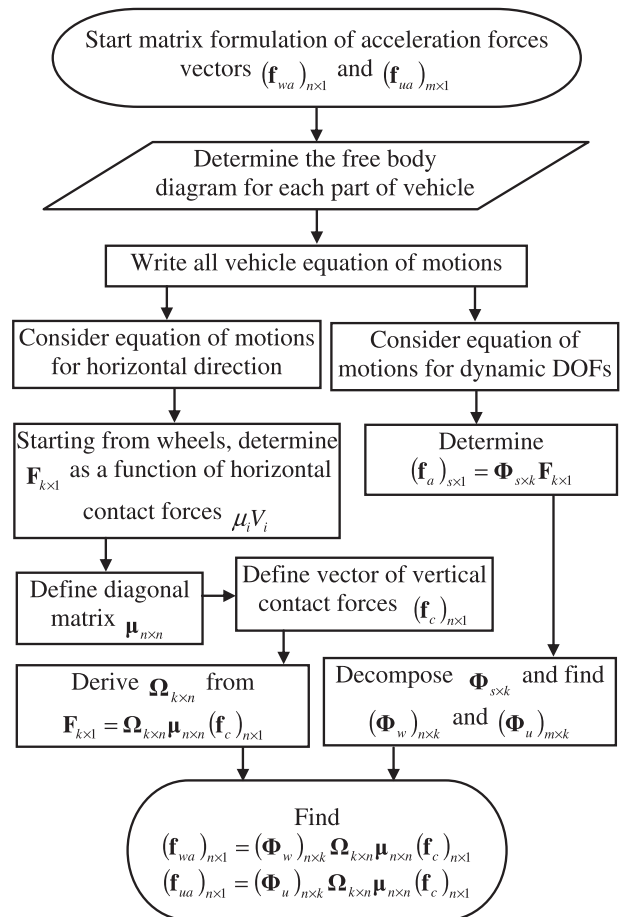


Fig. 3. Flowchart for formulation of acceleration forces.

3. Formulation of vertical contact forces

As stated in the preceding section, the effect of horizontal acceleration can be treated as external forces. Therefore, the equation of motion including the effect of acceleration is written by adding a new term to the external forces of the equation of motion of vehicles with constant velocity available in literature [12] as follows:

$$\begin{bmatrix} \mathbf{m}_{uu} & \mathbf{m}_{uw} \\ \mathbf{m}_{wu} & \mathbf{m}_{ww} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{d}}_u \\ \ddot{\mathbf{d}}_w \end{Bmatrix}_{t+\Delta t} + \begin{bmatrix} \mathbf{c}_{uu} & \mathbf{c}_{uw} \\ \mathbf{c}_{wu} & \mathbf{c}_{ww} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{d}}_u \\ \dot{\mathbf{d}}_w \end{Bmatrix}_{t+\Delta t} + \begin{bmatrix} \mathbf{k}_{uu} & \mathbf{k}_{uw} \\ \mathbf{k}_{wu} & \mathbf{k}_{ww} \end{bmatrix} \begin{Bmatrix} \mathbf{d}_u \\ \mathbf{d}_w \end{Bmatrix}_{t+\Delta t} = \begin{Bmatrix} \mathbf{f}_{ue} \\ \mathbf{f}_{we} \end{Bmatrix}_{t+\Delta t} + \begin{bmatrix} \mathbf{I}_u \\ \mathbf{I}_w \end{bmatrix} \mathbf{f}_{c,t+\Delta t} + \begin{Bmatrix} \mathbf{f}_{ua} \\ \mathbf{f}_{wa} \end{Bmatrix}_{t+\Delta t} \quad (9)$$

where the last term $\langle \mathbf{f}_{ua}, \mathbf{f}_{wa} \rangle_{t+\Delta t}^T$ represents the effect of vehicle horizontal acceleration. Eq. (9) is the equation of motion for two sets of vehicle DOFs, the upper part or non-contact part $\mathbf{d}_u$, and the wheel DOFs $\mathbf{d}_w$. The subscript $t + \Delta t$ represents the end of the time step t with the time increment Δt . The matrices of mass, damping, and stiffness are divided into four sub-matrices. Other external forces are those on upper part DOFs $\mathbf{f}_{ue}$ and wheel DOFs $\mathbf{f}_{we}$. Vertical contact forces are also represented with the vector $\mathbf{f}_{c,t+\Delta t}$. Matrices $\mathbf{I}_u$ and $\mathbf{I}_w$ transform the contact forces from contact points to the vehicle DOFs. Apparently, $\mathbf{I}_u$ is a zero matrix, since the contact forces are only acting on wheels and $\mathbf{I}_w$ is a unit matrix for 2-D models.

Then, Eq. (9) is expanded through a similar procedure available in literature [12], and the effect of acceleration forces $\mathbf{f}_{ua,t+\Delta t}$ and $\mathbf{f}_{wa,t+\Delta t}$ is included in the following:

$$\bar{\mathbf{p}}_{c,t+\Delta t} = \mathbf{I}_w^{-1} (\Psi_{wu} \Psi_{uu}^{-1} (\mathbf{f}_{ue,t+\Delta t} + \mathbf{f}_{ua,t+\Delta t}) - (\mathbf{f}_{we,t+\Delta t} + \mathbf{f}_{wa,t+\Delta t})) \quad (10)$$

which is called the vector of external forces and can be rearranged as:

$$\bar{\mathbf{p}}_{c,t+\Delta t} = \mathbf{p}_{c,t+\Delta t} + \mathbf{I}_w^{-1} (\Psi_{wu} \Psi_{uu}^{-1} \mathbf{f}_{ua,t+\Delta t} - \mathbf{f}_{wa,t+\Delta t}) \quad (11)$$

where $\mathbf{p}_{c,t+\Delta t}$ includes the effect of external forces $\mathbf{f}_{ue,t+\Delta t}$ and $\mathbf{f}_{we,t+\Delta t}$. In addition, Ψ_{wu} and Ψ_{uu} are constant matrices determined based on the vehicle mass, damping and stiffness matrices and factors of Newmark- β method and also time increment assumed, which are given elsewhere [12]. Moreover, Ψ_{wu} and Ψ_{uu} are $n \times m$ and $m \times m$ matrices, where n denotes the number of wheels and m represents the number of DOFs of upper part of the vehicle model.

The acceleration forces $\mathbf{f}_{ua,t+\Delta t}$ and $\mathbf{f}_{wa,t+\Delta t}$ in Eq. (11) are substituted with outputs of the flowchart illustrated in Fig. 3,

$$\bar{\mathbf{p}}_{c,t+\Delta t} = \mathbf{p}_{c,t+\Delta t} + \mathbf{I}_w^{-1} (\Psi_{wu} \Psi_{uu}^{-1} \Phi_u - \Phi_w) \Omega \mu \mathbf{f}_{c,t+\Delta t} \quad (12)$$

Using $\bar{\mathbf{p}}_{c,t+\Delta t}$, the contact force formula for vehicles with constant velocity [12] is modified as follows for vehicles experiencing acceleration:

$$\mathbf{f}_{c,t+\Delta t} = \mathbf{m}_c \ddot{\mathbf{d}}_{c,t+\Delta t} + \mathbf{c}_c \dot{\mathbf{d}}_{c,t+\Delta t} + \mathbf{k}_c \mathbf{d}_{c,t+\Delta t} + \bar{\mathbf{p}}_{c,t+\Delta t} + \mathbf{q}_{c,t} \quad (13)$$

where $\mathbf{d}_{c,t+\Delta t}$ is the vector of contact point displacement, and $\mathbf{m}_c$, $\mathbf{c}_c$, and $\mathbf{k}_c$ are called contact matrices which are constant [12]. Eq. (13) is modified as following applying Eq. (12),

$$\begin{aligned} & (\mathbf{I} - \mathbf{I}_w^{-1} (\Psi_{wu} \Psi_{uu}^{-1} \Phi_u - \Phi_w) \Omega \mu) \mathbf{f}_{c,t+\Delta t} \\ & = \mathbf{m}_c \ddot{\mathbf{d}}_{c,t+\Delta t} + \mathbf{c}_c \dot{\mathbf{d}}_{c,t+\Delta t} + \mathbf{k}_c \mathbf{d}_{c,t+\Delta t} + \mathbf{p}_{c,t+\Delta t} + \mathbf{q}_{c,t} \end{aligned} \quad (14)$$

where $\mathbf{I}$ is the unit matrix. For simplicity, let an acceleration parameter $[\mathbf{z}]$ be defined as:

$$\mathbf{z} = (\mathbf{I} - \mathbf{I}_w^{-1} (\Psi_{wu} \Psi_{uu}^{-1} \Phi_u - \Phi_w) \Omega \mu)^{-1} \quad (15)$$

All terms in Eq. (15) are time-independent. Finally, the contact forces $\{\mathbf{f}_c\}_{t+\Delta t}$ are obtained as

$$\mathbf{f}_{c,t+\Delta t} = \hat{\mathbf{m}}_c \ddot{\mathbf{d}}_{c,t+\Delta t} + \hat{\mathbf{c}}_c \dot{\mathbf{d}}_{c,t+\Delta t} + \hat{\mathbf{k}}_c \mathbf{d}_{c,t+\Delta t} + \hat{\mathbf{p}}_{c,t+\Delta t} + \hat{\mathbf{q}}_{c,t} \quad (16)$$

where matrices with hat-sign are in their transformed form of the corresponding matrices and vectors shown in Eq. (13), each pre-multiplied by the acceleration parameter $\mathbf{z}$ (e.g. $\hat{\mathbf{m}}_c = \mathbf{z} \mathbf{m}_c$). Eq. (16) is the new formulation of vertical contact forces including the effect of vehicle horizontal acceleration.

The acceleration parameter $\mathbf{z}$ is an $n \times n$ matrix where n denotes the number of wheels. This parameter is constant throughout the analysis and is calculated only at the beginning of the analysis. It represents the initial redistribution of vertical contact forces once the horizontal deceleration is applied. This is clear in Eq. (14), where $\mathbf{z}$ is pre-multiplied by the right-hand-side of that equation which is the formula for vertical contact forces when no horizontal acceleration is present. The matrix $\mathbf{z}$ is not a diagonal matrix, implying that the new values of vertical contact forces of each wheel after the redistribution are affected by other contact forces, as well. However, calculations showed that the effect is usually small. Another property of $\mathbf{z}$ is that its determinant is equal to one, which could be attributed to the fact that it represents only the redistribution of vertical contact forces while the summation of total contact forces remains the same. In addition, it should be noted that prior to the application of horizontal acceleration, $\mathbf{z}$ is equal to a unit matrix. From numerical point of view, it is as if $\mu = 0$ in Eq. (15).

4. A VBI element for vehicles experiencing sudden braking

For the case of vehicles with constant velocity, the equilibrium is satisfied via driving horizontal forces and vertical contact forces that transfer the vehicle weights to the bridge. On the other hand, if the vehicle experiences acceleration or deceleration, additional horizontal contact forces will be present at the contact points. However, in the finite element formulation, having structural matrices (i.e. mass, damping, and stiffness) of the bridge, it is required to find the nodal forces induced by those contact forces shown in Fig. 4 to be able to solve the bridge's equations of motion. The consistent nodal forces f_{bcj} , $j = 1-6$, shown in Fig. 4 can be determined from contact forces by using interpolation functions $\mathbf{N}_c^v$ and $\mathbf{N}_c^h$ as follows [21]:

$$\mathbf{f}_{bc} = \mathbf{V} \mathbf{N}_c^v + \mathbf{H} \mathbf{N}_c^h \quad (17)$$

The interpolation vector $\mathbf{N}_c^v$ can be calculated by Hermitian cubic polynomials, while $\mathbf{N}_c^h$ is usually considered as a linear function. All the entries in $\mathbf{N}_c^v$ are set to zero except those regarding vertical DOFs. This ensures that there would be only vertical resultant nodal forces from vertical contact forces V . Similarly, all entries of the vector $\mathbf{N}_c^h$ are set to zero except those due to horizontal DOFs in order to relate horizontal contact force H to horizontal nodal forces f_{bc2} and f_{bc5} . The interpolation functions $\mathbf{N}_c^v$ and $\mathbf{N}_c^h$ are calculated based on the location of the wheel on the element which is a function of time due of the vehicle movement.

By applying the new formula for vertical contact forces (Eq. (16)) in the method developed by Azimi et al. [14] the new formulation for VBI element considering the effect of acceleration is obtained. After relating the contact forces to nodal forces in Eq. (17),

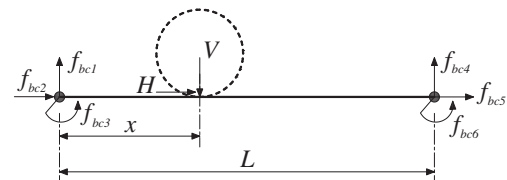


Fig. 4. A 2-D VBI element for vehicles with acceleration.

it is required to relate the displacement of the contact point to those of the nodal ones. For a 6-DOF beam element shown in Fig. 4, in order to obtain contact displacement d_c from nodal displacement vector $\mathbf{d}_b$, an interpolation function $\mathbf{N}_c^{vr}$ is used:

$$d_c = (\mathbf{N}_c^{vr})^T \mathbf{d}_b \quad (18)$$

where the interpolation function $\mathbf{N}_c^{vr}$ is determined using Hermitian cubic polynomials including the effect of the rotational DOFs in addition to the vertical DOFs. Also, the superscript T stands for the transpose of the matrix within the parentheses shown. By taking derivative of Eq. (18) with respect to time and using the chain rule, the velocity of the contact point is obtained:

$$\dot{d}_c = \frac{d(\mathbf{N}_c^{vr})^T}{dt} \mathbf{d}_b + (\mathbf{N}_c^{vr})^T \dot{\mathbf{d}}_b \quad (19)$$

Similar to $\mathbf{N}_c^v$ and $\mathbf{N}_c^h$, $\mathbf{N}_c^{vr}$ is a function of x , the location of the wheel, which by itself is a function of time; therefore:

$$\frac{d(\mathbf{N}_c^{vr})^T}{dt} = \frac{d(\mathbf{N}_c^{vr})^T}{dx} \times \frac{dx}{dt} = (\mathbf{N}_c^{vr})_{,x}^T \times v \quad (20)$$

where the comma denotes the differentiation with respect to the coordinates specified, and v is the vehicle velocity. Hence,

$$\dot{d}_c = v(\mathbf{N}_c^{vr})_{,x}^T \mathbf{d}_b + (\mathbf{N}_c^{vr})^T \dot{\mathbf{d}}_b \quad (21)$$

Similarly, the second derivative of the contact point is obtained as:

$$\ddot{d}_c = (\mathbf{N}_c^{vr})^T \ddot{\mathbf{d}}_b + 2v(\mathbf{N}_c^{vr})_{,x}^T \dot{\mathbf{d}}_b + (v^2(\mathbf{N}_c^{vr})_{,xx}^T + a(\mathbf{N}_c^{vr})_{,x}^T) \mathbf{d}_b \quad (22)$$

The last step is to obtain the VBI element equation of motion. In general, the beam element that is directly in contact with traversing wheels (i.e. VBI element) can be written as [14]:

$$\mathbf{m}_{bi} \ddot{\mathbf{d}}_{bi,t+\Delta t} + \mathbf{c}_{bi} \dot{\mathbf{d}}_{bi,t+\Delta t} + \mathbf{k}_{bi} \mathbf{d}_{bi,t+\Delta t} = \mathbf{f}_{bi,t+\Delta t} + \mathbf{f}_{bci,t+\Delta t} \quad (23)$$

where $\mathbf{m}_{bi}$, $\mathbf{c}_{bi}$, and $\mathbf{k}_{bi}$ are the i th VBI element mass, damping, and stiffness matrices, respectively. In addition, the vector $\mathbf{d}_{bi}$ and its derivatives are due to nodal displacements of the corresponding VBI element, $\mathbf{f}_{bci}$ represents the nodal forces induced by contact forces according to Eq. (17), and finally $\mathbf{f}_{bi}$ includes the effect of other external nodal forces that may be applied on the VBI element regardless of the moving vehicle.

If one substitutes $\mathbf{f}_{bci}$ of Eq. (17) into Eq. (23) and also applies Eq. (16) in Eq. (17) considering Eqs. (18), (21), and (22), the new VBI element equation of motion is formulated as:

$$\mathbf{m}_{bi} \ddot{\mathbf{d}}_{bi,t+\Delta t} + \mathbf{c}_{bi} \dot{\mathbf{d}}_{bi,t+\Delta t} + \mathbf{k}_{bi} \mathbf{d}_{bi,t+\Delta t} = \mathbf{f}_{bi,t+\Delta t} - \sum_{j=1}^n (\hat{\mathbf{m}}_{cij}^* \ddot{\mathbf{d}}_{bj} + \hat{\mathbf{c}}_{cij}^* \dot{\mathbf{d}}_{bj} + \hat{\mathbf{k}}_{cij}^* \mathbf{d}_{bj}) - \hat{\mathbf{p}}_{ci,t+\Delta t}^* - \hat{\mathbf{q}}_{ci,t}^* \quad (24)$$

where matrices with an asterisk incorporate the effect of the vehicle contact forces for the VBI element,

$$\hat{\mathbf{m}}_{cij}^* = (\mathbf{N}_{ci}^h \mu_i + \mathbf{N}_{ci}^v) \hat{\mathbf{m}}_{cij} (\mathbf{N}_{cj}^{vr})^T \quad (25a)$$

$$\hat{\mathbf{c}}_{cij}^* = (\mathbf{N}_{ci}^h \mu_i + \mathbf{N}_{ci}^v) \left(2v \times \hat{\mathbf{m}}_{cij} (\mathbf{N}_{cj}^{vr})_{,x}^T + \hat{\mathbf{c}}_{cij} (\mathbf{N}_{cj}^{vr})^T \right) \quad (25b)$$

$$\hat{\mathbf{k}}_{cij}^* = (\mathbf{N}_{ci}^h \mu_i + \mathbf{N}_{ci}^v) \left(v^2 \times \hat{\mathbf{m}}_{cij} (\mathbf{N}_{cj}^{vr})_{,xx}^T + (a \times \hat{\mathbf{m}}_{cij} + v \times \hat{\mathbf{c}}_{cij}) (\mathbf{N}_{cj}^{vr})_{,x}^T + \hat{\mathbf{k}}_{cij} (\mathbf{N}_{cj}^{vr})^T \right) \quad (25c)$$

and the equivalent nodal loads are:

$$\hat{\mathbf{p}}_{ci,t+\Delta t}^* = (\mathbf{N}_{ci}^h \mu_i + \mathbf{N}_{ci}^v) \hat{\mathbf{p}}_{ci,t+\Delta t} \quad (26a)$$

$$\hat{\mathbf{q}}_{ci,t}^* = (\mathbf{N}_{ci}^h \mu_i + \mathbf{N}_{ci}^v) \hat{\mathbf{q}}_{ci,t} \quad (26b)$$

Eqs. (24)–(26) are the new formulations for VBI element incorporating the effect of vehicle sudden deceleration on the vehicle and bridge responses. Knowing the asterisk matrices for the proposed VBI element, the structural properties (mass, damping, and stiffness) of the bridge can be assembled and updated in each time increment to solve the whole bridge equation of motion. Damping matrix is calculated by Rayleigh method using a linear relationship between mass and stiffness damping [13]. It should be mentioned that, since unconditionally-stable Newmark- β method with constant acceleration in each time increment is used, the vehicle acceleration a in Eq. (25c) is calculated at the beginning of each time increment using Eq. (6). The flowchart for the step-by-step numerical procedure to analyze a bridge traversed by a moving vehicle with sudden deceleration is shown in Fig. 5.

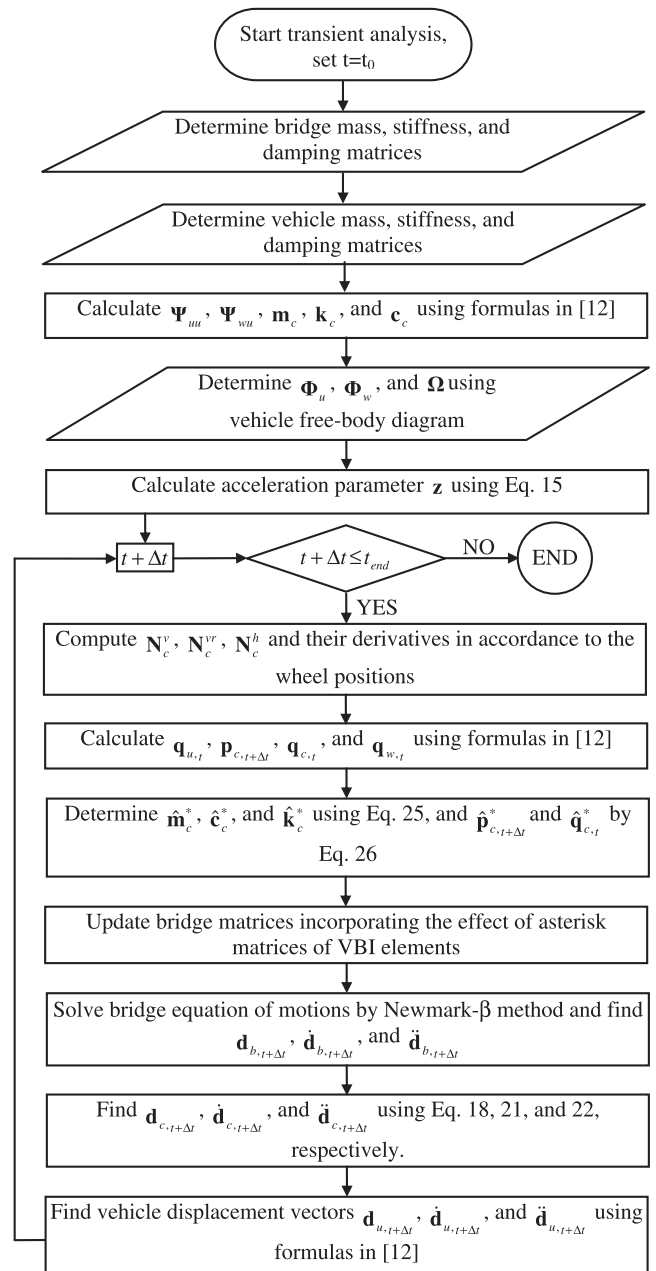


Fig. 5. Flowchart for the time-history dynamic VBI analysis.

5. Limitations and assumptions

The suspension units (dashpots and springs) of vehicle models used in the development of VBI elements will be assumed to retain linear properties. Majority of the vehicle models used by researchers in the analysis of bridge dynamics represented suspension systems using linear models. This is particularly common in the VBI analysis using vehicle models with dozens of DOFs, such as those used in the modeling of high-speed trains. However, there are few studies that adopted nonlinear properties (e.g. [9]). In addition, the effects of road roughness or track irregularities are not included in the proposed model. These effects could be added to the proposed formulation as a subject of future studies.

In the presence of vehicle horizontal acceleration, the formulation can be written based on either acceleration or friction coefficient. Since the sudden braking of vehicles with sliding is considered in this paper, the Coulomb kinetic friction can be used. Therefore, the contact forces are formulated based on friction coefficient, whose effect is incorporated in the so-called acceleration parameter shown in Eq. (15). In other cases where no pure sliding occurs, it is very difficult to specify the friction coefficients of the system. Many simplifying models have been proposed to include friction in multibody dynamic systems [22]. However, the developed VBI element can be used in such cases if an approximate response is desired.

6. Numerical verification

In order to perform the time-history analysis of the vehicle–bridge interaction, a computer program was written in MATLAB® [19] using the formulations developed in this paper. Since there were no models available in literature that presents the numerical solution for bridge–vehicle dynamic of vehicles with sudden deceleration including sliding, the developed formulation was first verified by a model having a vehicle with constant velocity, and then the effect of incorporating acceleration into the model is studied. In order to carry out the verification, vehicle horizontal acceleration was substituted with zero. The numerical verification is mainly done to verify the developed module written using MATLAB®. Hence, a half-car planar vehicle model used by Esmailzadeh and Jalili [23] shown in Fig. 6 is used. Each tire is modeled by a spring–dashpot under the wheel. The tire DOFs are in direct contact with the bridge without any mass. In the original study [23], no DOF was considered for tires; their effects were included in a formulation developed. In their model, there were a total of 6-DOF used for the vehicle model. However, two extra DOFs with zero mass are required in the locations of tires (see Fig. 6), to include the tire stiffness and damping in the vehicle model of this study. The bridge properties are: length $L = 100$ m, Young's modulus

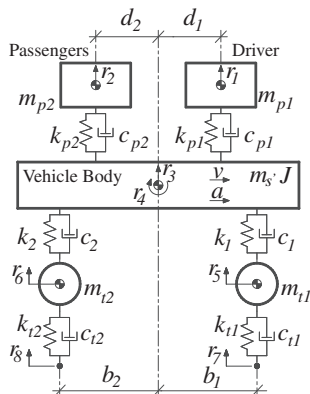


Fig. 6. A half-car planar vehicle model (adapted from [23]).

$E = 207$ GPa, moment of inertia $I = 0.174$ m⁴, damping $c = 1750$ N s/m, and mass per unit length $\rho = 20$ ton/m. The vehicle properties are tabulated in Table 1.

To formulate the acceleration parameter $\mathbf{z}$, required for the developed VBI element, vehicle characteristic matrices Φ_w , Φ_u and Ω need to be specified. Following the flowchart illustrated in Fig. 3, the geometry matrix Φ_u for upper DOFs is obtained as:

$$\Phi_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ h_1 & h_1 & h_2 & h_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

while Φ_w due to tire DOFs is a 2×6 zero matrix. Then, using equilibrium equations of horizontal forces, the dimensionless matrix of equilibrium Ω is obtained,

$$\Omega = \begin{bmatrix} \frac{m_{p1}}{M_t} & \frac{m_{p1}}{M_t} \\ \frac{m_{p2}}{M_t} & \frac{m_{p2}}{M_t} \\ 1 - \frac{m_{w1}}{M_t} & -\frac{m_{w1}}{M_t} \\ -\frac{m_{w2}}{M_t} & 1 - \frac{m_{w2}}{M_t} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (28)$$

In addition, vehicle stiffness matrices and the vector of self-weight are given in Appendix A. Then, the time-history analysis of the vehicle–bridge interaction is performed using the developed computer program for three constant velocities of 56, 72, and 88 km/h. Midspan deflection is shown in Fig 7, and driver bouncing in Fig. 8 illustrating very close results confirming the validity of the VBI element developed.

7. Numerical examples

In order to demonstrate the capability, versatility, and significance of the new VBI element, two examples are studied. The first example is a simple beam subjected to a moving rigid beam model supported by two identical spring–dashpot units. The second example is an open-deck plate-girder railway bridge with a simply-supported span traversed by a 4-axle train model. The bridge midspan deflection and acceleration are calculated as bridge responses. The bridge midspan deflection is a governing response in the calculation of the dynamic amplification factor or the bridge impact factor; and the bridge midspan acceleration is a criterion used to verify the stability of track in high-speed railways. As for vehicle responses, vehicle vertical acceleration is obtained, as it is an important criterion concerning passenger comfort in the design of high-speed railway bridges.

7.1. Rigid beam model traversing a simple beam

A rigid beam comprised of two DOFs supported by two wheels is the first example which is the same as the vehicle model shown

Table 1

Properties of the half-car planar vehicle model (adapted from [23]).

m_s	1794.4 (kg)	k_1	66824.4 (N/m)
m_{t1}	87.15 (kg)	k_2	18615.0 (N/m)
m_{t2}	140.4 (kg)	$k_{t1} = k_{t2}$	101,115 (N/m)
$m_{p1} = m_{p2}$	75 (kg)	$k_{p1} = k_{p2}$	14,000 (N/m)
J	3443.1 (kg m ²)	C_1	1190 (N s/m)
b_1	1.271 (m)	C_2	1000 (N s/m)
b_2	1.716 (m)	$c_{t1} = c_{t2}$	14.6 (N s/m)
d_1	0.481 (m)	C_{p1}	50.2 (N s/m)
d_2	1.313 (m)	C_{p2}	62.1 (N s/m)

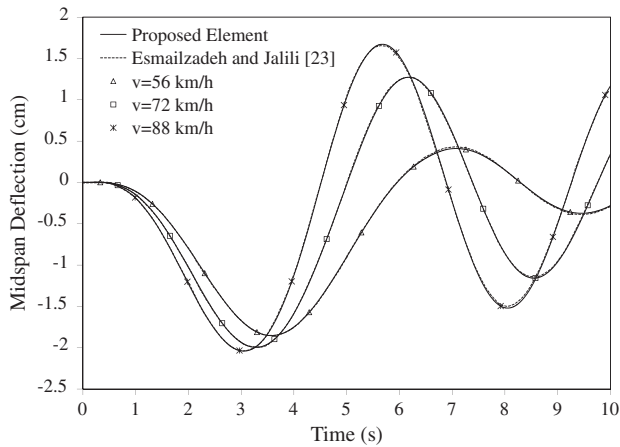


Fig. 7. Displacement at bridge midspan for half-car planar model.

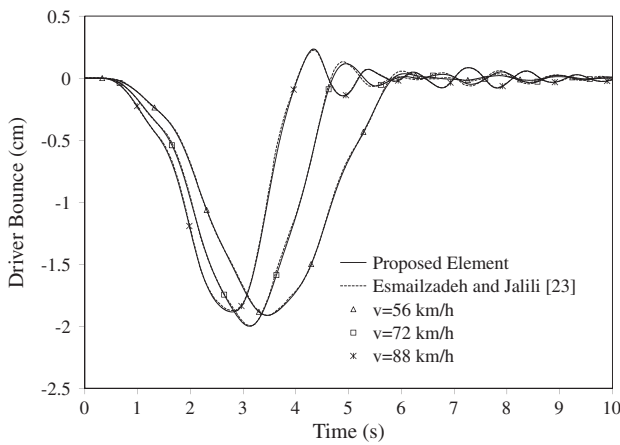


Fig. 8. Driver bouncing for half-car planar model.

in Fig. 1. This model was used by Yang and Wu [12]. Bridge properties are: Young's modulus $E = 2.943$ GPa, Poisson's ratio $\nu = 0.2$, moment of inertia $I = 8.65$ m⁴, mass per unit length $\rho = 36$ ton/m, and beam length $L = 30$ m. The moving beam properties are: sprung mass $m_c = 540$ ton, wheel masses are neglected as zero, rotary mass $I_c = 13,800$ ton m², and suspension spring constant $k_{v1} = k_{v2} = 41,350$ kN/m. No damping is included in the system. Half wheel distance $d_1 = d_2 = 7.5$ m. The length of each Euler–Bernoulli bridge elements is 3 m. Vehicle characteristic matrices required for the calculation of acceleration parameter $\mathbf{z}$ are presented in Eqs. (5) and (7). Finally, the VBI analysis is conducted and the results are compared with those presented by Yang and Wu [12] in the two following sub-sections.

7.1.1. Vehicle responses and discussions

The first response to be studied is the vertical contact forces. The proposed VBI element can predict the fluctuation in the vertical contact forces caused by the rotary or pitching moment. This moment is generated from horizontal vehicle acceleration as discussed in Section 2. Results of vertical contact forces for front and rear wheels are shown in Figs. 9 and 10, respectively. The forces are normalized with respect to the static vertical contact forces. As expected, at the beginning of braking, the contact force of the front wheel increases and the rear wheel decreases due to the rotary moment. After which, the vehicle experiences the oscillations exerted by the rotary moment resulting in a pitching movement with alterations in the vertical contact forces. A comparison between results obtained from the two models demonstrates

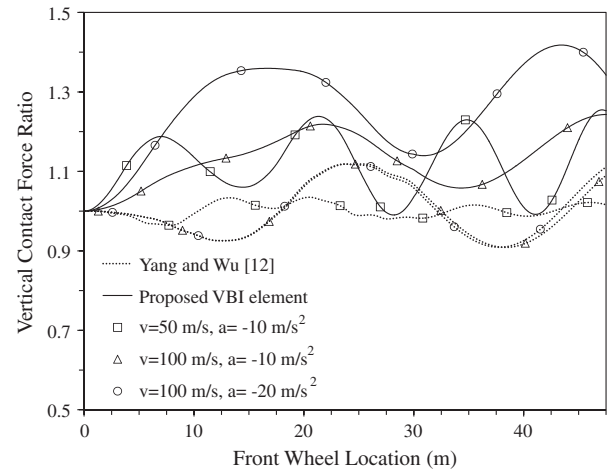


Fig. 9. Ratio of vertical contact forces over static contact forces in the front wheel of the rigid beam model.

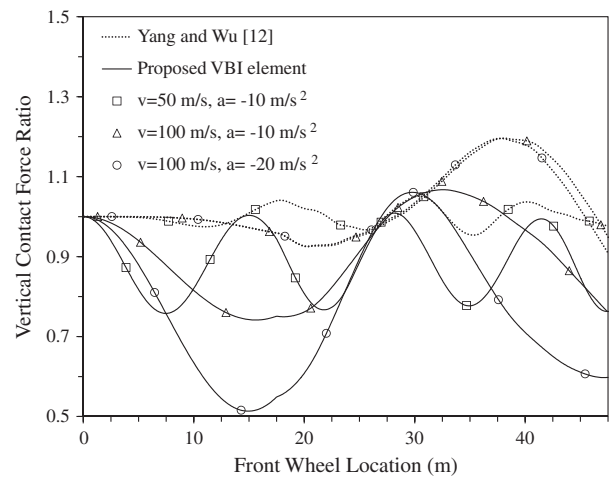


Fig. 10. Ratio of vertical contact forces over static contact forces in the rear wheel of the rigid beam model.

significant differences in contact forces. For instance, considering the front wheel of the vehicle with the initial speed of 100 m/s and the initial deceleration of -20 m/s², the new model results in 42% increase, while Yang and Wu [12] model shows only 10% increase. There are two main reasons for the differences between the results obtained from the new element with those presented by Yang and Wu [12]. The first reason is attributed to the effect of rotary moments which are added in the current model, and the second reason is that the vehicle horizontal acceleration is constant in the model of Yang and Wu [12]. Despite the friction coefficient is constant in the new VBI element, it does not necessarily imply that the vehicle horizontal acceleration is constant. It should be noted that the friction coefficient required for the analysis of the new VBI element is obtained from the ratio of the initial acceleration (i.e. 10 or 20 m/s²) over the gravitational acceleration.

Since friction coefficients are constant for this model, alterations in the vertical contact forces, considering Eq. (6), may introduce alterations in vehicle horizontal acceleration. Fig. 11 shows the vehicle horizontal deceleration resulting from variation in vertical contact forces. The figure is in the normalized form where the horizontal deceleration was divided by corresponding $\mu \cdot g$ value, where μ is the friction coefficient and g is the gravitational acceleration. The vehicle deceleration for the Yang and Wu [12] model is constant which is the line of normal deceleration equal to unity in Fig. 11.

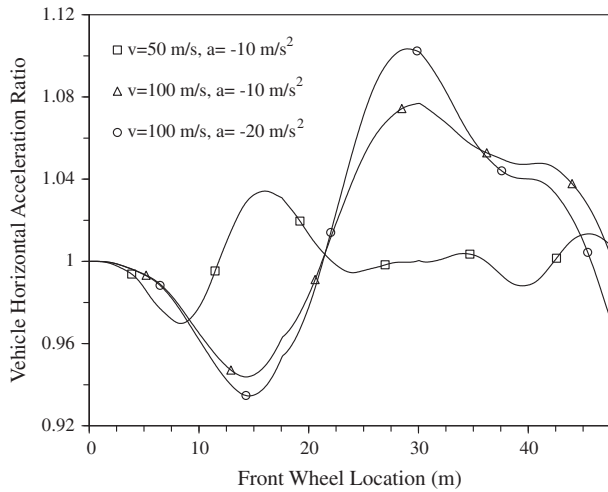


Fig. 11. Ratio of horizontal deceleration over initial deceleration of the rigid beam model.

Fig. 11 demonstrates that in addition to the vehicle horizontal deceleration, the vehicle initial speed is also important. The response pattern of the vehicles with 50 and 100 m/s and -10 m/s^2 are completely different; however for the vehicles with the same vehicle velocity of 100 m/s the response pattern is similar. As shown in Fig. 11, the vehicle experiences up to 10.5% increase in the horizontal vehicle deceleration for the case of 100 m/s and -20 m/s^2 . However, for another case of 50 m/s and -10 m/s^2 the increase is only 3.5%. It also can be concluded that the variation in the horizontal deceleration is much less than vertical contact forces comparing the percentage of increases in Fig. 11 with those in Figs. 9 and 10.

Less change in horizontal deceleration is attributed to the fact that when one wheel is experiencing higher contact forces the other one has an inverse behavior. To explain these differences, the vertical contact forces of this simple symmetric vehicle can be simulated with three components. As shown in Fig. 12, these three components are (A) two equal static forces F_1 ; (B) equal but reverse couple F_2 resulting from the vehicle pitching moments and also rotary moments caused by horizontal deceleration; and finally, (C) components of two equal forces F_3 corresponding to the vehicle vertical solid bouncing with no pitching. All the previous contact forces in each component category are equal due to the symmetry in the vehicle model.

Considering only component (A), assuming that there are no pitching and bouncing effects (e.g. two small moving mass models on a very stiff and even surface); if this model experiences braking, there would be no changes in the total contact forces of $2F_1$, hence the horizontal acceleration will remain constant. Moreover, components (A) and (B) together correspond to the vehicle model moving on a very stiff and even surface. If this vehicle experiences horizontal acceleration, only one rotary moment will be present and no solid vertical bouncing will appear. Since the forces in the components (B) are the same but in the reverse direction, the total contact forces in this case is also constant equal to $2F_1$; therefore, the horizontal acceleration remains constant, as well. Finally,

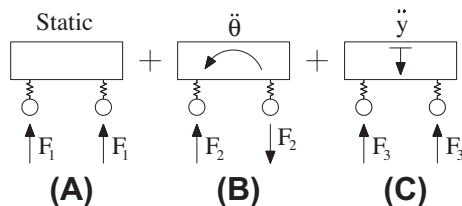


Fig. 12. Simplified illustration of the vehicle model dynamics.

consider all components together which corresponds to the vehicle model moving on a bridge or a non-stiff or uneven surface. In this case due to the vertical movement of wheels, the car body will also bounce vertically producing vertical acceleration in the car body. Since these two forces are in the same direction, the total contact force in this case is $2F_1 + 2F_3$ which is different from $2F_1$ and will result in alteration in the horizontal acceleration.

Regarding the free body diagrams of vehicle model shown in Fig. 2, consider only those equilibrium equations due to vertical bouncing of the vehicle (Eq. (1b) and Eq. (2b)) rewritten as follows:

$$\Delta V_1 + SP_{w1} - m_{w1}\ddot{r}_3 = 0; \quad \Delta V_2 + SP_{w2} - m_{w2}\ddot{r}_4 = 0 \quad (29a)$$

$$SP_{w1} + SP_{w2} + m_c\ddot{r}_1 = 0 \quad (29b)$$

where ΔV_i are due to changes in the vertical contact forces caused by vehicle vertical bouncing. Omitting suspension forces SP_{wi} , the alteration in the total contact force ΔV of the vehicle is obtained,

$$\Delta V = \Delta V_1 + \Delta V_2 = m_{w1}\ddot{r}_3 + m_{w2}\ddot{r}_4 + m_c\ddot{r}_1 \quad (30)$$

Since the wheel masses are neglected in this model and they are set to zero, ΔV will only be equal to $m_c\ddot{r}_1$. This implies that, for this particular example, the variation of the total vertical contact forces are linearly related to vehicle vertical acceleration; consequently, the vehicle horizontal acceleration is linearly related to the vehicle vertical acceleration. This can be confirmed from a comparison of Fig. 12 with results obtained for vehicle vertical acceleration presented in Fig. 13.

7.1.2. Bridge responses

Midspan deflection for various vehicle speeds and accelerations is shown in Fig. 14. Using the new VBI element, the midspan deflection rapidly increases due to higher front wheel contact forces. Differences of the new VBI element and Yang and Wu [12] model can reach up to 40% for vehicle initial velocity of 100 m/s and initial deceleration of -20 m/s^2 . One reason of such increase is the distance between wheels compared with the bridge span, which is about half of the bridge span. By decreasing the ratio of wheel spacing over the bridge span, the effect of the alteration in vertical contact forces on the bridge midspan deflection decreases since front and rear wheels will further counteract the effect of each other.

Another important parameter is the bridge midspan acceleration which is essential in the design of nonstructural components installed on the bridge or for the stability design of the ballast of the ballasted railway bridges for high-speed railways. The bridge midspan acceleration is shown in Fig. 15 for rigid beam model. Despite the significant differences between vertical contact forces ob-

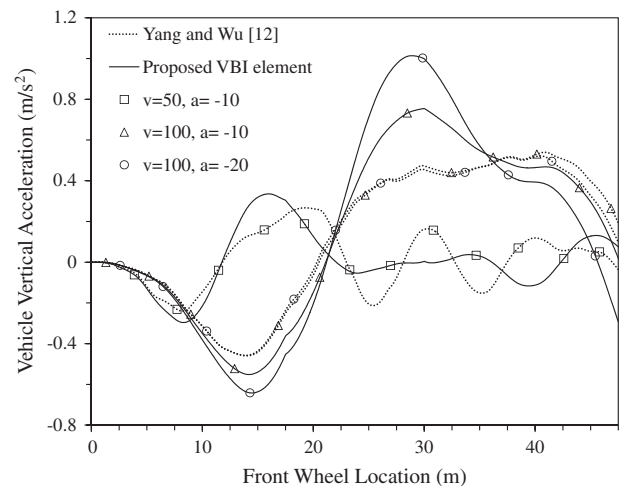


Fig. 13. Vehicle vertical acceleration of the rigid beam model.

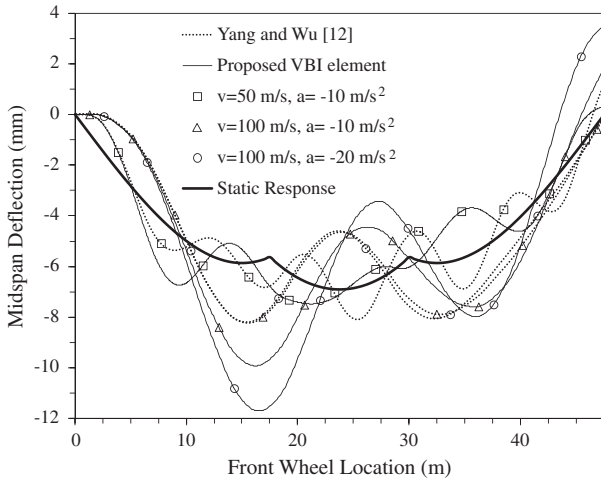


Fig. 14. Bridge midspan deflection of the rigid beam model.

tained by Yang and Wu [12] and the proposed element shown in Figs. 9 and 10, the bridge midspan accelerations are much closer. The comparison of results obtained from the proposed element for the initial velocity of 50 and 100 m/s and initial horizontal acceleration of -10 and -20 m/s², implies that the effect of vehicle velocity is much higher than vehicle horizontal deceleration. However, the effect of vehicle horizontal deceleration may not be neglected.

7.2. A train model of 10-DOFs

In order to study the vehicle–bridge interaction of a train model experiencing sudden braking, a model comprising four axels, two bogies and a car body is considered as shown in Fig. 16. Notations are similar to those in Fig. 2. Following the generalized procedure of formulation of acceleration forces shown in Fig. 3, the geometry matrix Φ_u for the upper part or non-contact DOFs is obtained as:

$$\Phi_u = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -h_1 & -h_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -h_2 & 0 & -h_3 & -h_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -h_2 & 0 & 0 & -h_3 & -h_3 \end{bmatrix} \quad (31)$$

where h_1 is the vertical distance between car body's center of gravity and horizontal internal forces interlocking between car body and

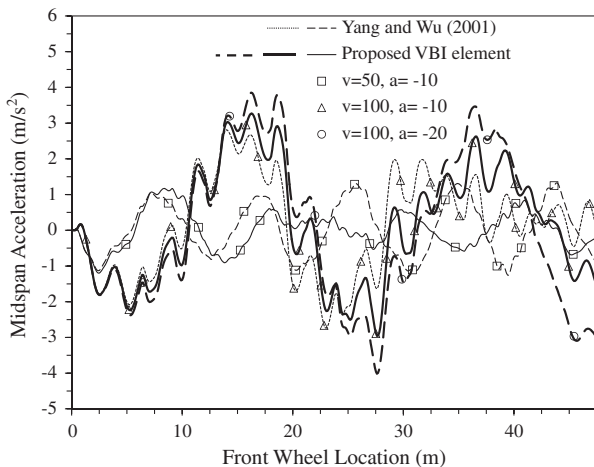


Fig. 15. Bridge midspan acceleration of the rigid beam model.

bogies. Those internal forces have a vertical distance of h_2 from center of gravity of bogies. Furthermore, h_3 is the vertical spacing between center of gravity of bogies and internal horizontal forces connecting bogies to wheels. In contrast to Φ_u , the matrix Φ_w due to wheel DOFs is a 4×6 zero matrix. Finally, following the flowchart of Fig. 3, the dimensionless equilibrium matrix Ω is determined,

$$\Omega = \begin{bmatrix} 1 - A_1 & 1 - A_1 & -A_1 & -A_1 \\ -A_2 & -A_2 & 1 - A_2 & 1 - A_2 \\ 1 - \frac{m_{w1}}{M_t} & -\frac{m_{w1}}{M_t} & -\frac{m_{w1}}{M_t} & -\frac{m_{w1}}{M_t} \\ -\frac{m_{w2}}{M_t} & 1 - \frac{m_{w2}}{M_t} & -\frac{m_{w2}}{M_t} & -\frac{m_{w2}}{M_t} \\ -\frac{m_{w3}}{M_t} & -\frac{m_{w3}}{M_t} & 1 - \frac{m_{w3}}{M_t} & -\frac{m_{w3}}{M_t} \\ -\frac{m_{w4}}{M_t} & -\frac{m_{w4}}{M_t} & -\frac{m_{w4}}{M_t} & 1 - \frac{m_{w4}}{M_t} \end{bmatrix} \quad (32)$$

where $A_1 = \frac{m_{w1} + m_{w2} + m_{b1}}{M_t}$ and $A_2 = \frac{m_{w3} + m_{w4} + m_{b2}}{M_t}$, and also M_t is the total vehicle mass.

The associated properties of the vehicle model are tabulated in Table 2 obtained from a 3D high-speed train of SKS series 300 model reported by Wu et al. [21]. Required vehicle stiffness matrices for numerical analysis using the new VBI element are given in Appendix A. The bridge model is a railway bridge reported by Foutch et al. [24], which is an open-deck plate-girder railway bridge with a simply-supported span of 15.24 m. The cross sectional area is 0.12684 m² and mass per unit length is 1428.64 kg/m. The moment of inertia is calculated as 0.05411 m⁴ based on the drawings available in the report. In addition, other required bridge properties are assumed: Young's modulus $E = 200$ GPa, damping ratio 2%, and Poisson's ratio 0.3. The dynamic interaction between the train model and the bridge were then undertaken according to the flowchart of Fig. 5, and the results are discussed in the following sub-sections.

7.2.1. Contact forces

The vertical contact forces for all wheels are shown in Fig. 17 for vehicle initial speeds of 50 and 100 m/s. Considering the vehicle model of Fig. 16, the vehicle wheels are numbered from right to left, where the first one at right is the first wheel and the last one at the very left is the fourth wheel. These designations are used in Fig. 17. The friction coefficient of $\mu = 0.1$ and 0.7 are used here representing two extreme cases of braking situations. The value of 0.7 for friction coefficient is extremely high and is the highest value reported by Olofsson and Telliskivi [25]. They concluded that, the friction coefficient varies between 0.5 and 0.6 for pure non-lubricated sliding tests which can reach 0.7 in some cases. If the rail is lubricated, the friction coefficient reduced to values between 0.2 and 0.4 .

Significant variations in the contact forces are observed for all vehicle speeds shown in Fig. 17. The maximum contact force in each case is affected slightly by the vehicle velocity and it is more a function of friction coefficient. The first and second wheels are

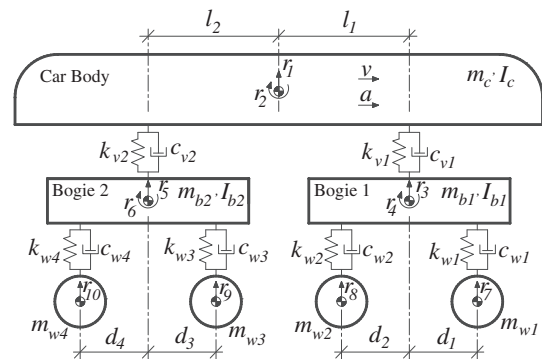


Fig. 16. Four-axle model of one car of a train.

Table 2
Properties of the train model [21].

m_c	41.75 (ton)
$m_{b1} = m_{b2}$	3.04 (ton)
$m_{w1} = m_{w2} = m_{w3} = m_{w4}$	1.78 (ton)
I_c	2080 (ton m ²)
$I_{b1} = I_{b2}$	3.93 (ton m ²)
$k_{v1} = k_{v2}$	1180 (kN/m)
$k_{w1} = k_{w2} = k_{w3} = k_{w4}$	530 (kN/m)
$c_{v1} = c_{v2}$	39.2 (kN s/m)
$c_{w1} = c_{w2} = c_{w3} = c_{w4}$	90.2 (kN s/m)
$l_1 = l_2$	8.75 (m)
$d_1 = d_2 = d_3 = d_4$	1.25 (m)
h_1	0.75 (m)
h_2	0.42 (m)
h_3	0.20 (m)

under bogie 1 and the other two wheels are under bogie 2 shown in Fig. 16. Due to the pitching moment on bogie 1, the first wheel experiences growth in the contact force and the other wheel is vice versa. Similar behavior is observed for the third and the forth wheel under bogie 2.

7.2.2. Bridge midspan deflection

The bridge midspan deflection is shown in Fig. 18 for vehicle initial speed of 50 and 100 m/s. Results for all cases are close. It is usually expected to have a direct relationship between the exerted forces on the bridge and the amount of resulting deflection. However, as shown in Fig. 17, significant fluctuations in contact forces are observed for all wheels with no exception; but the maximum midspan deflections for all cases are only slightly different.

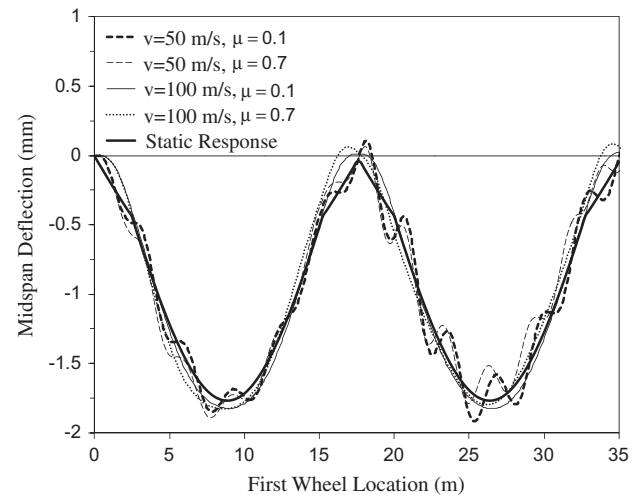


Fig. 18. Midspan deflection of the train model.

The reason is attributed to the spacing of the forces and how they are applied on the bridge. The spacing between the first wheel and the second wheel is 2.5 m. Therefore compared to the bridge length which is 15.24 m, the first and second wheels are relatively close to each other. Although the contact force in the first wheel is growing, it is decreasing in the second wheel. Hence, the resultant force varies much less than an individual wheel.

Results for the summation of the vertical contact forces for the first and last two wheels are shown in Fig. 19 for the vehicle initial

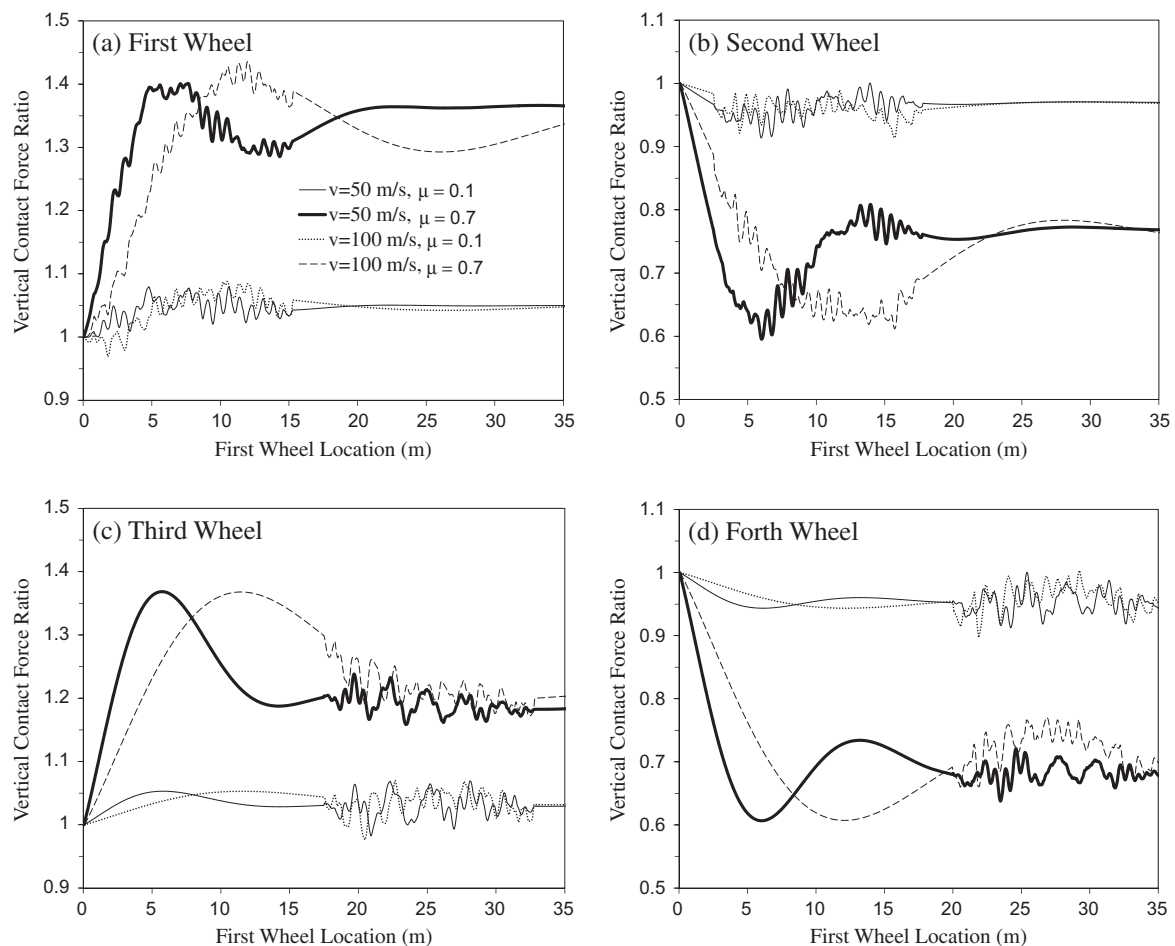


Fig. 17. Ratio of vertical contact forces over static contact forces of the train model.

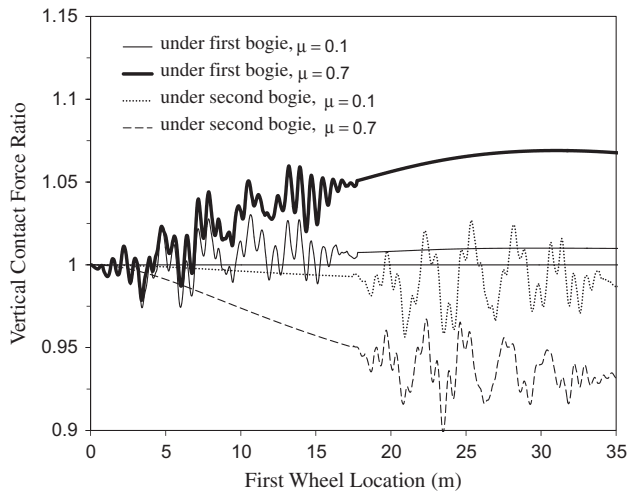


Fig. 19. Ratio of the summation of vertical contact forces under each bogie over corresponding static contact forces of the train model, for initial vehicle speed of 50 m/s.

speed of 50 m/s. For two wheels under bogie 1 (i.e. the first and second wheels) shown in Fig. 19, the increase in contact forces is only around 7% and for the bogie 2, the decrease is around 10%. Main reason of these alterations is the pitching moment exerting on the vehicle body itself. The vehicle body pitching moment is resisted by the inertial effect of the vehicle body rotary inertia and

two vertical spring-dampers supporting the vehicle body. Although, the amount of the pitching moment is larger compared to that for the two bogies, the rotary inertia of the car body is much higher than bogies; consequently, the pitching of the car body is much less than that for bogies shown in Fig. 20. Another reason of these changes shown in Fig. 19 is the bouncing of the car body and also bogies.

7.2.3. Vehicle horizontal deceleration

Vehicle horizontal deceleration is depicted in Fig. 21 for two levels of vehicle speeds and two levels of friction coefficients. The deceleration is obtained using Eq. (6) for each friction coefficient of 0.1 and 0.7. In addition to the friction coefficients that are constant for each analysis, the total vehicle mass M_t is also constant throughout the analysis; therefore considering Eq. (6) and as discussed before, the vehicle horizontal acceleration or deceleration is a function of summation of all vertical contact forces which does not change noticeably. As shown in Fig. 21, the maximum change in the horizontal deceleration is almost 2% which is very low and will not be felt by the passengers sitting on the train.

7.2.4. Bridge midspan acceleration

The bridge midspan acceleration is demonstrated in Fig. 22 for two vehicle initial speeds of 50 m/s and 100 m/s. For the stability of ballasted or non-ballasted tracks of high-speed railways, EN 1990/A2 [26] sets limits of 0.35 g and 0.5 g respectively, where g is the gravitational acceleration. As shown in Fig. 22, the peak mid-

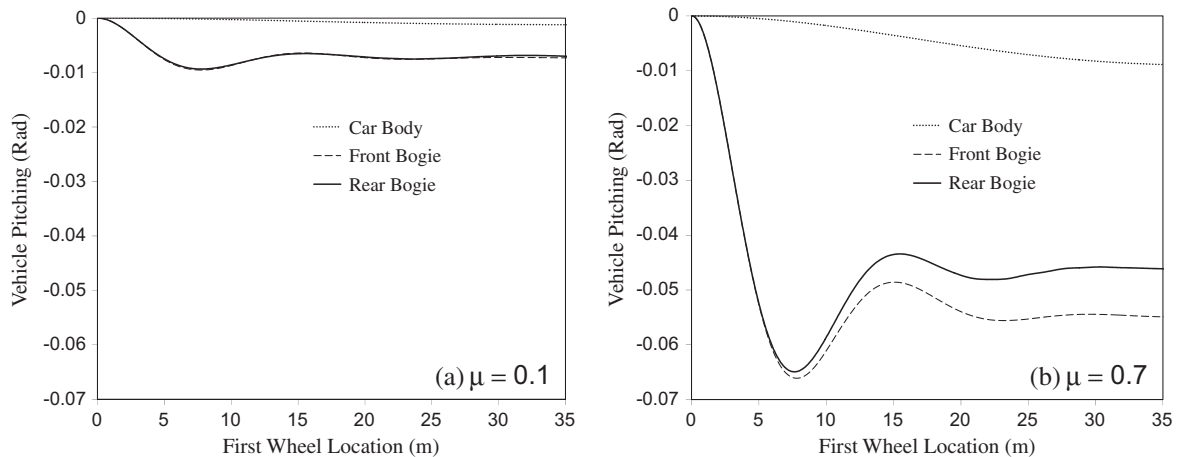


Fig. 20. Vehicle pitching of various parts of the train model, for initial vehicle speed of 50 m/s.

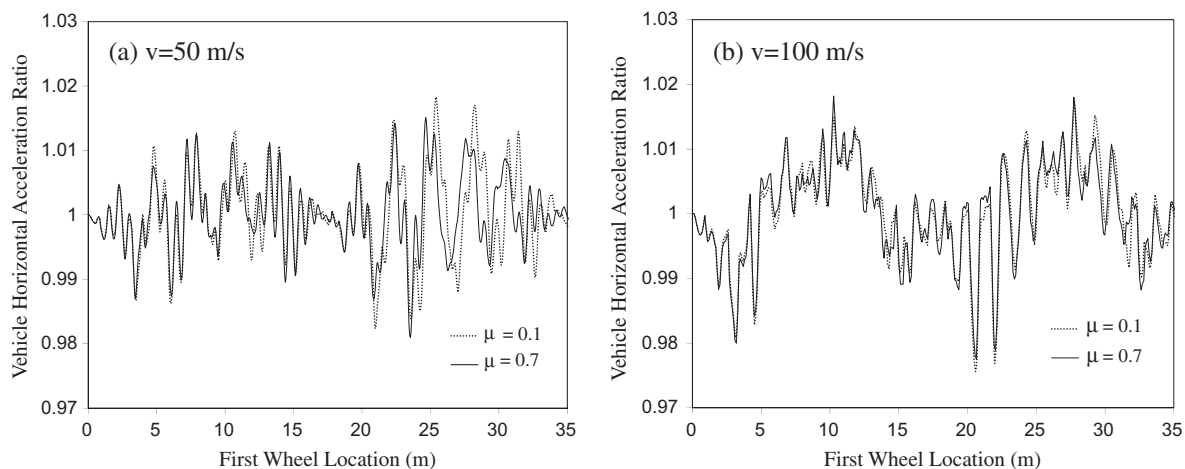


Fig. 21. Ratio of vehicle horizontal deceleration over initial deceleration of the train model for initial vehicle speeds of (a) 50 m/s, (b) 100 m/s.

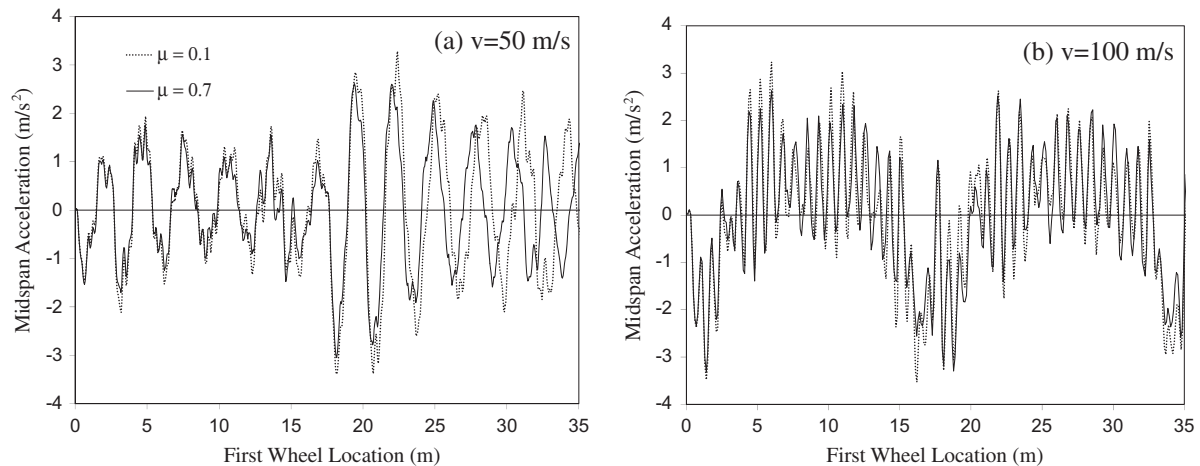


Fig. 22. Bridge midspan acceleration of the train model for initial vehicle speeds of (a) 50 m/s, and (b) 100 m/s.

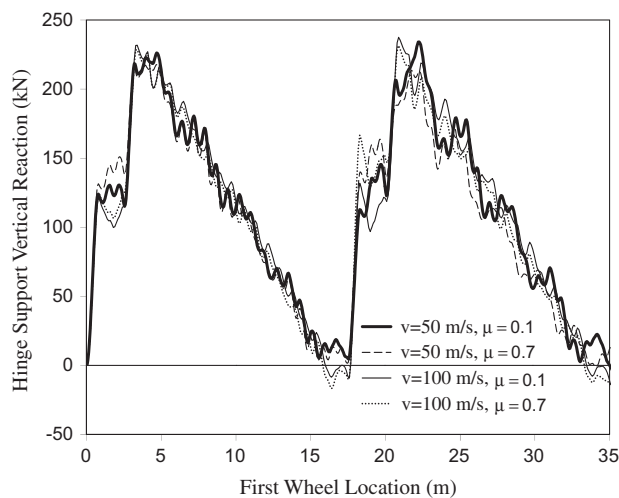


Fig. 23. Hinge support vertical reaction of the train model, for initial vehicle speed of 50 m/s.

span acceleration of both cases is about 3.5 m/s^2 which is less than the specified limit of $0.5 g$ for open-deck railway bridges. Similar to the rigid beam model, the vehicle velocity plays a governing role on the midspan acceleration shown in Fig. 22, not the vehicle horizontal deceleration.

7.2.5. Hinge support reaction

Fig. 23 shows the hinge support vertical reaction which is not affected noticeably by the severity of braking, and the peak values are only 5% different. The response can be divided into two parts in Fig. 23; the first part from zero to 18 m and the second part from 18 m to 35 m. The first part is basically the response due to the first two wheels and the second part is due to the last two wheels. Since the fluctuations of the last two wheels are more than the first two wheels shown in Fig. 19, the second part of Fig. 23 demonstrates more differences between $\mu = 0.1$ and 0.7 compared to the first part of the figure. Furthermore, the peak values of the hinge support reaction for initial speed of 50 m/s are 10% different in the second part of Fig. 23 which may be considered in the design practices.

8. Summary and conclusions

In this study, a numerical VBI (vehicle–bridge interaction) element for the calculation of VBI problems that accounts for vehicles experiencing sudden deceleration with sliding is proposed. A key

step was to numerically formulate acceleration forces based on vertical contact forces. Acceleration forces are those inertial forces generated by horizontal vehicle acceleration in the considered DOFs of the vehicle model. A new factor called acceleration parameter is introduced in the formulation of the vertical contact forces simulating the effect of the horizontal acceleration and is constant throughout the analysis convenient for numerical analysis. It represents the initial redistribution of vertical contact forces once the horizontal deceleration is applied. It is not a diagonal matrix and its determinant is equal to one. The general common formulation for the proposed VBI element is kept similar to VBI elements proposed by Azimi et al. [14] and Yang and Wu [12] for brevity and easier comparison. Using this element, one can model variations in the vertical contact forces resulted from horizontal vehicle acceleration, and can obtain various bridge and vehicle dynamic responses.

The effect of the proposed VBI element was first studied for a sample 4-DOF vehicle model called rigid beam model. The results presented by Yang and Wu [12] were compared with those obtained from new VBI element. Large differences were observed for vertical contact forces, vehicle vertical acceleration, and also bridge midspan deflection. The main reason is attributed to the variation in the vertical contact forces due to pitching moment which was included in the formulation of the new VBI model. However, the effect of the new VBI element is not prominent for bridge midspan acceleration.

Studying a train model of 10-DOF, having two levels of medium and high braking situations, shows that the vertical contact forces may be significantly affected and they can be modified by about $\pm 50\%$ for an extreme braking situation. This local effect could have implications on the design of the rail/track structure. On the other hand, such extreme braking situations have minor effect on the mid-span deflection mainly due to the small arm between the two wheels under each bogie compared to the bridge span. Moreover, the bridge midspan acceleration is affected mainly by the vehicle velocity not the acceleration for this model, similar to rigid beam model. In addition, the vehicle horizontal acceleration may increase support reactions, which may be considered in the design practice. For the studied train model, the support reaction was augmented by 10%.

Acknowledgement

The authors would like to acknowledge the support provided by the Canadian National Railway (CN) in the form of a fellowship to the first author.

Appendix A

Vehicle stiffness matrices and the vector of self-weight $\mathbf{f}_{we}$ for two vehicle models are given in this appendix. Damping matrices are constructed the same way that stiffness matrices are written, and mass matrices are written by diagonal matrices. Letter m denotes mass, k is for stiffness, c for damping, and g for gravitational acceleration. Corresponding matrices for the half-car planar vehicle model shown in Fig. 6 are as follows:

$$\mathbf{k}_{uu} = \begin{bmatrix} k_{p1} & 0 & -k_{p1} & -k_{p1}d_1 & 0 & 0 \\ 0 & k_{p2} & k_{p2} & k_{p2}d_2 & 0 & 0 \\ -k_{p1} & -k_{p2} & k_{p1} + k_{p2} + k_1 + k_2 & k_{p1}d_1 - k_{p2}d_2 + k_1b_1 - k_2b_2 & -k_1 & -k_2 \\ -k_{p1}d_1 & k_{p2}d_2 & k_{p1}d_1 - k_{p2}d_2 + k_1b_1 - k_2b_2 & k_{p1}d_1^2 + k_{p2}d_2^2 + k_1b_1^2 + k_2b_2^2 & -k_1b_1 & k_2b_2 \\ 0 & 0 & -k_1 & -k_1b_1 & k_1 + k_{t1} & 0 \\ 0 & 0 & -k_2 & k_2b_2 & 0 & k_2 + k_{t2} \end{bmatrix} \quad (\text{A.1a})$$

$$\mathbf{k}_{wu} = (\mathbf{k}_{uw})^T = \begin{bmatrix} 0 & 0 & 0 & 0 & -k_{t1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_{t2} \end{bmatrix} \quad (\text{A.1b})$$

$$\mathbf{k}_{ww} = \begin{bmatrix} k_{t1} & 0 \\ 0 & k_{t2} \end{bmatrix} \quad (\text{A.1c})$$

$$\mathbf{f}_{we,t+\Delta t} = -g \left\{ \begin{array}{l} \frac{m_s b_2 + m_{p1}(d_1 + b_2) + m_{p2}(b_2 - d_2)}{b_1 + b_2} + m_{t1} \\ \frac{m_s b_1 + m_{p2}(d_2 + b_1) + m_{p1}(b_1 - d_1)}{b_1 + b_2} + m_{t2} \end{array} \right\} \quad (\text{A.2})$$

and for the train model shown in Fig. 16:

$$\mathbf{k}_{uu} = \begin{bmatrix} k_{v1} + k_{v2} & k_{v2}l_2 - k_{v1}l_1 & -k_{v1} & 0 & -k_{v2} & 0 \\ k_{v2}l_2 - k_{v1}l_1 & k_{v1}l_1^2 + k_{v2}l_2^2 & k_{v1}l_1 & 0 & -k_{v2}l_2 & 0 \\ -k_{v1} & k_{v1}l_1 & k_{v1} + k_{w1} + k_{w2} & k_{w2}d_2 - k_{w1}d_1 & 0 & 0 \\ 0 & 0 & k_{w2}d_2 - k_{w1}d_1 & k_{w1}d_1^2 + k_{w2}d_2^2 & 0 & 0 \\ -k_{v2} & -k_{v2}l_2 & 0 & 0 & k_{v2} + k_{w3} + k_{w4} & k_{w4}d_4 - k_{w3}d_3 \\ 0 & 0 & 0 & 0 & k_{w4}d_4 - k_{w3}d_3 & k_{w3}d_3^2 + k_{w4}d_4^2 \end{bmatrix} \quad (\text{A.3a})$$

$$\mathbf{k}_{wu} = (\mathbf{k}_{uw})^T = \begin{bmatrix} 0 & 0 & -k_{w1} & k_{w1}d_1 & 0 & 0 \\ 0 & 0 & -k_{w2} & -k_{w2}d_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{w3} & k_{w3}d_3 \\ 0 & 0 & 0 & 0 & -k_{w4} & -k_{w4}d_4 \end{bmatrix} \quad (\text{A.3b})$$

$$\mathbf{k}_{ww} = \begin{bmatrix} k_{w1} & 0 & 0 & 0 \\ 0 & k_{w2} & 0 & 0 \\ 0 & 0 & k_{w3} & 0 \\ 0 & 0 & 0 & k_{w4} \end{bmatrix} \quad (\text{A.3c})$$

$$\mathbf{f}_{we,t+\Delta t} = -g \left\{ \begin{array}{l} \left(m_{b1} + \frac{l_2}{l_1 + l_2} m_c \right) \frac{d_2}{d_1 + d_2} + m_{w1} \\ \left(m_{b1} + \frac{l_2}{l_1 + l_2} m_c \right) \frac{d_1}{d_1 + d_2} + m_{w2} \\ \left(m_{b2} + \frac{l_1}{l_1 + l_2} m_c \right) \frac{d_4}{d_3 + d_4} + m_{w3} \\ \left(m_{b2} + \frac{l_1}{l_1 + l_2} m_c \right) \frac{d_3}{d_3 + d_4} + m_{w4} \end{array} \right\} \quad (\text{A.4})$$

References

- [1] Yang Y-B, Yau J-D, Wu Y-S. Vehicle-bridge interaction dynamics with applications to high-speed railways. Singapore: World Scientific Publishing Co.; 2004.
- [2] Xia H, Zhang N, De Roeck G. Dynamic analysis of high speed railway bridge under articulated trains. Comput Struct 2003;81:2467–78. [http://dx.doi.org/10.1016/S0045-7949\(03\)00309-2](http://dx.doi.org/10.1016/S0045-7949(03)00309-2).
- [3] Zhang Q-L, Vrouwenvelder A, Wardenier J. Numerical simulation of train-bridge interactive dynamics. Comput Struct 2001;79:1059–75. [http://dx.doi.org/10.1016/S0045-7949\(00\)00181-4](http://dx.doi.org/10.1016/S0045-7949(00)00181-4).
- [4] Yang F-H, Fonder GA. An iterative solution method for dynamic response of bridge-vehicles systems. Earthquake Eng Struct Dyn 1996;25:195–215. [http://dx.doi.org/10.1002/\(SICI\)1096-9845\(199602\)25:2<195::AID-EOE547>3.0.CO;2-R](http://dx.doi.org/10.1002/(SICI)1096-9845(199602)25:2<195::AID-EOE547>3.0.CO;2-R).
- [5] Frýba L. Vibration of solids and structures under moving loads. Groningen, Netherlands: Noordhoff International Publishing; 1972.
- [6] Stanišić MM. On a new theory of the dynamic behavior of the structures carrying moving masses. Ingenieur-Archiv 1985;55:176–85. <http://dx.doi.org/10.1007/BF00536412>.
- [7] Inbanathan MJ, Weiland M. Bridge vibrations due to vehicle moving over rough surface. J Struct Eng 1987;113:1994–2008. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(1987\)113:9\(1994\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(1987)113:9(1994)).
- [8] Neves SGM, Azevedo AFM, Calçada R. A direct method for analyzing the vertical vehicle-structure interaction. Eng Struct 2012;34:414–20. <http://dx.doi.org/10.1016/j.engstruct.2011.10.010>.
- [9] Hwang E-S, Nowak AS. Simulation of dynamic load for bridges. J Struct Eng 1991;117:1413–34. [http://dx.doi.org/10.1061/\(ASCE\)0733-9445\(1991\)117:5\(1413\)](http://dx.doi.org/10.1061/(ASCE)0733-9445(1991)117:5(1413)).
- [10] Frýba L. Dynamics of railway bridges. London (UK): Thomas Telford; 1996.
- [11] Green MF, Cebon D. Dynamic interaction between heavy vehicles and highway bridges. Comput Struct 1997;62:253–64. [http://dx.doi.org/10.1016/S0045-7949\(96\)00198-8](http://dx.doi.org/10.1016/S0045-7949(96)00198-8).
- [12] Yang Y-B, Wu Y-S. A versatile element for analyzing vehicle-bridge interaction response. Eng Struct 2001;23:452–69. [http://dx.doi.org/10.1016/S0141-0296\(00\)00065-1](http://dx.doi.org/10.1016/S0141-0296(00)00065-1).

- [13] Chopra AK. Dynamic of structures: theory and applications to earthquake engineering. Englewood Cliffs, New Jersey: Prentice-Hall; 2001.
- [14] Azimi H, Galal K, Pekau OA. A modified numerical VBI element for vehicles with constant velocity including road irregularities. Eng Struct 2011;33:2212–20. <http://dx.doi.org/10.1016/j.engstruct.2011.03.012>.
- [15] Law S-S, Zhu X-Q. Bridge dynamic responses due to road surface roughness and braking of vehicle. J Sound Vibr 2005;282:805–30. <http://dx.doi.org/10.1016/j.jsv.2004.03.032>.
- [16] Ju S-H, Lin H-T. A finite element model of vehicle–bridge interaction considering braking and acceleration. J Sound Vibr 2007;303:46–57. <http://dx.doi.org/10.1016/j.jsv.2006.11.034>.
- [17] Garg VK, Dukkipati RV. Dynamics of railway vehicle systems. Toronto, Canada: Academic Press; 1984.
- [18] Majka M, Hartnett M. Effects of speed, load and damping on the dynamic response of railway bridges and vehicles. Comput Struct 2008;86:556–72. <http://dx.doi.org/10.1016/j.compstruc.2007.05.002>.
- [19] MATLAB®. The Math Works Inc., Massachusetts; 2010.
- [20] AREMA, American Railway Engineering and Maintenance-of-way Association. Manual of recommended practice. Steel structures, Washington (DC); 2010 [chapter 15].
- [21] Wu Y-S, Yang Y-B, Yau J-D. Three-dimensional analysis of train-rail-bridge interaction problems. Vehicle Syst Dyn 2001;36:1–35. <http://dx.doi.org/10.1076/vesd.36.1.1.3567>.
- [22] Pfeiffer F, Glocker C. Multibody dynamics with unilateral contacts. New York (NY): John Wiley & Sons; 1996.
- [23] Esmailzadeh E, Jalili N. Vehicle–passenger–structure interaction of uniform bridges traversed by moving vehicles. J Sound Vibr 2003;260:611–35.
- [24] Foutch DA, Tobias DH, Otter DE, Lopresti JA, Uppal AS. Experimental and analytical investigation of the longitudinal loads in an open-deck plate girder railway bridge. Pueblo, CO., USA: Association of American Railroads, Report No. R-905; 1997.
- [25] Olofsson U, Telliskivi T. Wear, friction and plastic deformation of two rail steels—full scale test and laboratory study. Wear 2003;254:80–93. [http://dx.doi.org/10.1016/S0043-1648\(02\)00291-0](http://dx.doi.org/10.1016/S0043-1648(02)00291-0).
- [26] EN 1990-A2, Eurocode: basis for structural design-Annex A2: applications for bridges. Brussels: CEN; 2005.