

A modified numerical VBI element for vehicles with constant velocity including road irregularities

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ABSTRACT

There has been a growing interest to model and analyze Vehicle–Bridge Interaction (VBI) of intricate vehicles on bridges. The objective of such an analysis is to realistically investigate the dynamic effects of moving vehicles particularly in case of high-speed trains, where the vehicle acceleration is a design criterion and should be calculated appropriately. One method of analysis is to eliminate the wheel degrees of freedom (DOF) that are in direct contact with the bridge surface resulting in a VBI element, which is a modified beam element acted upon directly by wheel(s) of a running vehicle. The contact force is the mutual force between the wheel and the bridge. The available formulation in the literature is used to formulate the contact forces, which are related to those in the beam element DOFs by the Hermitian interpolation functions. Considering suitable interpolation functions between the beam element displacement vector and those for contact points and also a new formulation for the contact points, a new formulation is proposed for the structural properties of the VBI element, resulting in a new element capable of capturing bridge and vehicle responses more realistically. A study is conducted on the model variables and their effects on the bridge dynamic amplification factor and also bridge and vehicle accelerations, in order to compare the new VBI method with the existing one. The studied parameters include vehicle and bridge damping, frequency parameter, system mass parameter, and a new parameter called vehicle mass parameter. Results generally demonstrate noticeable differences particularly for high speed vehicles. In addition, it is observed that the effect of shear deformations in a simply-supported bridge might not be negligible and should be considered for moving vehicle analysis. For double girder open-deck steel railway bridges, the difference in midspan deflection of models including or excluding shear deformation can vary from 18% to 8% for 4.0 m and 30.0 m spans, respectively, for a sample vehicle.

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1. Introduction

Application of high-speed trains has been growing all over the world particularly in Japan, China, France, Germany, and Spain. Currently however, there is only one high-speed route in North America which is the Acela Express in Boston. In Canada, the implementation of new routes is under discussion in order to connect Edmonton to Calgary, as well as a route from Windsor to Québec city. The riding comfort is a key factor in the design of the high-speed railways; hence, it is required to model a vehicle capable of obtaining the maximum acceleration of the car body as the design criterion for riding comfort. Having an elaborate vehicle model necessitates the application of sophisticated methods of analysis for the Vehicle–Bridge Interaction (VBI). This was the subject of much research over the past decade, noting that methods

based on numerical analysis have been used by the majority of researchers.

The main objective of this paper is to present a modified two-dimensional VBI element for a constant velocity vehicle. A VBI element is a bridge element acted upon directly by the vehicle wheel. The bridge may have other elements that have no imposed wheel, which are treated as regular beam elements. In the current study, the analysis type used for the VBI problem is the time history analysis with suitable time increments. The calculation of the unknowns at the end of the time increment is often done by an iterative algorithm verifying convergence [1–4]. The convergence criterion is questionable and also for the analysis of a series of vehicles may be considerably time-consuming. However, Yang and Wu [5] presented a method which requires no iteration in the time increments yet may be considered adequately accurate. They proposed a new element which utilizes the concept of dynamic condensation method. They condensed the contact point DOFs, which resulted in a new VBI element with contact mass, damping, and stiffness matrices.

The overall bridge structural matrices are updated in each time increment using VBI elements and regular beam elements. The

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vertical contact force at the end of the time step was formulated on the basis of the vehicle equation of motion and wheels DOFs. The formulation for the contact forces presented in the aforementioned paper is used in the current study; however, a modified version of the VBI element is proposed compared to that proposed by Yang and Wu [5]. The contact forces are related to those in the beam element DOFs by Hermitian interpolation functions. Suitable interpolation functions between the beam element displacement vector and those for contact points are considered. The velocity and the acceleration of the contact point are then obtained by the first and second derivation of the corresponding displacement vector. The result is a new formulation proposed for the structural properties of a VBI element capable of capturing bridge and vehicle responses more realistically. To obtain the bridge response, VBI elements are assembled together with the original beam elements in global coordinates and the bridge equation of motion is solved by the Newmark- β method.

In order to investigate the effect of the new VBI element, a study on the model variables is conducted. Non-dimensional parameters used by other researchers [6,4] are used. A new parameter called vehicle mass parameter, which is the ratio of the wheel mass to the vehicle body mass, is introduced and considered. A simply-supported Euler–Bernoulli beam is used in the finite element formulation [7] which is based on the analogous properties of a plate girder railway bridge [8]. The effect of the vehicle can be modeled as the moving force, moving mass [9], or moving system model [10]. The moving force method is considered as the most common; however, a moving system method is considered more accurate which includes the vehicle suspension system. The linear properties for the suspension systems were applied in most studies particularly for trains. However, simplified nonlinear properties were also used for trucks [11]. A sprung mass model with a linear suspension system is used for the analysis in this research.

2. Modified VBI element for vehicles with constant velocity

A VBI element is a beam element acted upon by the wheel(s) of the running vehicle. In the two-dimensional (2-D) analysis of a vehicle with constant speed, there is no horizontal acceleration in the vehicle, implying that no horizontal contact forces are present and only the vertical contact forces are transferring vehicle loads to the bridge. The equation of motion for a beam element traversed by the i th wheel at the end of the time increment ($t + \Delta t$) can be shown as:

$$[m_{bi}] \{\ddot{d}_{bi}\}_{t+\Delta t} + [c_{bi}] \{\dot{d}_{bi}\}_{t+\Delta t} + [k_{bi}] \{d_{bi}\}_{t+\Delta t} = \{f_{bi}\}_{t+\Delta t} - \{f_{bci}\}_{t+\Delta t} \quad (1)$$

where $[m_{bi}]$, $[c_{bi}]$, and $[k_{bi}]$ are the mass, damping and stiffness matrices of the beam element. $\{d_{bi}\}$ is the nodal displacement vector of this element and $\{f_{bi}\}$ is the vector of the nodal external forces. $\{f_{bci}\}_{t+\Delta t}$ is the vector of the equivalent nodal forces resulting from the $V_{i,t+\Delta t}$ contact force.

Vertical contact forces $V_{i,t+\Delta t}$ are entries of the corresponding vector $\{f_c\}_{t+\Delta t}$ which can be formulated as [5]

$$\{f_c\}_{t+\Delta t} = [m_c] \{\ddot{d}_w\}_{t+\Delta t} + [c_c] \{\dot{d}_w\}_{t+\Delta t} + [k_c] \{d_w\}_{t+\Delta t} + \{p_c\}_{t+\Delta t} + \{q_c\}_t \quad (2)$$

where $[m_c]$, $[c_c]$, and $[k_c]$ are called the contact matrices and they are all known. $\{d_w\}$ is the displacement vector of the wheels which is unknown. $\{p_c\}_{t+\Delta t}$ is the effect of external forces on the vehicle DOFs at the end of the time increment and is known. And finally, $\{q_c\}_t$ represents the effect of the vehicle displacement vector and its derivatives at the beginning of the time increment which are known in a step-by-step analysis [5].

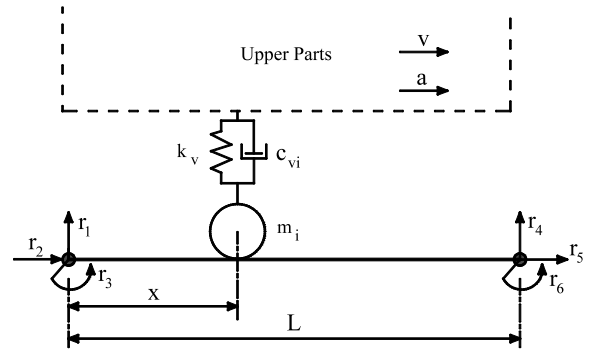


Fig. 1. 6-DOF VBI element for two-dimensional analysis.

To relate the contact force V to the corresponding element nodal forces $\{f_b\}$, an interpolation function $\{N_c\}$ is used

$$\{f_b\} = \{N_c\} V \quad (3)$$

The interpolation functions are chosen depending on the method of modeling and the dynamic degrees of freedom (DOFs) considered for the bridge elements. For the case that a lumped mass model is used, an interpolation function $\{N_c^v\}$ incorporating only vertical DOFs is sufficient; i.e. all the entries in $\{N_c^v\}$ are set to zero except those regarding vertical DOFs. This ensures that there would only be vertical resultant nodal forces. Nevertheless, for a model based on consistent mass, an interpolation function $\{N_c^{vr}\}$ is needed which includes the effect of the rotational DOFs in addition to the vertical DOFs. In this study the lumped mass model is used. Hermitian cubic polynomials can be used as the interpolation functions. $\{N_c^v\}$ or $\{N_c^{vr}\}$ are calculated based on the location of the wheel on the element, x , which is a function of time. For a two-dimensional 6-DOF Euler–Bernoulli beam element composed of vertical, horizontal, and rotational DOFs at each node, the corresponding interpolation functions are as follows

$$\{N_c^v\} = \langle 1 - 3\bar{x}^2 + 2\bar{x}^3, 0, 0, 3\bar{x}^2 - 2\bar{x}^3, 0, 0 \rangle^T \quad (4a)$$

$$\{N_c^{vr}\} = \langle 1 - 3\bar{x}^2 + 2\bar{x}^3, 0, x(1 - 2\bar{x} + \bar{x}^2), 3\bar{x}^2 - 2\bar{x}^3, 0, x(\bar{x}^2 - \bar{x}) \rangle^T \quad (4b)$$

where $\bar{x}$ is the local coordinate of the contact point on the element, i.e. $\bar{x} = x/L$ where L is the element length (see Fig. 1).

The wheel displacement vector $\{d_w\}$ can be related to the contact displacement $\{d_c\}$ of the bridge by the following constraint equation [12]

$$\{d_w\} = [\Gamma] \{d_c\} + \{r\} \quad (5)$$

where $[\Gamma]$ is a constant transformation matrix and $\{r\}$ represents the effect of the road irregularities. $\{r\}$ usually is defined based on the power spectral density functions which by themselves are functions of x , the location of the vehicle on the bridge. Therefore, the first derivative of the wheel displacement vector is obtained as

$$\{\dot{d}_w\} = [\Gamma] \{\dot{d}_c\} + \frac{dx}{dt} \{r\}_{,x} = [\Gamma] \{\dot{d}_c\} + v \{r\}_{,x} \quad (6)$$

where the comma denotes the differentiation with respect to the coordinates specified, and v is the velocity of the vehicle. By taking another derivative, we have

$$\{\ddot{d}_w\} = [\Gamma] \{\ddot{d}_c\} + a \{r\}_{,x} + v^2 \{r\}_{,xx} \quad (7)$$

where a is the vehicle acceleration which is zero for the case of the constant vehicle speed. Substituting Eqs. (5)–(7) in Eq. (2) and also defining new transformed contact matrices $[\bar{m}_c]$, $[\bar{k}_c]$, and

$[\bar{c}_c]$ instead of $[\Gamma][m_c]$, $[\Gamma][k_c]$, and $[\Gamma][c_c]$ respectively, a new formulation for the contact force is obtained

$$\{f_c\}_{t+\Delta t} = \{\bar{m}_c\} \{\ddot{d}_c\}_{t+\Delta t} + [\bar{c}_c] \{\dot{d}_c\}_{t+\Delta t} + [\bar{k}_c] \{d_c\}_{t+\Delta t} + \{p_c\}_{t+\Delta t} + \{q_c\}_t + \{r_c\}_{t+\Delta t} \quad (8)$$

where $\{r_c\}_{t+\Delta t}$ is defined as a new parameter representing the effect of the road irregularities in the contact force which is

$$\{r_c\}_{t+\Delta t} = v^2 [m_c] \{r\}_{,xx} + v [c_c] \{r\}_{,x} + [k_c] \{r\} \quad (9)$$

Then, it is required to relate the displacement of the contact point to that of the nodal ones. This is usually done in finite element analysis by applying the interpolation functions. For a 6-DOF beam element shown in Fig. 1, to obtain contact displacement d_c from nodal displacement vector $\{d_b\}$, an interpolation function $\{N_c^{vr}\}$ represented in Eq. (4b) should be used which includes the effect of the rotational DOFs in addition to the vertical DOFs

$$d_c = \{N_c^{vr}\} \{d_b\} \quad (10)$$

Velocity of the contact point can be determined using the chain rule by taking the first derivative of Eq. (10) with respect to time

$$\dot{d}_c = \frac{d\{N_c^{vr}\}}{dt} \{d_b\} + \{N_c^{vr}\} \{\dot{d}_b\} \quad (11)$$

According to Eq. (4b), $\{N_c^{vr}\}$ is a function of x , which is the location of the wheel and by itself is a function of time. Therefore

$$\frac{d\{N_c^{vr}\}}{dt} = \frac{d\{N_c^{vr}\}}{dx} \times \frac{dx}{dt} = \{N_c^{vr}\}_{,x} \times v \quad (12)$$

where the comma denotes the differentiation with respect to the coordinates specified, and v is the velocity of the vehicle. Hence, the first derivative of the displacement of the contact point is

$$\dot{d}_c = v \{N_c^{vr}\}_{,x} \{d_b\} + \{N_c^{vr}\} \{\dot{d}_b\} \quad (13)$$

similarly, the acceleration of the contact point is obtained as

$$\ddot{d}_c = \{N_c^{vr}\} \{\ddot{d}_b\} + 2v \{N_c^{vr}\}_{,x} \{\dot{d}_b\} + (v^2 \{N_c^{vr}\}_{,xx} + a \{N_c^{vr}\}_{,x}) \{d_b\} \quad (14)$$

where a is the acceleration of the vehicle. However, for the case of the constant velocity, the acceleration is zero. Using Eqs. (8), (10), (13) and (14), the equation of motion for the bridge element (Eq. (1)), traversed by the i th wheel, can be reformulated as

$$\begin{aligned} & [m_{bi}] \{\ddot{d}_{bi}\}_{t+\Delta t} + [c_{bi}] \{\dot{d}_{bi}\}_{t+\Delta t} + [k_{bi}] \{d_{bi}\}_{t+\Delta t} \\ & = \{f_{bi}\}_{t+\Delta t} - \sum_{j=1}^n \left([m_{cij}^*] \{\ddot{d}_{bj}\} + [c_{cij}^*] \{\dot{d}_{bj}\} \right. \\ & \quad \left. + [k_{cij}^*] \{d_{bj}\} \right) - \{p_{ci}^*\}_{t+\Delta t} - \{q_{ci}^*\}_t - \{r_{ci}^*\}_{t+\Delta t} \end{aligned} \quad (15)$$

where n is the total number of wheels and the matrices with an asterisk are calculated using contact matrices and interpolation vectors as

$$[m_{cij}^*] = \{N_{ci}^v\} \bar{m}_{cij} \{N_{cj}^{vr}\} \quad (16a)$$

$$[c_{cij}^*] = \{N_{ci}^v\} \left(2v \times \bar{m}_{cij} \{N_{cj}^{vr}\}_{,x} + \bar{c}_{cij} \{N_{cj}^{vr}\} \right) \quad (16b)$$

$$\begin{aligned} [k_{cij}^*] &= \{N_{ci}^v\} \left(v^2 \times \bar{m}_{cij} \{N_{cj}^{vr}\}_{,xx} \right. \\ & \quad \left. + v \times \bar{c}_{cij} \{N_{cj}^{vr}\}_{,x} + \bar{k}_{cij} \{N_{cj}^{vr}\} \right) \end{aligned} \quad (16c)$$

and the equivalent nodal loads are

$$\{p_{ci}^*\}_{t+\Delta t} = \{N_{ci}^v\} p_{ci,t+\Delta t}; \quad \{q_{ci}^*\}_t = \{N_{ci}^v\} q_{ci,t} \quad (17)$$

similarly, the equivalent vector of road irregularities is

$$\{r_{ci}^*\}_{t+\Delta t} = \{N_{ci}^v\} r_{ci,t+\Delta t} \quad (18)$$

In the prior three formulas, parameters with i or j are the entries of the corresponding vectors or matrices, e.g. $\bar{m}_{cij}$, $\bar{k}_{cij}$, and $\bar{c}_{cij}$ are entries of the i th row and j th column of the transformed contact matrices $[\bar{m}_c]$, $[\bar{k}_c]$, and $[\bar{c}_c]$, respectively. Eqs. (15)–(18) comprise the formulations for the proposed VBI element including modified mass, damping, and stiffness matrices. Using these Eqs., the new modifications can be easily consulted compared with the VBI element proposed by Yang and Wu [5]. This element incorporates explicitly the effect of the moving vehicle with the asterisk matrices and vectors. The overall bridge structural matrices (mass, damping, and stiffness) can be assembled using Eqs. (15)–(18), and are updated in each time increment.

3. Working procedure of the VBI analysis

The step-by-step time history numerical procedure to analyze a bridge traversed by a moving vehicle with constant velocity is as follows:

- Construct the structural matrices (i.e. mass, damping, and stiffness) for the bridge free of any vehicle. The Rayleigh method is used to calculate the damping matrix. In addition, determine the structural matrices for the vehicle model.
- Define t_{end} and x_{end} and also Δt . The parameter x_{end} is needed to determine whether the vehicle is still on the bridge or not. This parameter can be specified for any wheel and its value is the vehicle length plus the bridge length.
- Consider initial conditions for the bridge elements and vehicle DOFs. Let $t = 0$.
- Perform the new time increment $t_{\text{new}} = t + \Delta t$. Stop the analysis if $t_{\text{new}} > t_{\text{end}}$ or the position of the first wheel is greater than x_{end} .
- Compute interpolation functions $\{N_c^v\}$ and $\{N_c^{vr}\}$ based on wheel positions using Eq. (4).
- Determine corresponding asterisk matrices $[m_{cij}^*]$, $[c_{cij}^*]$, and $[k_{cij}^*]$ using Eq. (16), and also asterisk vectors $\{p_{ci}^*\}_{t+\Delta t}$, $\{q_{ci}^*\}_t$, and $\{r_{ci}^*\}_{t+\Delta t}$ by Eqs. (17) and (18).
- Update the global bridge structural matrices using asterisk values calculated in the previous step.
- Solve the bridge equation of motion by the Newmark- β method and find the bridge displacement vector for time $t + \Delta t$.
- Find $\{d_w\}$, $\{\dot{d}_w\}$, and $\{\ddot{d}_w\}$ for time $t + \Delta t$ using Eqs. (5)–(7).
- Calculate the displacement vector for the upper part of the vehicle, which is not in direct contact with the bridge, at time $t + \Delta t$.
- Go to Step “d” for the new time increment.

4. Numerical verifications

4.1. Moving sprung mass model

Consider a simply supported girder subjected to a moving sprung mass as illustrated in Fig. 2. This case was used for verification by many researchers such as Majka and Hartnett [4], Zhang et al. [2], and Yang and Wu [5]. The required data are: Young's modulus $E = 2.87$ GPa, Poisson's ratio $\nu = 0.2$, moment of inertia $I = 2.90$ m⁴, mass per unit length $m = 2303$ kg/m, girder length $L = 25$ m, sprung mass $M_v = 5.75$ ton, wheel mass $M_w = 0$, and suspension spring constant $k_v = 1595$ kN/m. No damping is included in the system. The span is divided into 10 Euler–Bernoulli beam elements. The results of the sprung mass system are compared in Fig. 3 with the analytical solution presented by Biggs [13] and the moving load model response. The results show a perfect match between the two methods. In addition, the moving load response is obtained using the same MATLAB code by using gravitational loading of the vehicle as the constant moving load.

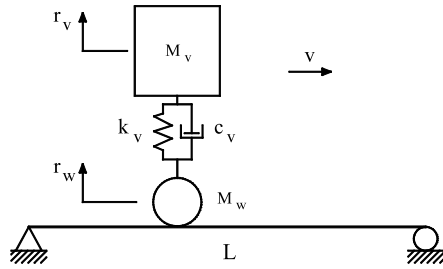


Fig. 2. Verification case 1, moving sprung mass model.

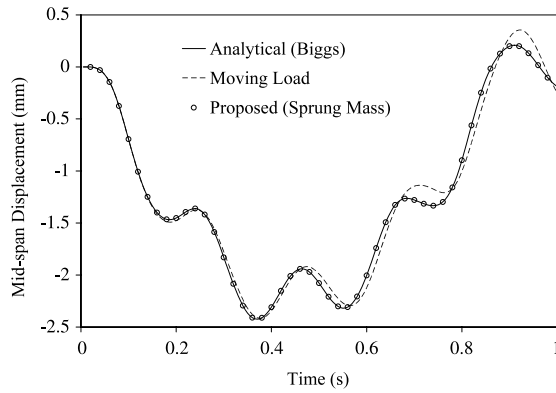


Fig. 3. Displacement at the beam mid-span, verification case 1.

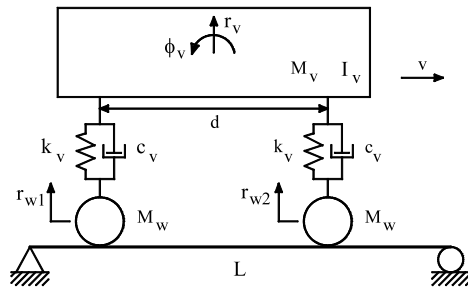


Fig. 4. Verification case 2, suspended rigid beam model.

4.2. Suspended rigid beam

A rigid beam comprised of two DOFs supported by two wheels is the second verification case. Each wheel is connected to the rigid beam by a spring and damper. This model shown in Fig. 4 was used by Yang and Wu [5]. Properties for the beam are: Young's modulus $E = 2.943$ GPa, Poisson's ratio $\nu = 0.2$, moment of inertia $I = 8.65$ m⁴, mass per unit length $m = 36$ t/m, and beam length $L = 30$ m. The moving beam properties are: sprung mass $M_v = 540$ ton, wheel masses are zero, rotatory mass $I_v = 13800$ t m², and suspension spring constant $k_v = 41350$ kN/m. No damping is included in the system. The wheel to wheel distance is 17.5 m. The vehicle speed is 27.78 m/s. The beam is modeled by 10 elements each 3 m. The result presented by Yang and Wu [5] is depicted in Fig. 5 together with results obtained from the proposed method and also from the moving load model. Again, a very close match is obtained for the proposed element and the existing element proposed by Yang and Wu [5].

5. Effect of model variables

5.1. Effect of the shear deformations on the VBI analysis

The way that bridges are being modeled is important in the final response of any analysis of the bridges including moving

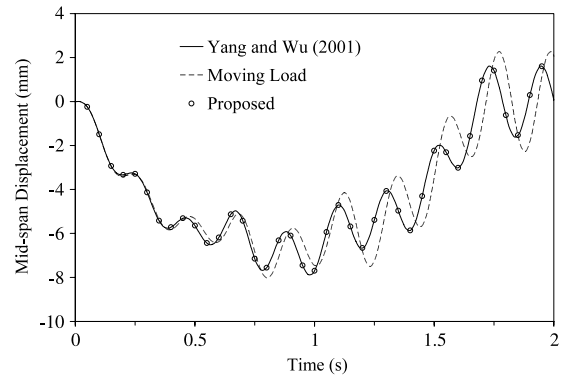


Fig. 5. Displacement at mid-span of the beam, verification case 2.

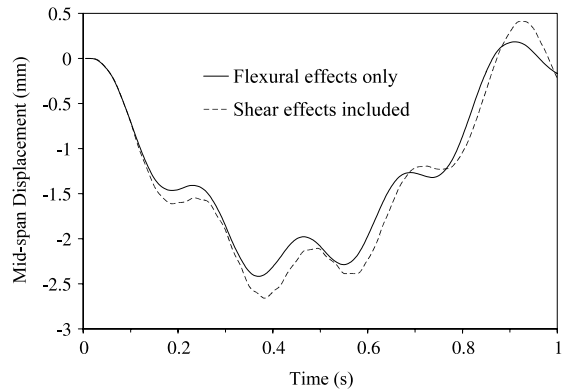


Fig. 6. Effect of the shear deformations on the sprung mass model; element length 1 m.

vehicles. Some models for simply-supported bridges consider only the flexural stiffness, while ignoring the effects of the shear deformations. To study this effect, a sample 20.0 m long steel girder that has a W840 × 226 section is assumed. The stiffness matrix is assembled by using the direct stiffness method and two types of element local stiffness matrices; the first type is only with flexural effects and the second also includes shear effects. The frequencies of the first three modes of vibration are compared with the results obtained from linear finite element model for three element lengths in Table 1.

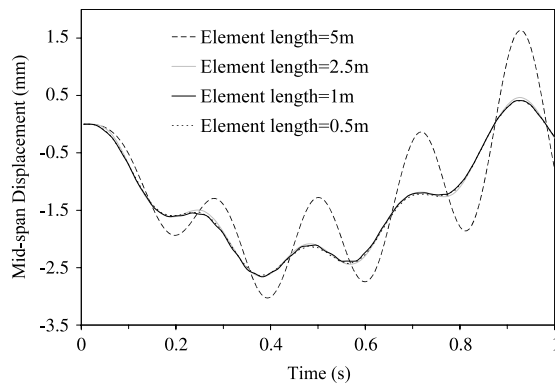
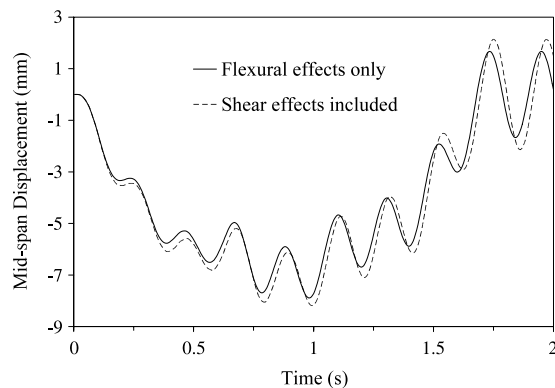
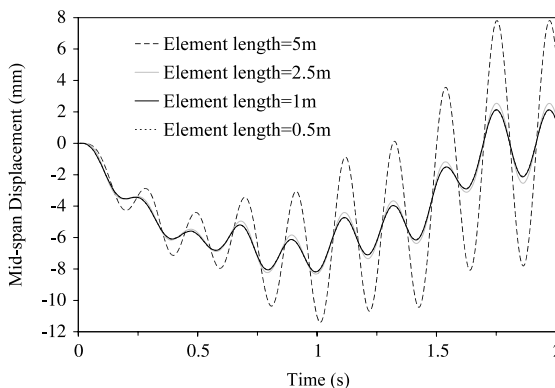
To investigate the effects of this type of element on the response of the bridge under moving vehicles, verification case 1 and case 2 are analyzed for both beam elements including and excluding shear effects for different element lengths. Results presented in Figs. 6 and 8 indicate that the effect of the shear deformation is not negligible and should be included in the stiffness matrix. Figs. 7 and 9 show that the response is affected drastically by long element lengths, however small element lengths may not be needed for the analysis since for a range of elements the results are similar but computational efforts are much higher for small elements.

The foregoing discussion concluded that the effect of the shear deformations should be included in the analysis. One of the key parameters affecting the contribution of the shear deformations is the bridge shear span-to-depth ratio. To study the significance of the effect of the bridge span, several double plate-girder open-deck railway bridges were designed following the requirements of AREMA (American Railway Engineering and Maintenance-of-Way Association) manual, Chapter 15 [14]. The moment of inertia and cross-sectional area of bridges were determined to sustain the Cooper-E 80 loading or alternate live load considering overall deflection and local buckling requirements. Appropriate impact loads and rocking effects were also added to the live loads. The summary of the bridge properties is tabulated in Table 2. These

Table 1

Modal frequencies for a simply-supported beam obtained from computer code compared to linear FEM.

	Linear FEM model			Flexural effects only			Flexure and shear effects		
	5 m	2.5 m	1 m	5 m	2.5 m	1 m	5 m	2.5 m	1 m
Mode 1	27.207	27.213	27.214	27.322	27.325	27.326	27.180	27.186	27.188
Mode 2	106.7	107.17	107.24	109.03	109.29	109.30	106.60	107.06	107.14
Mode 3	228.34	234.73	235.51	241.37	245.76	245.93	228.12	234.50	235.28

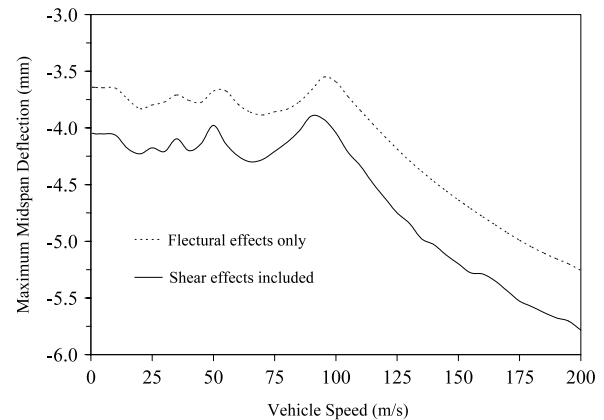
**Fig. 7.** Effect of the element length on the sprung mass model including shear effects.**Fig. 8.** Effect of the shear deformations on the suspended rigid beam model; element length 1 m.**Fig. 9.** Effect of the element length on the suspended rigid beam model.

bridges were traversed by a sample vehicle with a sprung mass model similar to Fig. 2 as a generic moving vehicle. The properties of the vehicle are based on parameters presented by Lei and Noda [15] which are due to a TGV (French high-speed train)

Table 2

Structural properties of the designed bridges.

Bridge span (m)	Moment of inertia (m ⁴)	Cross-section area (m ²)	Shear area (m ²)
4	0.001020	0.019464	0.00896
6	0.002806	0.032080	0.01440
8	0.006384	0.047040	0.02160
10	0.011170	0.060144	0.02730
12	0.016496	0.066252	0.03198
14	0.024240	0.075100	0.03640
16	0.031300	0.085232	0.04200
18	0.042020	0.093080	0.04676
20	0.057560	0.100720	0.05320
25	0.105540	0.121720	0.06580
30	0.183480	0.153360	0.07560

**Fig. 10.** The effect of the shear deformations on a 12 m long bridge for various vehicle speeds.

locomotive, similar to an Amtrak Acela Express high-speed train operating between Boston and Washington. Mass of the car, bogie, and wheel-set are 53.5 ton, 3.26 ton, and 2.06 ton, respectively. The wheel-set spacing is 14 m, and vertical stiffness and damping of the primary suspension are 3.28×10^6 N/m and 90 kN/s/m.

The designed bridges were modeled by different element lengths and the sample vehicle with different velocities ranging from 0.5 to 200 m/s. Results revealed similar behavior for all bridges with various element lengths. Midspan deflection, as the reference response to evaluate bridge dynamics, was calculated for two types of bridge models which are first comprised of elements with flexural effects and second by elements including shear deformations in the stiffness matrix. The difference between these two modeling schemes was observed almost identical for vehicles with various speeds. Fig. 10 shows a typical response for maximum midspan deflection calculated due to two different modeling methods.

As expected, the shorter spans experienced more differences between results obtained by bridge models including or excluding shear deformations demonstrated in Fig. 11. For shorter spans, the difference increases rapidly (to reach about 18% for 4.0 m span) while for longer spans the difference reduces to be around 8%.

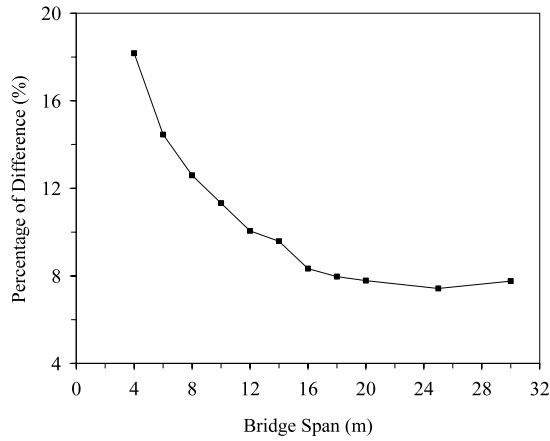


Fig. 11. The percentage of difference for midspan deflection response regarding models including shear deformation effects with respect to models including flexural effects only for various bridge spans.

5.2. Evaluation of the new VBI element via important dimensionless parameters

Parameters to be investigated are frequency parameter γ , system mass parameter κ , vehicle mass parameter μ , and the damping of the vehicle and the bridge. Frequency parameter γ is the ratio of the natural frequency of the vehicle to that for the bridge. System mass parameter κ is the total vehicle mass to the bridge mass.

In order to study the influence of the dimensionless parameters, a railway bridge model is used reported by Foutch et al. [8]. The bridge is considered as an open deck plate girder railway bridge with a simply-supported span of 15.24 m. The cross sectional area is 0.12684 m^2 and mass per unit length is 1428.64 kg/m . The moment of inertia is calculated as 0.05411 m^4 based on the drawings available in the report. In addition, other required bridge properties are assumed which are Young's modulus $E = 200 \text{ GPa}$, damping ratio 2%, and Poisson's ratio 0.3. The same vehicle model (TGV locomotive) is used in this section as well, with the sprung mass model (see Fig. 2).

To carry out the analysis, the dynamic amplification factor (DAF), as the ratio of the mid-span displacement of the dynamic analysis to the static one, is calculated as the most important parameter in the bridge response. In addition to DAF, two other responses are evaluated in this study, namely maximum bridge and vehicle acceleration. The bridge acceleration is also important in the design of bridges especially ballasted railway bridges, and vehicle acceleration is a design criterion in the design of high speed trains governing the passenger comfort. Results are presented in subsequent figures. Each figure illustrates the effect of each parameter on the bridge response and also shows a comparison of the new proposed element with the existing one proposed by Yang and Wu [5].

The effect of the bridge damping is shown in Figs. 12–14, and for the vehicle damping, results are presented in Figs. 15–17. The effect of the vehicle damping on the bridge responses is minor, i.e. bridge acceleration and DAF, while the effect of bridge damping is much more, in particular for the bridge acceleration. However, the effect of the vehicle damping is more influential on the vehicle response compared to the bridge damping.

The comparison between the new proposed element and Yang and Wu [5] element indicates that the new method results in smaller values of DAF for both bridge and vehicle damping. This effect is more evident for vehicle acceleration. In addition, the general trend shows the growing effect of the proposed element for higher speeds. On the other hand, more results are obtained for

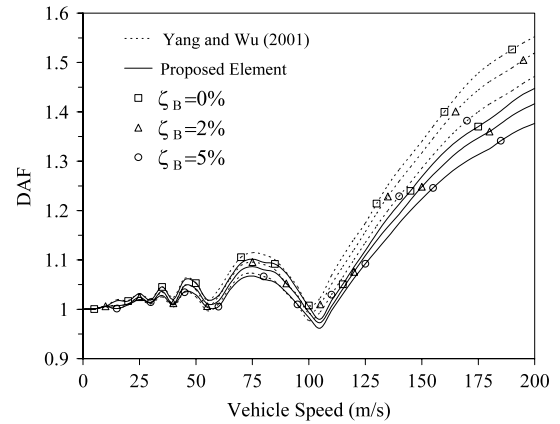


Fig. 12. The comparison of the effect of the bridge damping ζ_B on DAF.

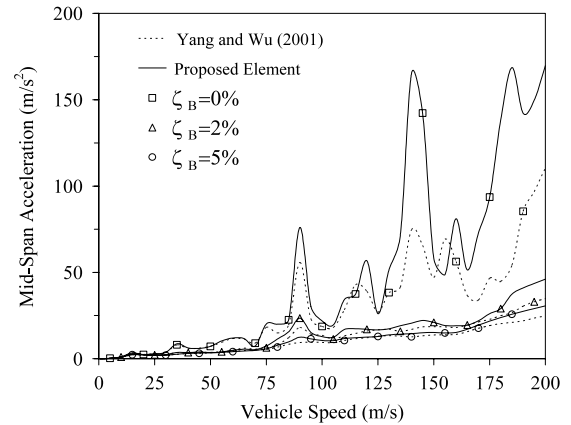


Fig. 13. The comparison of the effect of the bridge damping ζ_B on the bridge acceleration.

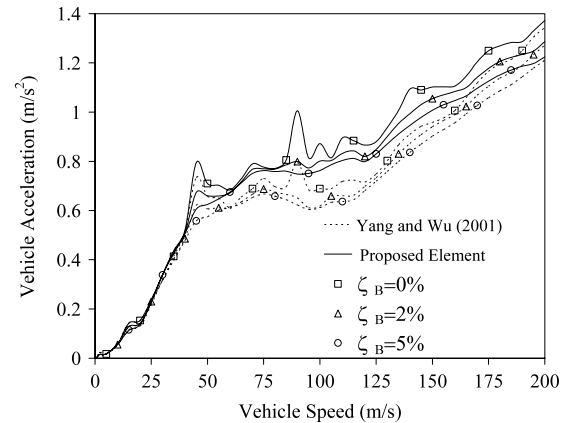


Fig. 14. The comparison of the effect of the bridge damping ζ_B on the vehicle acceleration.

the proposed element compared with the Yang and Wu [5] element regarding the vehicle or bridge acceleration. The important point is that the general trends of the effect of all parameters are almost the same for both Yang and Wu [5] and the proposed elements.

The effect of the ratio of the first mode vehicle frequency over the bridge natural frequency (i.e. parameter γ) is shown in Figs. 18–20. In order to incorporate changes in the frequency parameter, only the stiffness of the moving sprung mass was varied such that the targeted γ was obtained. The results indicate that the bridge acceleration is not influenced substantially by the alteration in the frequency parameter. However, γ has a considerable effect on DAF and vehicle acceleration even for low speeds.

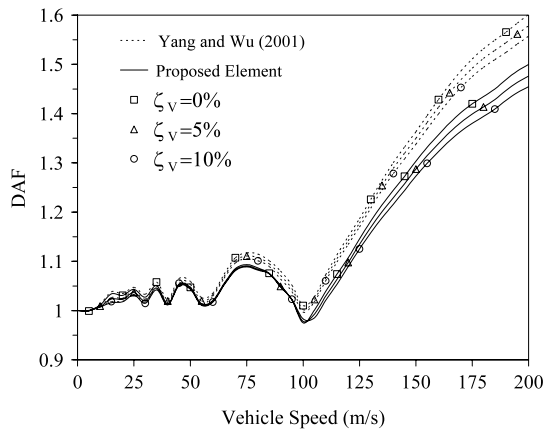


Fig. 15. The comparison of the effect of the vehicle damping ζ_V on DAF.

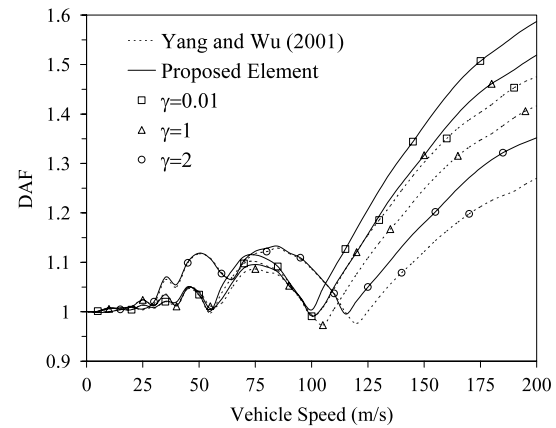


Fig. 18. The comparison of the effect of the frequency parameter γ on DAF.

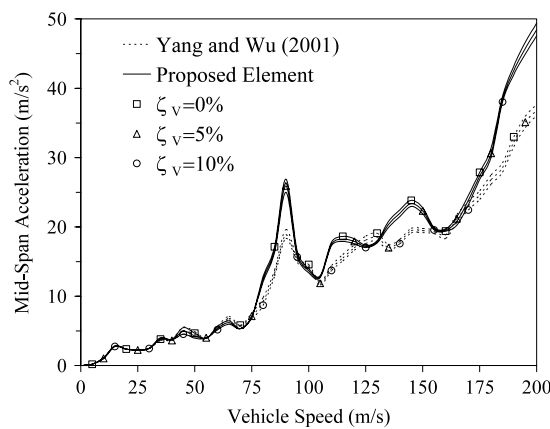


Fig. 16. The comparison of the effect of the vehicle damping ζ_V on the bridge acceleration.

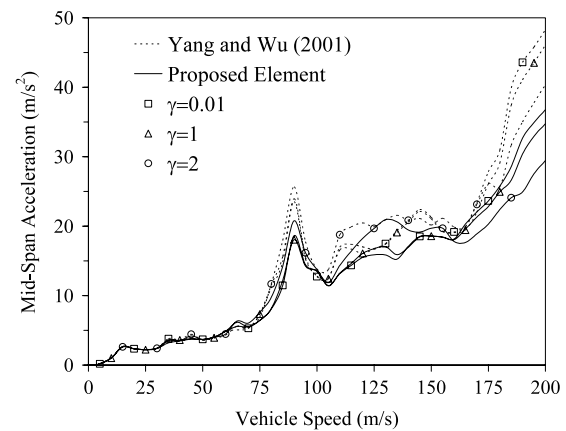


Fig. 19. The comparison of the effect of the frequency parameter γ on the bridge acceleration.

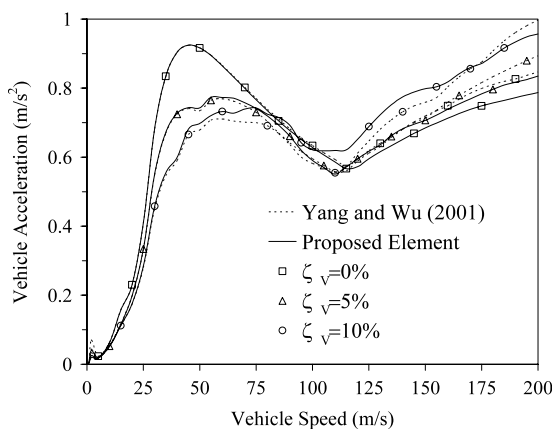


Fig. 17. The comparison of the effect of the vehicle damping ζ_V on the vehicle acceleration.

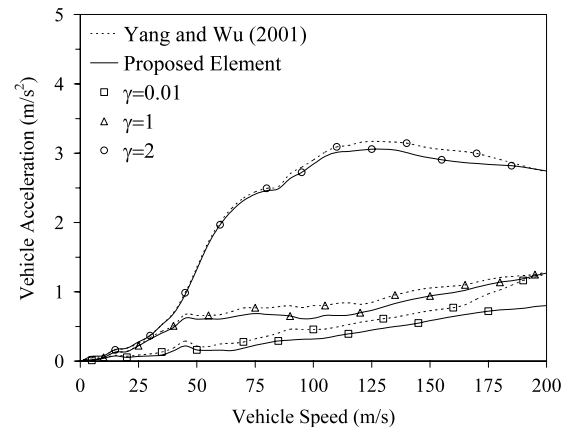


Fig. 20. The comparison of the effect of the frequency parameter γ on the vehicle acceleration.

The results, calculated based on the proposed and also Yang and Wu [5] elements, specify that the difference rises while vehicle speed increases. This may be attributed to the fact that the proposed element incorporates more realistically the effect of dynamic parameters such as vehicle velocity which have more effect in higher speeds. However, the two methods provide very close results for vehicle acceleration. In contrast to the bridge or vehicle damping, the new proposed element results in greater responses for DAF and smaller values for bridge and vehicle acceleration due to changing frequency parameter.

The last parameter studied is the system mass ratio κ which is the ratio of the vehicle mass over the bridge mass. It has significant effect on the bridge or vehicle acceleration as shown in Figs. 22 and 23. It is interesting to note that, for comparatively low speeds, the rate of vehicle acceleration is substantially increased when κ is increased and the effect is less for higher speeds. The results obtained for bridge acceleration is as expected and it is ascending with almost the same ratio as κ value. The most interesting observation may be attributed to the changes in the dynamic amplification factor (DAF) with respect to κ values as seen in Fig. 21. DAF is less for greater κ parameters which is true for a

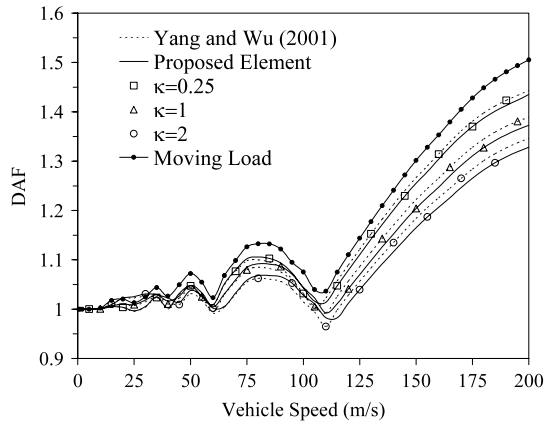


Fig. 21. The comparison of the effect of the system mass parameter κ on DAF.

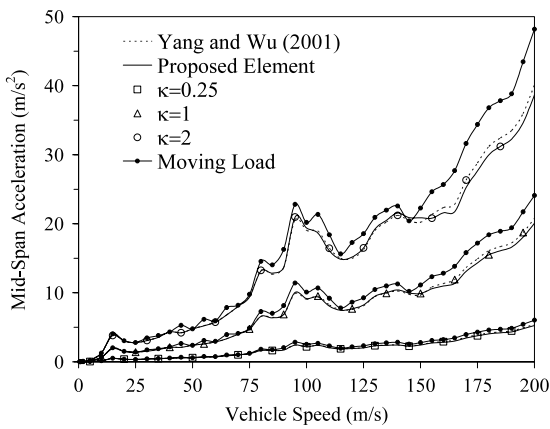


Fig. 22. The comparison of the effect of the system mass parameter κ on the bridge acceleration.

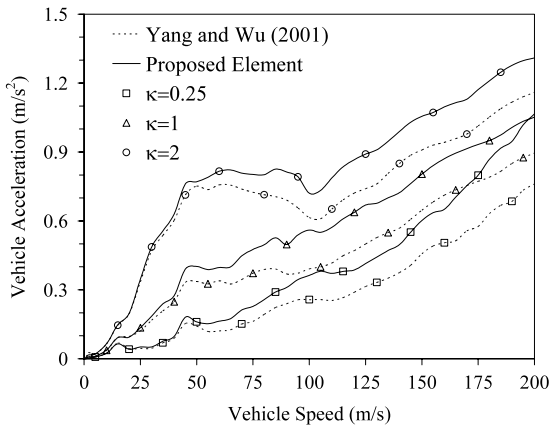


Fig. 23. The comparison of the effect of the system mass parameter κ on the vehicle acceleration.

great range of vehicle speeds (more than 40 m/s) representing a moderate to high speed vehicle. The results, obtained for different κ values, determine that two elements, i.e. the proposed and Yang and Wu [5] elements, result in similar responses even for high speeds. However, variations are greater for vehicle acceleration in particular for higher speeds. In addition, moving load analysis gives the upper bound responses compared to the moving sprung mass model shown in Figs. 21 and 22.

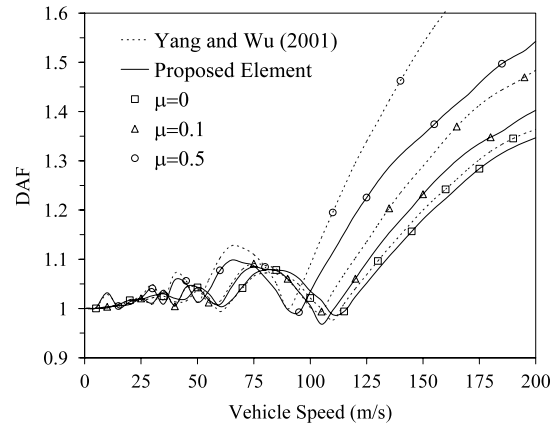


Fig. 24. The comparison of the effect of the vehicle mass parameter μ on DAF.

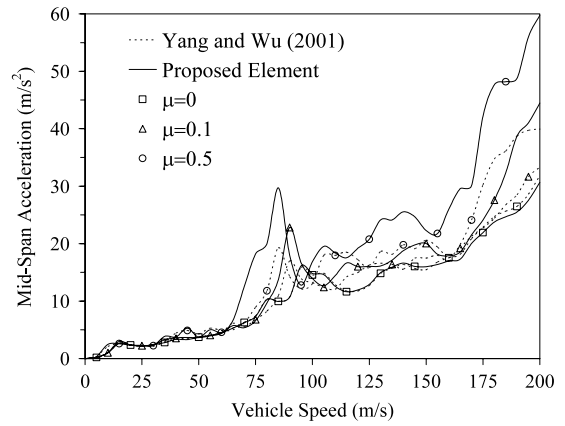


Fig. 25. The comparison of the effect of the vehicle mass parameter μ on the bridge acceleration.

5.3. Effect of vehicle mass parameter

A new parameter, called vehicle mass parameter, is proposed for the study of the model variables. This parameter, denoted as μ , is defined as the wheel mass m_w to the vehicle body mass m_{vb}

$$\mu = \frac{m_w}{m_{vb}} \quad (19)$$

Since the wheel mass is constant for a particular vehicle, the upper and lower bound of this parameter is due to the maximum loaded vehicle or an empty one, respectively. However, fully loaded vehicles are usually used for the bridge design yet there are different μ values for various vehicles (e.g. cars, trucks, and trains) which affect the discrepancy of the bridge dynamic response with respect to the static analysis. In contrast to the bridge design, fully loaded passenger cars may not result in maximum design criteria for the passenger comfort which is evaluated by the vehicle acceleration. Partially loaded trains have higher μ values, since the mass of the car body is less whilst the wheel mass is the same as for the fully loaded train.

To study the effect of this parameter on the dynamic response, the same bridge and vehicle model used in Section 5.2 is utilized here as well. Figs. 24–26 show the sensitivity of the response due to various values of this parameter and also verify the different responses obtained from proposed modified VBI element and Yang and Wu [5] element. Similar to the bridge and vehicle damping ratio, smaller results for DAF are obtained from the proposed element compared to the Yang and Wu [5] element and greater responses for vehicle and bridge acceleration.

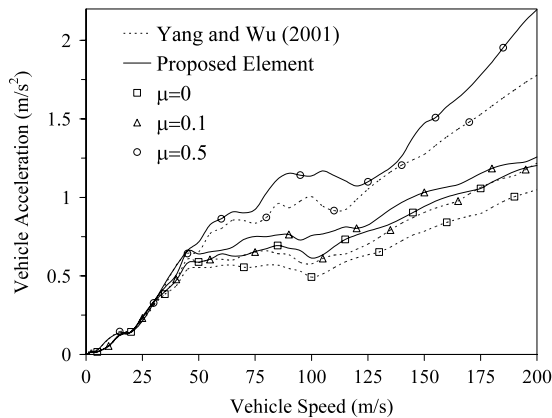


Fig. 26. The comparison of the effect of the vehicle mass parameter μ on the vehicle acceleration.

The differences between the two methods associated with higher values of μ are significant for DAF in high-speed vehicles. Accordingly, Fig. 24 demonstrates that the effect of the wheel mass is overestimated in the Yang and Wu [5] element which can result in great DAF values for $\mu = 0.5$ in high-speed vehicles. All responses of the bridge and vehicle shown in Figs. 24–26 tend to increase when μ is augmented. Figs. 25 and 26 imply that the greater acceleration in both bridge and vehicle is obtained for vehicles with less weight having higher μ values. Therefore, using fully loaded vehicles will not always result in upper bound responses for bridge and vehicle acceleration particularly for high-speed vehicles.

6. Conclusions

In this study, a new modified VBI (Vehicle–Bridge Interaction) element for the calculation of complicated VBI problems that incorporates more realistically the effect of the dynamic parameters (e.g. vehicle speed, etc.) is proposed. The general common notations are kept the same as the VBI element proposed by Yang and Wu [5] for easier comparison between the two methods. The new formulation and mathematical background are presented comprehensively in Section 2. The final applicable formulas for numerical analysis as the properties of the new VBI element are summarized in Eqs. (15)–(18).

In order to investigate the effect of the new VBI element compared with the VBI element proposed by Yang and Wu [5], a study on the influence of some important parameters was carried out. Parameters available in the literature considered in this study were vehicle and the bridge damping, total vehicle mass to the bridge mass κ , and the ratio of the vehicle first mode frequency over the bridge natural frequency γ . In addition, a new parameter μ was introduced in this study which is the ratio of the wheel mass to the vehicle body mass. The dynamic amplification factor, as the ratio of the mid-span displacement of the dynamic analysis to the static one, was considered as the most important bridge response for the study on the model variables. Bridge and vehicle accelerations were also obtained from the analysis. The conclusions generally imply that, the new VBI element results in different DAF values increasing when vehicle speed increases. For vehicle and bridge acceleration, different responses are observed for all vehicle speeds. These differences can be attributed to the effect of dynamic parameters (e.g. vehicle speed) presented in the formulation proposed for the new modified VBI element.

In addition, studying the effect of dimensionless parameters demonstrates that the greatest sensitivity in both methods is observed for the vehicle mass parameter μ for any vehicle speed and also for frequency parameter γ regarding high speed vehicles. Results indicate that using fully loaded vehicles having less μ values will not always result in upper bound responses for bridge and vehicle acceleration particularly for high-speed vehicles. Finally, the analyses showed that the dynamic amplification factor (DAF) is not always increasing with vehicle velocity and can be even less than one for medium speed vehicles. This region of decreasing DAF can be shifted if longer bridges with different natural frequency are applied.

The effect of the shear deformation and the element length was also studied. It demonstrated that the response is affected drastically by long element lengths, however small element lengths may not be needed for the analysis since for elements smaller than 2.5 m (for the bridge examples analyzed) the results are similar, and also computational efforts are much higher for small elements. In addition, it was shown that shear deformations should be included in the construction of the stiffness matrix and they are not negligible in the VBI analysis. By the passage of a sample vehicle (French high speed locomotive) over a series of designed double plate-girder open-deck railway bridges, the difference between models including and excluding shear deformations were measured up to 18% for 4.0 m span bridge and around 8% for spans longer than 16.0 m.

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