

A fatigue stress-life damage accumulation model for variable amplitude fatigue loading based on virtual target life



Ihab El Aghoury, Khaled Galal *

Department of Building, Civil and Environmental Engineering, Concordia University, Montréal, Québec, Canada

ARTICLE INFO

Article history:

Received 16 March 2012
Revised 26 February 2013
Accepted 1 March 2013
Available online 20 April 2013

Keywords:

Fatigue
Damage model
Variable amplitude
Stress-life
Virtual Target Life Curves (VTLCs)

ABSTRACT

Fatigue of structural elements has been a major cause of many catastrophic failures of railway and high-way steel bridge. In this research, a new stress-life damage accumulation model is proposed, calibrated and validated using two different experimental works available in the literature. The newly proposed model (called the VTLC, for Virtual Life Target Curves) introduces a time dimension to the conventional S–N curves. The model has the advantage that it can simulate the effect of non-uniform fatigue loading. The VTLC model was then verified using a case study of a riveted single-track railway bridge that was subjected to variable amplitude loading. The numerical simulation shows that the proposed VTLC model is capable of predicting the remaining fatigue life of the studied structural detail under variable amplitude loading.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Most of the available data regarding fatigue life is based on experimental work conducted on specimens subjected to constant amplitude cyclic loading (stress or strain). During the service life of any structure, it is expected that the structure would be subjected to various random loads that could have a constant- or random-loading pattern. The current practice for quantifying the accumulated damage at each stress (or strain) level, and consequently the fatigue life of the structural element, is to use these available constant amplitude experimental data. Unfortunately, experiments using variable amplitude loading are expensive, time-consuming and having much uncertainties about the in-service loading spectrum [1]. Moreover, variable amplitude loading is more difficult to formulate compared to constant amplitude stress fatigue. These constraints resulted in scarceness in research work that studies multi-stress level fatigue. The objective of this research is to propose a simple and reliable damage model that is capable of predicting the remaining fatigue life of structural steel details subjected to variable amplitude loading.

In 1945, Miner [2] expressed the concept of damage accumulation in a very simple linear trend. This rule is still the most popular rule used due to its simplicity. Unfortunately, this damage model has many deficiencies. This rule fails to predict the effect of load history sequence. Experimental data indicate that the sequence

in which various stress levels are applied has significant influence on the fatigue behavior of materials. It is also widely known that Miner's damage sum to failure is greater than unity for low-to-high tests and less than unity for high-to-low tests [3]. Another major limitation of Miner's rule is that it is stress-level independent, in that its damage trend follows only one function (which is linear) for any stress level.

Corten et al. [4] performed hundreds of experimental tests on cold-drawn steel wires by applying different loading blocks with different stress levels. Their experimental test program had several series of two-stress level loading sequences repeated until failure. They described damage in a phenomenological hypothesis that represents the nucleation of microscopic voids which develop into cracks. The damage is given in terms of number of damaged nuclei and the rate of damage propagation as a power function of the number of cycles that applies for two stress level loading blocks histories.

With over 20 years of testing, Manson and his co-workers (e.g. [5,6]) proposed several conceptually-different damage models, and continuously updated them to correlate with their considerable test result data. Manson et al. [5] first introduced the concept of Double Linear Damage Rule (DLDR). After which, their work was extended to develop the concept of Damage Curve Approach (DCA) and later the Double Damage Curve Approach (DDCA) [6]. These models were in good agreement with experimental work in low cycle fatigue under two loading levels. These damage models are all load-level dependent, but do not account for the load interaction effect and small amplitude cycle damage. Also determining the Knee point (which is a new concept introduced in Manson's

* Corresponding author. Tel.: +1 5148482424.

E-mail address: galal@bcee.concordia.ca (K. Galal).

work and represents the intersection between the two straight lines of the bilinear curves) is a difficult procedure that could be inconvenient for regular design procedure.

Many researchers agreed that the errors in the Linear Damage Rule (LDR) life predictions are not necessarily due to the linear summation of damage but due to the assumption of damage-rate independence of loading levels [7]. Based on experimental results, Subramanyan [8] introduced a modified damage model of several damage iso-lines having a knee point or pivot near the fatigue limit of the material. This approach has problems at stresses near the fatigue limit due to the singularity at the knee point. Later, Hashin et al. [9] presented a discussion of the S–N curve rotation approaches and performed analytical calculations along with experimental work. The predicted results were in good agreement with test data. Later, Leipholz [10] revived the concept of a modified S–N curve, named S–N' curve, which accounts for load interaction effects.

2. Proposed damage model

The aim of the proposed damage model is to alleviate some of the shortcomings in the previously mentioned available damage models. The damage model will incorporate the effects of stress-level dependency and load sequence effects while still maintaining a simple form (which is one of the main advantages of the widely used Palmgren–Miner rule).

In this model, a phenomenological approach for estimating the level of fatigue damage will be used. Material damage can be described as the loss in expected life by accumulation of loading cycles under certain stress levels. A new concept is proposed which will be called the “Virtual Target Life Curves” (VTLCs). In this approach, it will be assumed that any material has a virtual (theoretically infinite) expected life that is, by definition, greater than the real failure life under constant amplitude loading. As constant amplitude fatigue loading proceeds, this expected life reduces in magnitude depending on the number of cycles, where the rate of deterioration keeps changing depending on the stress level. In cases of variable amplitude fatigue loading, some sudden jumps of damage (and consequently corresponding loss in expected life) will occur due to overloading effects resulting from the increase from low stress levels to high stress levels (and vice versa). The magnitudes of these jumps depend on the ratio of the two stress levels. Fig. 1 shows that the concept of Virtual Target Life Curves – in case of constant amplitude loading – will lead to the regular S–N curve (which represents the number of constant amplitude cycles that leads to failure of a laboratory tested element).

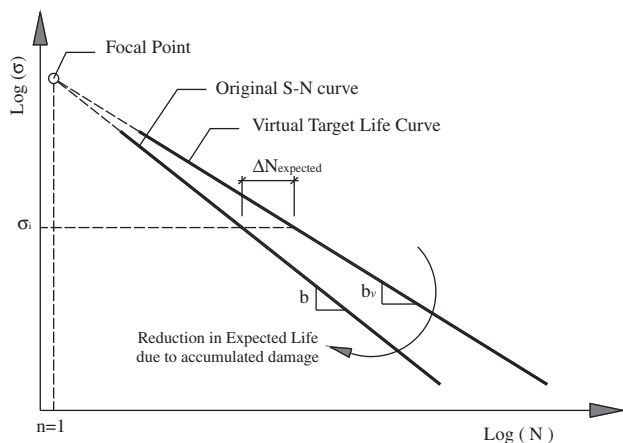


Fig. 1. The proposed virtual target life concept.

It is important to note the difference between the proposed approach and other approaches in the literature that are based on modifying the S–N curve. Previous approaches such as Manson's early work [5] and Subramanyan [8] were based on shifting the so-called virgin material curve as a starting point to account for cumulative damage in cases of variable amplitude loading. Using the term *virgin material curve* for the S–N curve could be misinterpreted as this curve represents in reality the failure of the specimen, whereas the term *virgin material* implies a totally unloaded material. In this study, a different approach is adopted; the material is supposed to endure until it reaches a virtual target life. The virtual life decreases as damage accumulates. The rate of decrease of this virtual life changes based on the loading history (including both: stress levels and loading sequences) and depends on the material. Fig. 2 illustrates the difference between Manson's approach, Subramanyan's approach and the VTLC approach.

The VTLC approach aims to give regular S–N curves (for constant amplitude excitations) a time- or loading- history dependency by making them dynamically moving according to the damage state. Moreover, it would be useful to have the S–N Curve as a way of monitoring the history of deterioration of a certain component in a structure as will be shown in the case study.

2.1. Derivation

The concept of fatigue damage has always been challenging as it is not easy to set a clear definition for damage or provide a physical correlation to it. The VTLC approach does not use physical damage as a failure criterion. The VTLC approach gives a different perspective for the concept of damage. It views damage rather than being an accumulative sum of cycles up to failure, to being a concept of accumulated loss in the expected life. Unlike the regularly used criterion for failure when damage factor reaches unity, the VTLC failure criterion is reached when the accumulated number of cycles reaches the virtual target life at a certain stress level. The new concept of the virtual target life will be explained later throughout this section.

Using S–N curves, at certain stress amplitude σ_a , the number of cycles to failure could be determined from the following equation:

$$N_f = \left(\frac{\sigma_a}{\sigma'_f} \right)^{1/b} = \left(\frac{\sigma'_f}{\sigma_a} \right)^{-1/b} \quad (1)$$

where b is the fatigue strength exponent and σ'_f is the fatigue strength coefficient of the material. Eq. (1) can be re-written as:

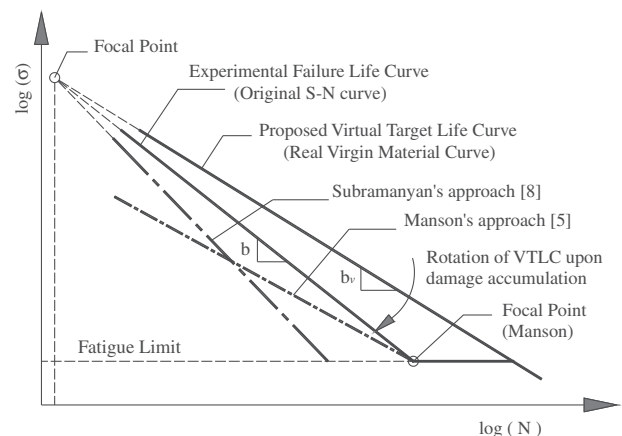


Fig. 2. Difference between previous S–N curve modification approaches [5,8] and the proposed model.

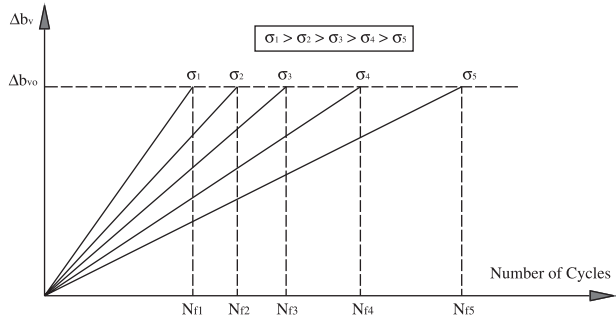


Fig. 3. Change in slope in multi-stress level loading using a linear trend.

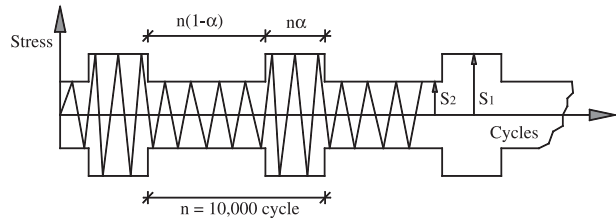


Fig. 4. Testing loading pattern by Corten et al. [4].

$$\log(\sigma_a) = b \log(N_f) + \log(\sigma'_f) \quad (2)$$

To construct a “Virtual Target Life” (VTL) curve from an existing S–N curve, it will be assumed that there is another curve with slope b_v , which has to be less than b . All VTL curves are assumed to meet at the same intercept at $N = 1$ cycle which means the equation will have the same constant term: $\log(\sigma'_f)$. Thus, the equation for any Virtual Target Life Curve would be:

$$\log(\sigma_a) = b_v \log(N_{VTL}) + \log(\sigma'_f) \quad (3)$$

Therefore, the virtual target life (VTL) of a component at a certain stress level i can be expressed by the following equation:

$$N_{VTL_i} = \left(\frac{\sigma_i}{\sigma'_f} \right)^{1/b_{vi}} = \left(\frac{\sigma'_f}{\sigma_i} \right)^{-1/b_{vi}} \quad (4)$$

The rate by which the VTLC approaches the original S–N curve is important. Corten et al. [4] proposed a stress dependency trend that schematically depends on the stress level following a power law. In order to simplify the damage calculation process, the trend of the

stress dependency was chosen to be linear. However, a new factor for excess overloading damage is introduced. Generally, the total change in slope Δb_i of a VTL curve from the original initial slope b_v at stress level i , can be expressed as:

$$\Delta b_i = r_i N_i^a \quad (5)$$

where a is the power chosen for the trend, N_i is the total equivalent number of accumulated cycles under stress level i and r can be determined from:

$$r_i = \Delta b_{v0} / N_{fi}^a \quad (6)$$

where Δb_{v0} is the total expected change in slope of the VTL curve in case of constant amplitude loading and N_{fi} is the number of cycles needed for failure under constant amplitude loading at stress level i .

$$N_i = N_{i-1} \left(\frac{N_{fi}}{N_{fi-1}} \right) \quad (7)$$

where N_i , N_{i-1} are the total equivalent accumulated number of cycles under the current and previous stress levels i and $i - 1$, respectively. N_{fi} , N_{fi-1} are the number of cycles to failure under the current and previous stress levels i and $i - 1$, respectively.

Thus it can be easily proved that by adding n_i cycles:

$$\Delta b_i = r_i (N_i + n_i)^a = \frac{\Delta b_{v0}}{N_{fi}^a} \left(N_{i-1} \frac{N_{fi}}{N_{fi-1}} + n_i \right)^a \quad (8)$$

To make the trend linear, taking $a = 1$ in Eq. (8)

$$\Delta b_i = \frac{\Delta b_{v0}}{N_{fi}} \left(N_{i-1} \frac{N_{fi}}{N_{fi-1}} + n_i \right) \quad (9)$$

This equation can be formulated in terms of stress ratio

$$\Delta b_i = \frac{\Delta b_{v0}}{N_{fi}} \left(N_{i-1} \left(\frac{S_{fi-1}}{S_{fi}} \right)^{-1/b} + n_i \right) \quad (10)$$

By using $a = 1$, the trend will become linear as shown in Fig. 3.

In the VTLC method, the change in slope can be interpreted as being the loss in expected life. Applying a constant amplitude loading at a certain stress level should decrease the life by a well-known value $\Delta N_{\text{expected}}$ corresponding to a change in slope of Δb_{v0} such that the moving target life curve coincides with the original S–N curve as shown in Fig. 1.

The total change in slope Δb_{v0} will be taken equal to the original S–N curve slope. This assumption is verified by the experimental

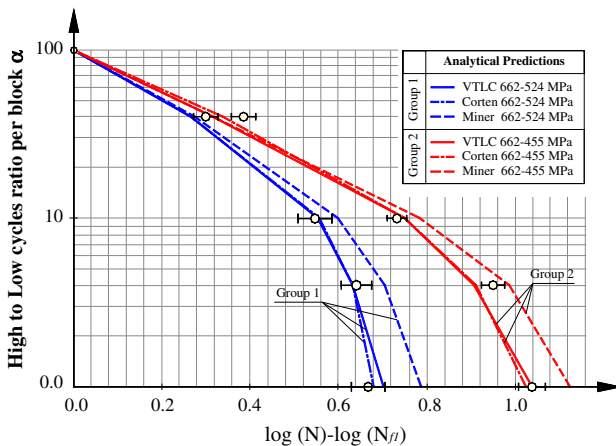


Fig. 5. Analytical predictions of fluctuating stress amplitude experiments with high stress of 662 MPa using three methods: proposed VTLC, Corten [4], and Miner [2]. (—○—) Experimental results [4].

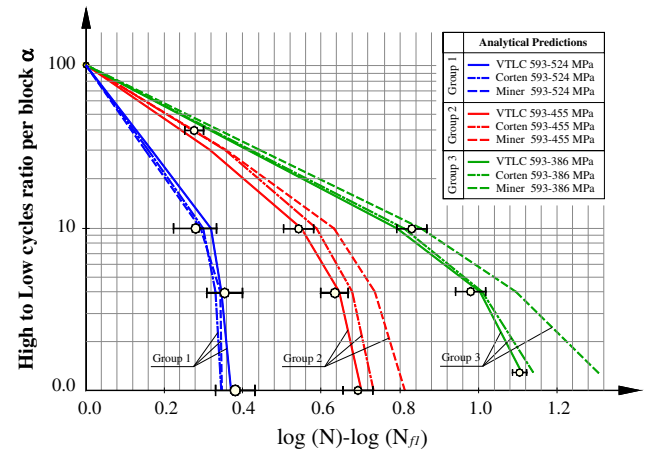


Fig. 6. Analytical predictions of fluctuating stress amplitude experiments with high stress of 593 MPa using three methods: proposed VTLC, Corten [4], and Miner [2]. (—○—) Experimental results [4].

Table 1

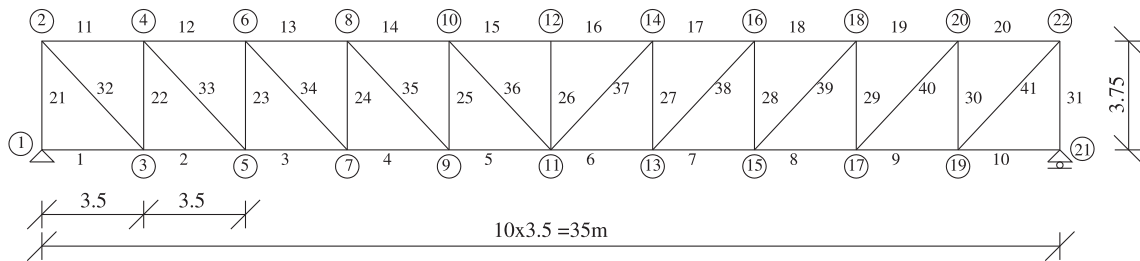
Testing stress levels and number of cycles for the tests conducted by Jeelani et al. [12].

Specimen number	Cumulative Stress								Total number of cycles
	Stage 1		Stage 2		Stage 3		Stage 4		
	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Cycles	Stress (MPa)	Cycles	
1	465	200,000	605	10,000	730	1000	800	1200	212,200
2	455	200,000	595	10,000	700	2000	875	600	212,600
3	490	10,000	630	10,000	770	1000	840	600	111,600
4	455	200,000	700	5000	595	800	770	1700	207,500
5	490	100,000	630	10,000	560	1000	700	4800	115,800
6	420	200,000	595	5000	490	3000	700	23,900	231,900

Table 2

Comparison of results using the VTLC and Miner's LDR.

Specimen number	Experimental cycles to failure	Estimated cycles (VTLC)	Estimated cycles (Miner)	VTLC/experimental	Miner/experimental	Error (VTLC) (%)	Error (Miner) (%)
1	212,200	211,713	211,245	0.9977	0.9955	0.23	0.45
2	212,600	212,512	212,137	0.9996	0.9978	0.04	0.22
3	111,600	110,891	110,858	0.9937	0.9934	0.63	0.66
4	207,500	206,521	205,837	0.9953	0.9920	0.47	0.80
5	115,800	115,733	112,672	0.9994	0.9730	0.06	2.70
6	231,900	231,900	213,483	1.0000	0.9206	0.00	7.94

**Fig. 8.** Structural system of the truss bridge showing the investigated member D-36 [14].**Table 3**

Values reported by Kühn et al. [14] for estimated damage from (a) 1895–1980, and (b) 1980–2000.

$\phi \cdot \sum k \cdot \Delta \sigma$ (N/mm ²)	n_i	m	N_i	$\frac{n_i}{N_i}$
(a) For the time period 1895–1980				
92.6	235,000	3	902,000	0.26053
82.8	369,000		1,261,000	0.29262
70.8	318,000		2,017,000	0.15766
64.3	18,000	5	2,693,000	0.00668
60.8	140,000		3,185,000	0.04396
45.9	880,000		9,615,000	0.09152
37.6	63,000	∞	26,067,000	0.00242
34.7	481,000		38,939,000	0.01235
23.6	90,000		∞	0.00000
20.4	784,000	∞	∞	0.00000
16.8	18,000		∞	0.00000
	3,396,000			$\sum \frac{n_i}{N_i} = 0.86775$
				$\frac{n_i}{N_i}$ (for 1 year)
(b) From 1980–2000				
70.8	20,000	3	2,017,000	0.009915716
45.9	65,000	5	9,615,000	0.00676027
20.4	41,000	∞	∞	0.00000
	$\sum n_i = 126,000$			$\sum \frac{n_i}{N_i} = 0.01668$

 m represents the slope of the S–N curve. n_i represents the actual number of cycles. N_i represents the number of cycles to failure.

convenient to combine the uncertainty in measurements by forming the ratio N/N_{f1} , which in terms of logarithm is equal to $\log(N) - \log(N_{f1})$. Each set of results represents 17 tested specimens.

The round circles for each set in the figures represent the mean value, while the error range bars represent the 95% confidence limits of the experimental results.

4.2. Validation Study 2

In order to further validate the proposed methodology, results from a multi-stress level experimental investigation performed by Jeelani et al. [12] were used. Fatigue tests were conducted on a direct tension compression machine which is equipped with an automatic hydraulic load maintainer. Six specimens made of AISC 4130 steel shown in Fig. 7. The test was done by cyclically loading the specimens under complete reversals and under four different stress levels in a low to high (L–H) loading pattern. Table 1 shows testing stress levels and number of cycles per stress level for each test.

The VTLC method was used for the life estimation using the value of p in Eq. (11) taken the same as in Eq. (13). Table 2 shows a comparison between the experimental results and the estimations using the VTLC and Miner's LDR. Results using the VTLC method showed a very good agreement with the experimental data resulting in better and more conservative life estimates than those obtained using Miner's LDR in fully reversed loading.

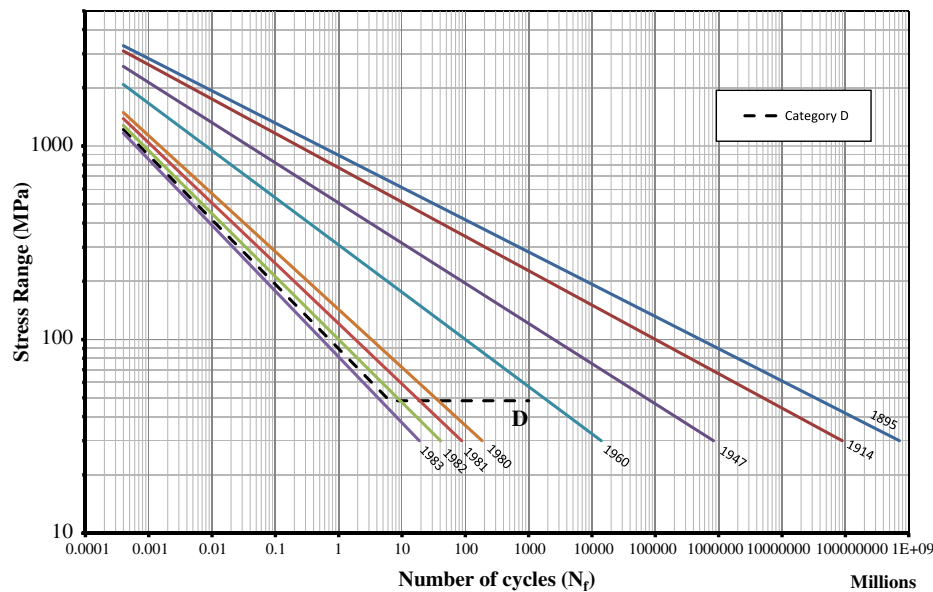
4.3. Observations and conclusions

From the above results it could be seen that the proposed VTLC approach captures different ratios of low to high (L–H) loading

Table 4

Chronologically ordered data from Kühn et al. [14] report.

	Train passing	$\varphi \cdot \sum k \cdot \Delta \sigma$	Number of cycles
1895–1913	Cargo wagon	23.6	90,000
	S 123 Locomotive	60.8	140,000
	Passenger wagon	20.4	173,000
1914–1946	S 32	82.8	369,000
	Cargo wagon	34.7	345,000
	Passenger wagon	20.4	301,000
1947–1959	S 38	92.6	235,000
	Cargo wagon	34.7	136,000
	Passenger wagon	20.4	119,000
1960–1980	E 362	70.8	318,000
	2E 362	64.3	18,000
	Cargo wagon	45.9	880,000
	E 320	37.6	63,000
	Passenger wagon	20.4	192,000
1980–2000 (Yearly)	E 362	70.8	20,000
	Cargo wagon	45.9	65,000
	Passenger wagon	20.4	41,000

**Fig. 9.** VTL curves for the investigated member D-36 showing the deterioration in fatigue life with years (assuming the detail is Category D).

patterns with a relatively small error compared to the non-conservative results obtained using Miner's LDR rule. Miner's LDR rule has good predictions for loading patterns with block stress ratio $r_b = S_2/S_1$ which is very close to constant amplitude loading (close to unity). Although the proposed VTLC method takes a linear (thus a more simple) trend in describing damage, it has very good agreement with life predictions that uses Corten's rule. This could be attributed to the fact that VTLC accounts for damage jumps that are caused by overloading through the term Δb_{Di} . As such, it could be said that the VTLC approach is a linear damage rule for measuring the loss in life that incorporates nonlinear effects arising from overloading in case of a variable fatigue loading pattern. It can be observed that for high values of the ratio α , the predicted life using either the Miner rule, Corten's rule or the proposed VTLC method is usually more conservative. This might be attributed to the crack closure phenomenon reported by Elber [13], which can be observed when the amount of high stress cycles becomes significantly more than the low stress cycles. This complex phenomenon could be further investigated and incorporated through the term Δb_{Di} in future work. Also, it should be noted that

it is interesting to explore the model for case of both variable stress and variable frequency patterns, but there was no available experimental data in the literature to correlate the model with.

5. Case study

As an example for the practical application of the developed damage model, the remaining fatigue life of a main member of a riveted single-track railway bridge built in Croatia in 1895 was investigated. This bridge was chosen because it was the only bridge (available to the authors during the course of this study) that had detailed data for its loading over about 100 years which was very important to the study. Detailed bridge data and loading was acquired from the report provided by Kühn et al. [14]. No climate data or corrosion data have been reported.

5.1. Geometry

The bridge is composed of two equal trusses. The structural system of each truss is shown in Fig. 8. In year 2000, after over

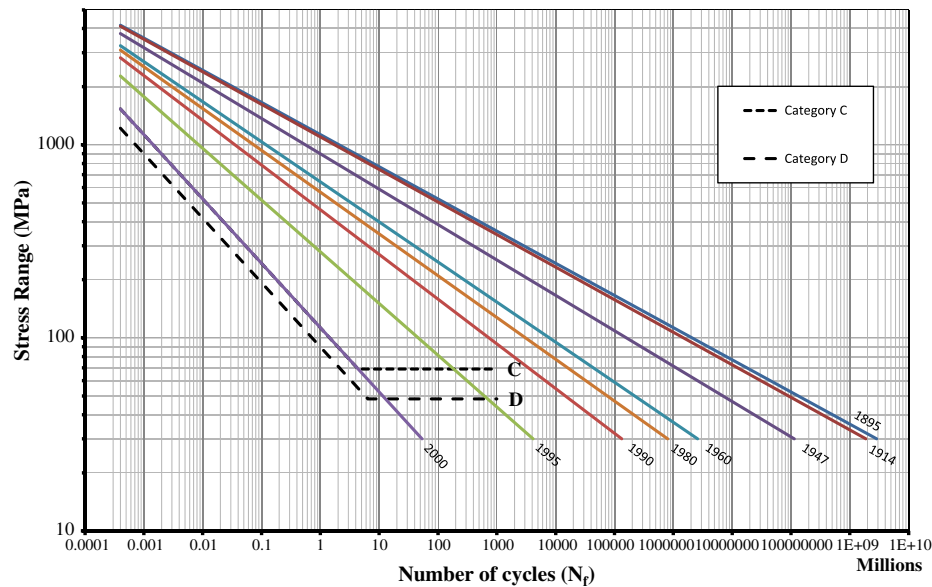


Fig. 10. VTL curves for the investigated member D-36 showing the deterioration in fatigue life with years (assuming the detail is Category C according to AASHTO [15]).

100 years of service, there was an initiative for assessment since the bridge has reached the end of its design working life. Therefore, an assessment was carried out to determine the residual service life of the bridge. The actual traffic at that time has reached the maximum capacity of the railway line. It was reported that construction of a new railroad is expected in the future (double track) [14].

Concerning the number of locomotives and wagons involved, a detailed explanation is provided in the report prepared by Kühn et al. [14]. Since the bridge has been in use for 105 years, there is a higher probability of failure due to fatigue than due to static overloading. Data about traffic load and about materials were obtained from the railways archives. The structural system of the main truss is shown in Fig. 8. It should be noted that each specific structural element has a different fatigue life. The diagonal element in the middle of the span, D-36, was estimated as the most critical in fatigue. The highest number of cycles and the highest tensile stress range are expected to occur in this element (ordinates of the shear influence line have positive and negative ordinates, and at their highest value). It was confirmed by reports of measurements by Kühn et al. [14] that the highest stress range was at this element. Member D-36 is 5.13 m long and has a cross-section of 4L 80 × 10 with a gross area of 60.4 cm² and a net area of 52.0 cm². It is connected at its ends by a riveted connection using rivets 16 mm in diameter. Detailed data on traffic such as number of trains each year, the type of trains and tons transported yearly was collected from the railways archives and available to do a Miner's sum. The analyzed element was built using rivets, which corresponds to the detail Category D in AASHTO [15].

5.2. Formation of the stress range spectrum

On the basis of the data reported by Kühn et al. [14], velocity measurements were taken along with corresponding stress ranges to calculate the dynamic amplification factor which was calculated to be about 1.17. The total secondary stresses were practically chosen to be 16% of the primary stresses. In that report, the calculations were based on the net cross-sectional area of the member D-36. The stress range spectrum for the time period from 1895 until 1980 was provided and corresponding fatigue damage was calculated using Miner's rule as shown in Table 3a. Another stress range spectrum was done for each year from 1980 to 2000 and the corresponding number of crossings as shown in Table 3b.

Kühn et al. [14] considered the studied detail having a fatigue category D according to AASHTO [15]. Calculations showed that there is no remaining design fatigue life at 1985. The member chosen for the analysis cannot be considered safe anymore after 90 years of service, although the bridge was reported to still be in service for more than 105 years.

The data from Kühn et al.'s report has been thoroughly investigated and rearranged chronologically year by year as shown in Table 4.

5.3. Fatigue life prediction using VTLC

By using the same detail category of the connection (category D, AASHTO [15]) as reported by Kühn et al., the proposed VTL curve for this member was constructed using an initial slope equal to double the original slope 3. By performing the procedure mentioned in Section 2.3 several VTL curves were plotted showing the loss in fatigue life of the studied member (which has a detail category D) throughout the years of service as shown in Fig. 9. In order to evaluate the effect of the choice of the category, the predicted fatigue life considering the case where the detail would be category C is studied. Several VTL curves showing the loss in fatigue life of the studied member (in case of AASHTO's category C detail) were plotted as shown in Fig. 10.

5.4. Discussion and conclusions

A fatigue critical member in railway truss bridge has been studied by Kühn et al. [14]. In the current study, the fatigue life of the same bridge member was investigated using the proposed VTLC method. The VTLC predictions showed that there is no remaining design fatigue life after year 1982, which is about 87 years of service. This estimation is more conservative than the 90 years predicted by Kühn et al. [14] using Miner's cumulative damage rule.

It is important to mention that the bridge was reported to still be in service for more than 105 years, while the VTLC estimations (as well as those by Kühn et al. [14]) were about 87 years as presented in Fig. 9. This difference could be attributed to the choice of AASHTO's detail category D for all riveted connections, which is – as mentioned before in the literature review – a lower bound for all riveted connections that could go up to detail category C or slightly more in some cases. If the same detail was to be considered as AASHTO's

category C detail, the predicted life using the proposed VTLC would have extended to the year 2000, which corresponds to 104 years of service life as shown in Fig. 10. This implies that there is a need to better categorize riveted connections and further classify them into more discrete categories.

6. Summary and conclusions

The proposed VTLCs (Virtual Life Target Curves) method provides a simple and a visually illustrative method for predicting the fatigue life for structural steel details subjected to variable amplitude fatigue loading. The proposed model introduces a time dimension to the conventional S–N curves. The model has the advantage that it can simulate the effect of non-uniform fatigue loading and can be easily adapted to take into account the effects of overloading. Moreover, the model is generic and is not limited to a certain loading pattern. The proposed stress-life damage accumulation model is calibrated and validated using experimental work available in the literature. It is shown that this method is capable of incorporating the effect of stress overloading on the fatigue life predictions. The proposed VTLC model was then verified using a case study of a riveted single-track railway bridge that was subjected to variable amplitude loading. Given the right fatigue category classification, the estimates of the VTLC method were shown to provide very good fatigue life estimates.

Acknowledgements

The financial support of the Canadian Network Railway (CN) through their awards and the Natural Sciences and Engineering Research Council of Canada (NSERC) are greatly appreciated.

References

- [1] Van Paepegem W, Degrieck J. Effects of load sequence and block loading on the fatigue response of fibre-reinforced composites. *Mech Adv Mater Struct* 2002;9:19–35.
- [2] Miner MA. Cumulative damage in fatigue. *J Appl Mech* 1945;67: A159–64.
- [3] Hwang W, Han KS. Cumulative damage models and multi-stress fatigue life prediction. *J Compos Mater* 1986;20.
- [4] Corten HT, Dolan TJ. London cumulative fatigue damage. In: International conference on fatigue of metals. Institute of Mechanical Engineering; 1956. p. 235–46.
- [5] Manson SS, Freche JC, Ensign SR. Application of double linear damage rule to cumulative fatigue. *ASTM STP* 1967;415:384–412.
- [6] Manson SS, Halford GR. Practical implementation of double linear damage rule and damage curve approach for treating cumulative fatigue damage. *Int J Fract* 1981;17.
- [7] Fatemi A, Yang LI. Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials. *Int J Fatigue* 1998;20:9–34.
- [8] Subramanyan A. Cumulative damage rule based on the knee point of the S–N curve. *ASME J Eng Mater* 1976;98:316–21.
- [9] Hashin Z, Rotem A. A cumulative damage theory of fatigue failure. *Mater Sci Eng* 1978;34:147–60.
- [10] Leipholz HH. On the modified S–N curve for metal fatigue prediction and its experimental verification. *Comput Struct* 1986;20(1–3):495–505.
- [11] Li ZX, Ko JM, Chan THT. Modelling of load interaction and overload effect on fatigue damage of steel bridges. *Fatigue Fract Eng Mater Struct* 2001;24: 379–90.
- [12] Jeelani S, Musial M. A study of cumulative fatigue damage in AISI 4130 steel. *J Mater Sci* 1986;21:2109–13.
- [13] Elber W. Fatigue crack closure under cyclic tension. *Eng Fract Mech* 1970;2: 37–45.
- [14] Kühn B, et al. Assessment of existing steel structures – recommendations for estimation of the remaining fatigue life. Luxembourg; EUR 23252 EN: Office for Official Publications of the European Communities; 2008.
- [15] AASHTO. AASHTO LRFD bridge design specifications (SI). Washington, DC: American Association of State Highway and Transportation Officials; 2007.