MATH 206 Self-Assessment ■ Duration: 1Hr 30Mins
Student Success Centre
Concordia University

1. Simplify the expression below. Do not use a calculator.
   a. \( \log 5 - \log (3^3 + 10) + \log \left( \frac{1}{5} \right) \)
   b. \( 5\sqrt{10} - \sqrt{90} + 10\sqrt{250} \)

2. Rationalize the denominator:
   \[
   \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}
   \]

3. Factor the following polynomials:
   a. \( 8x^3 + 27 \)
   b. \( 4x^2 + 8x + 4 \)

4. Solve the following equations:
   a. \( \ln(4x - 7) = 3 \)
   b. \( \frac{x}{x^2 - 25} + \frac{5}{x - 5} = \frac{25}{x + 5} \)

5. Solve the following inequities, express your answer using set notation or interval notation:
   a. \( \frac{2}{3} \leq \frac{x - 5}{2} \leq \frac{4}{5} \)
   b. \( 3|3x - 2| \leq 27 \)

6. Find the distance between the points \((0, 6)\) and \((-2, -3)\). Also, show which one is closer to \((0, 0)\).

7. Find the domain and range of the functions (do not graph):
   a. \( f(x) = \sqrt{-2x + 10} \)
   b. \( g(x) = |x - 2| - 2 \)
   c. \( h(x) = \frac{1}{\sqrt{x - 1}} \)
8. Sketch the graph of the function \( f(x) = 2 \ln(x + 3) \) starting from the graph of the function \( g(x) = \ln x \) and using appropriate transformations.

9. Let \( f(x) = \frac{x}{x - 1} \) and \( f(x) = \frac{x^2}{x + 1} \), find:
   a. \( fg \)
   b. \( \frac{f}{g} \)

10. Tyrik invests $20000, some in stocks and the rest in bonds. If he invests twice as much in stocks as he does in bonds, how much does he invest in each?

11. Find the inverse of the function \( f(x) = \frac{x - 3}{2x + 5} \). Find the vertical and horizontal asymptotes of \( f(x) \).

12. The number \( N \) of bacteria present in a culture at time \( t \) (in hours) obeys the law of uninhabited growth

\[
N(t) = 100e^{0.001t}
\]

   a. Determine the number of bacteria at \( t = 0 \) hours.
   b. What is the population after 10 hours?
   c. When will the number of bacteria doubles?

**NOTE (REFERENCES):**
Questions in this document have been selected from a final exam at Concordia University.
ANSWER KEY:

1. 
   a. $52\sqrt{10}$
   b. $\log\left(\frac{1}{37}\right) \text{ OR } -\log 37$

2. $-5 - 2\sqrt{6}$

3. 
   a. $(2x + 3)(4x^2 + 6x + 9)$
   b. $4(x + 1)^2$

4. 
   a. $x = \frac{e^3 + 7}{4}$
   b. $x = \frac{-150}{19}$

5. 
   a. $\frac{19}{3} \leq x \leq \frac{33}{5}$
   b. $\frac{-7}{3} \leq x \leq \frac{11}{3}$

6. Distance between two points = $\sqrt{85}$; Point $(-2, -3)$ is closer to $(0, 0)$.

7. 
   a. $\text{Domain} = [5, \infty) \quad ; \quad \text{Range} = [5, \infty)$
   b. $\text{Domain} = (-\infty, \infty) \quad ; \quad \text{Range} = [-2, \infty)$
   c. $\text{Domain} = (1, \infty) \quad ; \quad \text{Range} = (0, \infty)$
8. \begin{align*}
y & = \ln x \\
y & = 2 \ln(x + 3)
\end{align*}

9. \begin{align*}
a. & \quad \frac{x^3}{x^2 - 1} \\
b. & \quad \frac{x + 1}{x^2 - x}
\end{align*}

10. \textbf{Bonds} = $6666.67 \quad ; \quad \textbf{Stocks} = $13333.34

11. \textbf{Inverse}: \quad f^{-1}(x) = \frac{-3 - 5x}{2x - 1}; \quad \textbf{VA}: x = -\frac{5}{2}; \quad \textbf{HA}: y = \frac{1}{2}

12. \begin{align*}
a. & \quad N(0) = 100 \textit{ bacteria} \\
b. & \quad N(10) = 101.005 \textit{ bacteria} \\
c. & \quad \text{At } t = 693.15 \textit{ hours}
\end{align*}