MATH 204 Self-Assessment ■ Duration: 1Hr

Student Success Centre

Concordia University

Instruction| For questions 1 - 3 only, choose one out of the provided options as answer.

1. Which of the following is a unit vector?

   a. (1,0,0,1)

   b. $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0, 0)$

   c. $(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}, 0, 0)$

   d. *None of the above*

2. Which of the following set is the basis of R^4?

   a. (1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)

   b. (1,0,0,1), (1,0,0,0), (0,0,1,1), (0,0,0,1)

   c. (1,0,0,0), (0,1,0,0), (0,0,1,0), (1,1,1,0)

   d. *None of the above*

3. Which of the following is a possible subspace of R^4?

   a. $A = (a, b, c, d)|(a + b + c + d) > 0$

   b. $B = (a, b, c, d)|(a * b) > 0$

   c. $C = (a, b, c)|(a + b + c) > 0$

   d. $D = (a, c, d)|(c * d) > 0$

4. Given the matrix below:

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 3 & 5 & 2 \\ 1 & -2 & -3 \end{pmatrix}$$

   a. Use Gaussian-Jordan elimination to find the inverse matrix of A.

   Hint| Your work should start with this: $\left[\begin{array}{ccc|ccc} 2 & 3 & -1 & 1 & 0 & 0 \\ 3 & 5 & 2 & 0 & 1 & 0 \\ 1 & -2 & -3 & 0 & 0 & 1 \end{array}\right]$

   b. Based on the result from **part a**, find the determinant of matrix A.

   Hint| You will need the echelon form of matrix A.

c. Given matrix $C = \begin{bmatrix} 1 \\ 8 \\ -1 \end{bmatrix}$. Find the solution for the equation $A * x = C$.

Hint| Use the inverse of matrix A and matrix multiplication.

5. Find the solution to the following system of equations:

$$\begin{cases} x_1 + x_2 - 2x_3 + x_4 = 5 \\ 2x_1 + 2x_2 - 3x_3 + 4x_4 = 3 \\ 3x_1 + 3x_2 - 4x_3 + 2x_4 = 1 \end{cases}$$

6. Given two vectors:

$$\vec{u} = \langle 2, 4, 5 \rangle \text{ and } \vec{v} = \langle 1, 0, 1 \rangle.$$

Find the following:

a. Their dot product $\vec{u} \cdot \vec{v}$.
b. The angle between $\vec{u}, \vec{v}$.
c. Their cross product $\vec{u} \times \vec{v}$.
d. The equation of the plane containing $\vec{u}, \vec{v}$ and the point $P(1, 3, -2)$.

7. Let $P_1 = (2, 3, 6)$, $P_2 = (1, -1, -2)$, $P_3 = (1, 4, -2)$ and $P_4 = (2, 0, 3)$.
   a. Find the area of the triangle made by the three vertices P_1, P_2, P_3.
   b. Find the volume of the parallelepiped made by the vectors $\overrightarrow{P_1P_2}, \overrightarrow{P_1P_3}, \overrightarrow{P_1P_4}$.

8. Given $matrix\ A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$ and the eigenvalues $\lambda_1 = 5\ (multiplicity = 1)$, $\lambda_2 = 3\ (multiplicity = 2)$. Find the eigenvectors of matrix A.

ANSWER KEY:

1. b.

〔The magnitude/norm of a unit vector must equal to one〕

2. a.

〔A member of a basis must be minimal. It cannot be expressed by the other members using linear expression〕

3. b.

〔A subspace is a subset of R^4 so that its members must have four elements. A subspace is a vector space, which means that, it must satisfy all of the axioms for a vector space. Typically, the sum of two members is a new member of the space and the product of a member with any constant is also a member of the space.〕

4.

a.	$Inv(A) = \left[\begin{array}{ccc\|ccc} 1 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{-5}{22} & \frac{-7}{22} \\ 0 & 0 & 1 & \frac{-1}{2} & \frac{7}{22} & \frac{1}{22} \end{array}\right]$
b.	$det(A) = 22$
c.	$X = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

5. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -9 \\ -7 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} s$

6.

a.	$\vec{u} \cdot \vec{v} = 7$
b.	$angle,\ \alpha = 42.45°$
c.	$\vec{u} \times \vec{v} = \langle 4, 3, -4 \rangle$
d.	$Equation\ of\ plane: 4x + 3y - 4z = 21$

7.

a.	$Area = 18.282\ square\ units$
b.	$Volume = 27\ cubic\ units$

8. For $\lambda_1 = 5$, $\lambda_2 = \lambda_3 = 3$, corresponding eigenvectors are:

$$\overrightarrow{v_1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ \overrightarrow{v_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix},\ \overrightarrow{v_3} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$