MATH 203 Self-Assessment ■ Duration: 1Hr 30Mins

Student Success Centre

Concordia University

Instruction| For questions 1 – 6 only, choose one out of the provided options as answer.

1. Evaluate $\lim_{x \to 2} \frac{\sqrt{2x+1}-3}{x^2-16}$.

   a. 0
   b. ∞
   c. $\frac{1}{48}$
   d. $\frac{1}{24}$

2. Evaluate $\lim_{x \to \infty} \frac{2x(x^2+4)^2}{(3x+4)(2x^2+1)^2}$.

   a. ∞
   b. $\frac{1}{6}$
   c. $-\frac{1}{6}$
   d. *Does not exist*

3. Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

   a. 2
   b. 0
   c. ∞
   d. 1

4. Find the derivative of $y = \left(2x + \frac{1}{x}\right)\ln(2x) + \sec x$.

   a. $y' = \left(2 - \frac{1}{x^2}\right)\ln(2x) + \frac{1}{x}\left(2x + \frac{1}{x}\right) + \sec x \tan x$
   b. $y' = \left(2 - \frac{1}{x^2}\right)2x + \frac{2}{x}\left(2x + \frac{1}{x}\right) + \csc x$
   c. $y' = \left(2 + \frac{1}{x^2}\right)\ln(2x) + \frac{2}{x}\left(2x + \frac{1}{x}\right) + \tan x$
   d. $y' = \left(2 - \frac{1}{x^2}\right)\ln(2x) + \left(2x + \frac{1}{x}\right)$

5. Find the derivative of $y = \arcsin^2 x$.

   a. $y' = 2\left(\frac{1-x^2}{\sqrt{1-x^2}}\right)\arcsin x$
   b. $y' = \left(\frac{2}{\sqrt{1-x^2}}\right)\arcsin x$
   c. $y' = \frac{1-x^2}{\sqrt{1-x^2}}$
   d. $y' = \frac{1}{\sqrt{1-x^2}}$

6. Find the derivative of $y = \frac{\arcsin^2 x}{1-x^2}$.

a. $y' = \dfrac{2\left(\frac{1-x^2}{\sqrt{1-x^2}}\right)\arcsin x - 2x\arcsin^2 x}{(1-x^2)^2}$

b. $y' = \dfrac{\left(\frac{1-x^2}{\sqrt{1-x^2}}\right)\arcsin x + x\arcsin^2 x}{(1-x^2)^2}$

c. $y' = \dfrac{2\left(\frac{1-x^2}{\sqrt{1-x^2}}\right)\arcsin x + 2x\arcsin^2 x}{(1-x^2)^2}$

d. $y' = \dfrac{2\left(\frac{1-x^2}{\sqrt{1-x^2}}\right)\arcsin x - 2x\arcsin^2 x}{(1-x^2)}$

7. Given the following functions.

$$f(x) = 2 + \frac{1}{x} \quad \text{and} \quad g(x) = \frac{1}{\sqrt{x^3}}$$

a. Find $fog(x)$ and its domain.

b. Find $gof(x)$ and its domain.

c. Find the inverse of $f(x)$. Determine its domain and range.

8. Find a and b such that the function $f(x)$ is continuous at every point.

$$f(x) = \begin{cases} -\dfrac{4}{x^2}, & x \le -2 \\ \dfrac{ax}{2} - b, & -2 < x \le 0 \\ x^2 - 2, & x > 0 \end{cases}$$

9. Use the definition of derivation to find the derivative of $f(x) = \sqrt{x+1}$.

Hint| $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

10. Let $f(x) = \frac{x^2-4}{2x}$.

a. Find all asymptotes of $f(x)$.

b. Find the intervals where $f(x)$ is increasing and where it is decreasing. Also, find the critical points (if any).

c. Find the intervals where $f(x)$ is concave upward and where it is concave downward. Also, find the inflection points (if any).

11. Sketch the graph of the function $f(x) = |2x - 4| + 1$.

***Hint*|** start from the graph of $f(x) = |x|$ and use appropriate transformations.

NOTE (REFERENCES):

Some questions in this document have been selected from final exams and midterms at Concordia University.

ANSWER KEY:

1. d.
2. b.
3. a.
4. a.
5. b.
6. c

7.

a.	$fog(x) = 2 + \sqrt{x^3}$; $\boldsymbol{domain} = (0, \infty)$
b.	$gof(x) = \frac{1}{\sqrt{2 + \left(\frac{1}{x}\right)^3}}$; $\boldsymbol{domain} = (-\infty, -\frac{1}{2}) \cup (0, \infty)$
c.	$f^{-1}(x) = \frac{1}{x-2}$; $\boldsymbol{domain} = \mathbb{R} - \{2\}$; $\boldsymbol{range} = \mathbb{R} - \{0\}$

8. $a = -1$ and $b = 2$

9. $f'(x) = \frac{1}{2\sqrt{x+1}}$

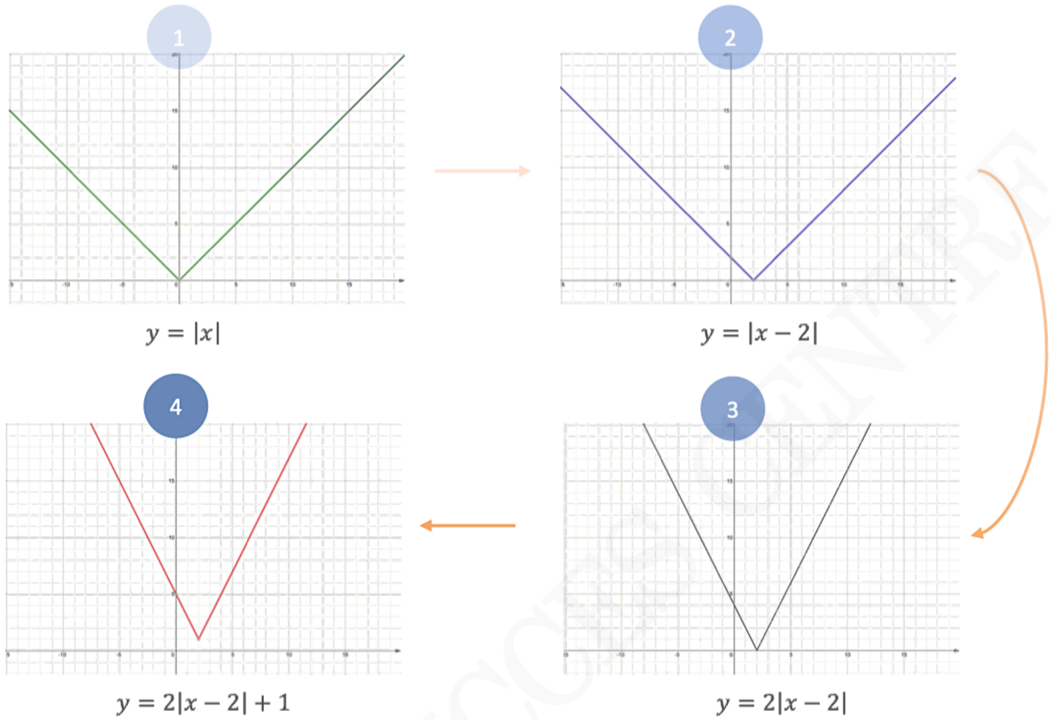
10.

a.	***Horizontal Asymptote***: none ***Vertical Asymptote***: $x = 0$
b.	***increasing***: $(-\infty, \infty)$; ***decreasing***: *never* ***critical points***: *none*
c.	***concave up***: $(-\infty, 0)$; ***concave down***: $(0, \infty)$ ***inflection points***: *none* (*since* $x = 0$ *is not part of the domain*)

11. Please find the answer on the next page.

$$y = |2x - 4| + 1 \quad \textit{is equivalent to} \quad y = 2|x - 2| + 1$$

Thus, the following sequence of transformations:



1. *Start with a basic function. Pick some reference points, for example (0,0).*
2. *Perform a horizontal shift to the right by 2 units.*
3. *Stretch (scale on the y-axis) by a factor of 2. Notice the y-intercept has changed from 2 to 4.*
4. *Perform a vertical shift upwards by 1 unit.*