Concordia University

School of Graduate Studies

DOCTORAL THESIS DEFENCE

The Oral Examination

for the Degree of Doctor of Philosophy of

Petr Zorin

in the

Department of Mathematics & Statistics

will take place on

Monday, August 1, 2016

in room LB 921-4, 1400 de Maisonneuve Blvd. W.

at 2:00 p.m.

Thesis Title:

Spectral comparison theorems in relativistic quantum mechanics

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Examining Committee:

TBA, Chair
Dr. Richard Hall (Mathematics & Statistics), Supervisor
Dr. Georgios Vatistas (Mechanical & Industrial Engr.)
Dr. Alina Stancu (Mathematics & Statistics)
Dr. Ron Stern (Mathematics & Statistics)

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Abstract

Spectral comparison theorems in relativistic quantum mechanics

Petr Zorin, Ph.D
Concordia University, 2016

The classic comparison theorem of quantum mechanics states that if the comparison potentials are ordered then the corresponding energy eigenvalues are ordered as well, that is to say if $V_a \leq V_b$, then $E_a \leq E_b$. The nonrelativistic Schrödinger Hamiltonian is bounded below and the discrete spectrum may be characterized variationally. Thus the above theorem is the direct consequence of the min–max characterization of the discrete spectrum [1, 2]. The classic comparison theorem does not allow the graphs of the comparison potentials to cross over each other. The refined comparison theorem for the Schrödinger equation [3] overcomes this restriction by establishing conditions under which graphs of the comparison potentials can intersect and still preserve the ordering of eigenvalues.

The relativistic Hamiltonian is not bounded below and it is not easy to define the eigenvalues variationally. Therefore comparison theorems must be established by other means than variational arguments. Attempts to prove the nonrelativistic refined comparison theorem without using the min–max spectral characterization suggested the idea of establishing relativistic comparison theorems for the ground states of the Dirac and Klein–Gordon equations [4, 5]. Later relativistic comparison theorems were proved for all excited states by the use of monotonicity properties [6]. In the present work, refined comparison theorems have now been established for the Dirac §4.2.1 and §4.2.2 [7] and Klein–Gordon §4.1.1 and §4.1.2 [8] equations. In the simplest one–dimensional case, the condition $V_a \leq V_b$ is replaced by $U_a \leq U_b$, where $U_i = \int_0^x V_i dt$, $x \in [0, \infty)$, and $i = a$ or b.

Special refined comparison theorems for spin–symmetric and pseudo–spin–symmetric relativistic problems [9], which also allow very strong potentials such as the harmonic oscillator §4.1.2, §4.2.1, and §4.2.2 [8, 10], are proved.