Concordia University

School of Graduate Studies DOCTORAL THESIS DEFENCE

The Oral Examination

for the Degree of Doctor of Philosophy of

Jing Zhang

in the

Department of Mathematics & Statistics

will take place on

Friday, April 1, 2016

in room LB 921-4, 1400 de Maisonneuve Blvd. W.

at 2:00 p.m.

Thesis Title:

Some topics on Dirichlet forms and non-symmetric Markov processes

Examining Committee:

TBA, Chair

Dr. Wei Sun (Mathematics & Statistics), Supervisor Dr. Wen-Fang Xie (Mechanical & Industrial Engr.) Dr. Xiaowen Zhou (Mathematics & Statistics) Dr. Arusharka Sen (Mathematics & Statistics)

External Examiner:

Dr. Toshihiro Uemura Department of Mathematics Kansai University, Japan

ABSTRACT

Some topics on Dirichlet forms and non-symmetric Markov processes

Jing Zhang, Ph.D.

Concordia University, 2016

In this thesis, we discuss three topics on Dirichlet forms and non-symmetric Markov processes.

First, we explore the analytic structure of non-symmetric Markov processes. Let U be an open set of \mathbb{R}^n , m a positive Radon measure on U, and $(P_t)_{t>0}$ a strongly continuous contraction sub-Markovian semigroup on $L^2(U;m)$. We give an explicit Lévy-Khintchine type representation of the generator A of $(P_t)_{t>0}$. If $(P_t)_{t>0}$ is an analytic semigroup, we give an explicit characterization of the semi-Dirichlet form \mathcal{E} associated with $(P_t)_{t>0}$.

Second, we consider the Dirichlet boundary value problem for a general class of second order non-symmetric elliptic operators L with singular coefficients. We show that there exists a unique, bounded continuous solution by using the theory of Dirichlet forms and heat kernel estimates. Also, we give a probabilistic representation of the non-symmetric semigroup generated by L.

Finally, we present new results on Hunt's hypothesis (H) for Lévy processes. These include a comparison result on Lévy processes which implies that big jumps have no effect on the validity of (H), a new necessary and sufficient condition for (H), and an extended Kanda-Forst-Rao theorem.