Concordia University

School of Graduate Studies

DOCTORAL THESIS DEFENCE

The Oral Examination

for the Degree of Doctor of Philosophy of

Patrick Meisner

in the

Department of Mathematics & Statistics

will take place on

Monday, August 22, 2016

in room LB 921-4, 1400 de Maisonneuve Blvd. W.

at 11:00 a.m.

Thesis Title:

Distribution of the Number of Points on Abelian Curves over Finite Fields

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Examining Committee:

Dr. Pablo Bianucci (Physics), Chair
Dr. Chantal David (Mathematics & Statistics), Supervisor
Dr. Mariana Frank (Physics)
Dr. Hershy Kisilevsky (Mathematics & Statistics)
Dr. Lea Popovic (Mathematics & Statistics)

External Examiner:

Dr. Yu-Ru Liu
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ABSTRACT

Distribution of the Number of Points on Abelian Curves over Finite Fields

Patrick Meisner, Ph.D.

Concordia University, 2016

Classical results due to Katz and Sarnak [8] show that if the genus is fixed and $q \to \infty$, then the number of points on a family of curves over $\mathbb{F}_q$ is distributed as the trace of a random matrix in the monodromy group associated to the family.

Every smooth projective curve C corresponds to a finite Galois extension of $\mathbb{F}_q[X]$. Therefore, some natural families to consider are the curves that correspond to a extensions with a fixed Galois group. This thesis involves determining the distribution of the families with fixed abelian Galois group, G, when q is fixed and the genus tends to infinity.

Several authors determined that the distribution for the family of prime-cyclic curves ($G = \mathbb{Z}/p\mathbb{Z}$, p a prime) [2],[3],[9] as well as for the family of biquadratic curves ($G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$) [10] is that of a sum of $q+1$ random variables. This thesis shows that if we fix *any* abelian group, the distribution will be that of $q+1$ random variables.

The above results deal only with the distribution for the coarse irreducible moduli space of the families. It has been shown in [1] that if you look at the whole (coarse) moduli space, the distribution is the same in the case of prime-cyclic curves. We are able to show that the distribution is the same for the coarse moduli space of curves with $G = (\mathbb{Z}/Q\mathbb{Z})^n$, Q a prime. Some work is done towards proving this true for all abelian groups.