Department of Mathematics & Statistics
Concordia University

MAST 661 (MATH 494/MAST 837), Sec. B
Selected Topics in Analysis
Topic: "Harmonic Analysis and applications"
Winter 2024

Instructor: Dr. G. Dafni, Office: LB 927-15 (SGW), Phone: 848-2424, Ext. 3216
Email: galia.dafni@concordia.ca

Schedule: Thursdays: 5:45-8:15 pm, in FG B030 (SGW).

Office Hours: (tentative) Mondays & Wednesdays, 5:15-6:30 pm, and by appointment.

Textbooks: The following textbooks are recommended:
Fourier Analysis: An Introduction, by E. M. Stein & R. Shakarchi.
An Introduction to Harmonic Analysis, by Y. Katznelson.
Harmonic Analysis from Fourier to Wavelets, by C. Pereyra and L. Ward.

Other texts: These references will be put on reserve in the library, or are available online:
Ten Lectures on Wavelets, by Ingrid Daubechies.
Fourier Series and Integrals, by H. Dym and H. P. McKean.
Fourier Analysis and its Applications, by G. B. Folland.
Real Analysis: Modern Techniques and Their Applications, by G. B. Folland.
Fourier Analysis, by T. W. Korner.
Wavelets and Operators, by Yves Meyer.
Real and Complex Analysis, by W. Rudin.
Functional Analysis, by W. Rudin.
Functional Analysis: Introduction to Further Topics in Analysis, by E. M. Stein & R. Shakarchi.
Introduction to Fourier Analysis on Euclidean Spaces, by E. M. Stein & G. Weiss.
Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals, by E. M. Stein,
Singular integrals and differentiability properties of functions, by E. M. Stein.
Trigonometric Series, by A. Zygmund.

Topics: 1. Fourier series on the circle

- Introduction: the origins of Fourier analysis (independent reading: Stein & Shakarchi 1)
- Convergence of Fourier series (basic: Stein & Shakarchi 2-3, extra: Katznelson I-II)

- The Hardy-Littlewood maximal function (extra: Katznelson III.2. Rudin R&C 7, Stein SI I)
- Applications of Fourier series (Stein & Shakarchi 4)
- Harmonic functions, Poisson integrals and the conjugate function (Katznelson III, Rudin 11, Stein & Weiss II)

2. Fourier transforms on the line and on R^n
   - Definition and properties of the Fourier transform, the Schwartz space (basic: Stein & Shakarchi 5-6, extra: Katznelson VI, Rudin R&C 9, Stein & Weiss I)
   - Tempered distributions (extra: Stein & Weiss I, Stein & Shakarchi FA 3, Rudin FA 6)
   - The Poisson Summation Formula (Stein & Shakarchi 5, Katznelson VI)
3. Wavelets
   - Haar wavelets (Pereyra & Ward, Ch. 9)
   - Multiresolution analysis (Pereyra & Ward, Ch. 10)
   - Calculating with wavelets (Pereyra & Ward, Ch. 11)
4. Additional topics if time permits/ student presentations:
   - Interpolation (Katznelson IV, Stein & Weiss V)
   - Singular integrals (Stein SI II, Stein & Weiss II, VI)
   - Littlewood-Paley Theory
   - Hardy spaces, BMO (Stein & Shakarchi FA 2, Katznelson III.3, Rudin 17, Stein HA III, IV)
   - Weak/weak* topologies; topological vector spaces, distributions (Rudin FA 1,6)
   - Applications to PDE, Sobolev spaces (Stein & Shakarchi 5-6, extra: Rudin FA 8)
   - Spherical harmonics (Stein & Weiss IV)
   - Fourier analysis on groups, the discrete Fourier transform (Stein & Shakarchi 7)
   - Applications to number theory (Stein & Shakarchi 8)

Pre-requisites: Previously or concurrently: real analysis/metric spaces (equivalent to MATH 464); measure theory (equivalent to MATH 467/669); basic complex analysis (equivalent to MATH 366).

Assignments: Homework will be assigned approximately once every two weeks, on Moodle, via Assignments, and submitted on Moodle. **Late homework will not be accepted.**

Exams: There will be an in-class 75 min midterm exam during the 6th or 7th week of classes.

Evaluation: Homework assignments 40%, Midterm exam 20%, Final project/presentation 40%.
PhD students will be required to do additional work (to be determined) compared with their MA/MSc classmates.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

Communication: Communication between the students and the instructor will take place in person and online via Zoom meetings, Moodle announcements and email messages. **Students are responsible for reading and taking note of all electronic communication from the instructor and the University.**

Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: https://www.concordia.ca/conduct/academic-integrity.html" *[Undergraduate Calendar, Sec 17.10.2]*

Behaviour
All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

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