Department of Mathematics & Statistics
Concordia University

MAST 235
Linear Algebra and Applications II
Winter 2024

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Class Schedule: Tuesdays & Thursdays, 11:45-13:00.

Office Hours: Thursdays, 13:30-15:00.

Textbook: *Linear Algebra:* An Introduction Using Maple, *Fred Szabo*, Harcourt/Academic Press e-book version, posted on the Moodle website for Mast 235 in PDF format at the discretion of the author. The book contains most of the Maple code required in this course and can be copied from the text and pasted into Maple worksheets with only minor changes.

Prerequisites: MATH 234 or equivalent is a prerequisite for this course.

Objectives: In this course, we enrich the linear algebra basics studied in Math 234 with algebraic and geometric concepts and properties such as distance, angles, and orthogonality. We extend the structure of real vector spaces and linear transformation to normed Euclidean spaces, inner product spaces, and linear transformations preserving norms and inner product.

This additional structure is used to study geometric and statistical objects such as angles, variances, covariances, standard deviations, and correlations. We then explore quadratic forms, the principal axis theorem, conic sections, and quadric surfaces.

After that, we introduce the concept of orthogonality, explores orthogonal projections and the Gram-Schmidt orthogonalization process. We apply these concepts and tools to define and study orthonormal bases of Euclidean vector spaces and the QR decomposition of matrices, orthogonal matrices, orthogonal subspaces, and linear transformations preserving the shape and size of geometric objects.

Applications: Whenever appropriate, the Thursday lectures are dedicated to the exploration of applications of linear algebra using the ideas of techniques covered in the preceding lectures. They include, whenever time allows, network problems, Leontief input-output models, linear systems, and interpolation polynomials, Vandermonde matrices and interpolation polynomials, Markov chains and matrix powers, discrete dynamical systems, population dynamics, least-squares approximations, matrices and finite graphs, determinants, singular value decomposition, and applications to image compression and principal component analysis.

Software: *MAPLE (version 17 or higher)* is required for this course. Academic pricing of the Maple software is offered to registered Concordia students.

Pedagogy: In this course, we use the symbolic and numeric computation system called *Maple* as our electronic organizer and computation system. However, no familiarity with Maple beyond the skills and experience with the system acquired in Mast 234 is required. Maple is used as a computational tool, ***not as an object of independent study***.

It should also be noted that in this course, we do not use pencils, paper, or handheld calculators. All work is electronic, and all coursework, including assessments and examinations, uses Maple.

Although the textbook for this course is provided in PDF format, Maple can import PDF, Mathematica, and other file formats with only minimal change.

Even though technology plays a significant role in real-world applications of linear algebra, as mentioned, the part of Maple in this course is primarily computational. Only a limited number of Maple features are used and will be explicitly documented when needed.

Assignments: Biweekly assignments based on the textbook are posted and submitted online in Moodle. Assignments count for 10% of the final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class are essential for success in this course. ***It should be noted that late assignments will not be accepted.***

Midterm Test: There will be ***one midterm examination*** based on the material of Lectures 1-6 which will contribute up to 30% to your final grade (see the Grading Scheme). It will be held in class on the **Thursday of Week 7**.

It is the Department's policy that tests missed for any reason, **including illness,** cannot be made up. If you missed the midterm because of illness, an appropriately authenticated Short-term absence form or valid medical note is required for the final exam to count for 85% of your final course grade. The remaining 15% will be from assignments and quizzes.

Final Exam: The final examination will be 3 hours closed-book Maple examination. Access to resources such as lecture notes, class notes, and similar material may be disabled.

Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office.

Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, ***not*** to the instructor.

Students must be available until the end of the final exam period. Conflicts due to travel plans **will not** be accommodated.

Important Note: **There are no supplemental exams for this course.**

Grading Scheme: The final grade will be based on the higher of (a) and (b) below:

(a) 10% for the assignments
5% for the best of the class quizzes
30% for the midterm test
55% for the final examination

(b) 10% for the assignments
5% for the best of the class quizzes
15% for the midterm test
70% for the final examination

Please note that there is no 100% final exam option in this course.

Since the term work contributes at least 30% to the final grade, active participation in classes and continuous work on the course material during the semester is essential for success in this course.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

COURSE CONTENTS

Weekly Theme	Weekly Topics	Textbook Pages
Week 1 **Mast 234 Review 1**	Interpolating polynomials Leontief input-output systems Markov chains Vandermonde matrices	P. 75 P. 218 P. 472 P. 99
Week 2 **Mast 234 Review 1**	Vector spaces Linear transformations Eigenvalues and eigenvectors The matrix diagonalization theorem	P. 281 P. 361 P. 415 P 457

Week 3 **Vector and Matrix Norms**	Euclidean norms, length and distance Frobenius matrix norm and distance Non-Euclidean norms Cosines and angles	P. 483 P. 486 P. 492 P. 489
Week 4 – Tuesday 1. **Norms continued** 2. **Quiz 1 Practice** Week 4 - Thursday	Non-Euclidean norms Discrete dynamical systems Systems with complex eigenvalues Review of Weeks 1 to 3 **Quiz 1 - Weeks 1 to 3**	P. 492 P. 486 P. 416
Week 5 **Inner Products**	Inner product spaces The Cauchy-Schwarz inequality The triangle inequality The Pythagorean theorem	P. 482 P. 505 P. 483 P. 541
Week 6 **Angles and Statistics**	Angles and correlation matrices Correlation matrices Variances and standard deviations	P. 514 P. 519 P. 516
Week 7 – Tuesday Week 7 - Thursday	Midterm Review **MIDTERM - Weeks 1 to 6**	
Week 8 **Principal Axes**	Quadratic forms Matrices of quadratic forms The principal axis theorem Conic sections	P. 523 P. 524 P. 527 P. 530
Week 9 **Gram-Schmidt Algorithm**	Orthogonal vectors Orthogonal projections Gram-Schmidt orthogonalization Orthonormal bases	P. 541 P. 545 P. 551 P. 556
Week 10 **Orthogonality** Week 10 - Thursday	Orthogonal matrices and subspaces Orthogonality of fundamental subspaces Self-adjoint linear transformations The spectral theorem **QUIZ 2 – Week 8 to 10**	P. 579 P. 585 P. 591 P. 598
Week 11 **QR Decomposition**	The QR matrix decomposition Normal equations and QR decomposition The method of least squares Fitting curves with the method of least squares	P. 562 P. 561 P. 605 P. 609
Week 12 **Singular Value Decomposition**	Singular values and singular vectors The singular value decomposition theorem	P. 619 P. 623

Academic Integrity and the Academic Code of Conduct

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: https://www.concordia.ca/conduct/academic-integrity.html" *[Undergraduate Calendar, Sec 17.10.2]*

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