

**MAST 235**  
Linear Algebra and Applications II  
*Fall 2023*

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**Class Schedule:** Tuesdays & Thursdays, 10:15-11:30 AM.

**Office hours:** Thursday: 12:30-14:00.

**Textbook:** There is no mandatory textbook for this course. All the material will be posted in the lecture and class work files on the MAST 235 Moodle site.

For additional reading and practice, most of the topics covered in this course can be found in the following complementary texts:

*Linear Algebra with Applications*, by W. Keith Nicholson, Open Texts by Lyryx.  
[lila1.lyryx.com/textbooks/OPEN\\_LAWA\\_1/marketing/Nicholson-OpenLAWA-2021A.pdf](http://lila1.lyryx.com/textbooks/OPEN_LAWA_1/marketing/Nicholson-OpenLAWA-2021A.pdf)

Fred F. Szabo, *Linear Algebra, An Introduction Using Maple*: Academic Press. This textbook will be posted on Moodle in PDF format.

*Linear Algebra, Theory and Applications*, by Ward Cheney & David Kincaid. The book is very good but expensive. A second-hand book of any edition can be used.

**Prerequisites:** Math 234 or equivalent is a prerequisite for this course.

**Objectives:** This course will focus on applications of the theory studied in Math 234 and its further development. There are two major concepts, on which this course is based: (a) *Linear Systems & Operators*, and their applications such as Economic Models or Dynamical systems, and (b) *Inner Product Spaces and Self Adjoint Operators*, leading to applications like Orthogonalization, Least Square solutions, and SVD diagonalization. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Algebra courses in your curriculum.

**Pedagogy:** Classes and all work in this course use the *MAPLE* as the tool, *not* object of study. The structure of classes includes lecture time on the theory, alternating with problem solving tasks done by students individually. Mathematical issues that arise during problem solving are discussed in class.

**Software:** *MAPLE (version 17 or higher)* is mandatory for this course. The Waterloo's Maplesoft is making *MAPLE* ("Student's edition", quite sufficient for the course) available to Concordia students at a special price. In this course the software is only used as a computational *tool*, *not as an object of study* in itself. All the tests, the final examination and the assignments are done using *MAPLE*.

**Assignments:** Assignments are given and submitted online through Moodle. Late assignments **will not** be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in this course.

**Midterm Test:** There will be **one midterm test** based on the material of Lectures 1-6 which will contribute up to 30% to your final grade (see the Grading Scheme). It will be held in class on **Tuesday, October 24, 2023**.

**NOTE:** It is the Department's policy that tests missed for any reason, **including illness**, cannot be made up. If you missed the midterm because of illness (**to be confirmed by a valid medical note**) the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and quizzes (see the **Grading Scheme**).

**Final Exam:** The Final Examination will be 3 hour closed-book Maple examination. Access to resources such as lecture notes, class notes, and similar material is disabled. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, *not* to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans **will not** be accommodated.

**NOTE:** There are no supplemental exams for this course.

**Grading Scheme:** The final grade will be based on the higher of (a) and (b) below:

(a) 10% for the assignments.  
5% for the best of 2 quizzes (written in class)  
30% for the midterm test  
55% for the final examination.

(b) 10% for the assignments  
5% for the best of 2 quizzes  
15% for the midterm test.  
70% for the final examination.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

**IMPORTANT:** **NOTE** that there is **NO "100% FINAL EXAM"** option in this course.

The term work contributes at least 30% to the final grade. Therefore, active participation in classes and continuous work on the course material **during** the semester is essential for success in this course.

## CONTENTS

Week	Lectures TOPICS	Sub-Topics considered	Complementary Reading
1	<b>Lecture 1</b> LINEAR SYSTEMS: NETWORKS	<ul style="list-style-type: none"> <li>• Systems of Linear Equations, Homogeneous: an <i>Overview</i></li> <li>• Network Problems</li> </ul>	Sections KN: 1.3, 1.4
2	<b>Lecture 2</b> LINEAR SYSTEMS: ECONOMIC MODELS	<ul style="list-style-type: none"> <li>• Economic Models <ul style="list-style-type: none"> <li>◦ Closed Leontieff Model</li> <li>◦ Open Leontieff Model</li> </ul> </li> </ul>	Sections KN: 2.8
3	<b>Lecture 3</b> <i>DETERMINANTS</i> PROPERTIES & APPLICATIONS	<ul style="list-style-type: none"> <li>• Determinants: overview and basic properties</li> <li>• Applications: <ul style="list-style-type: none"> <li>◦ Polynomial Interpolation Problem</li> <li>◦ Vandermonde matrix</li> </ul> </li> </ul>	Sections KN: 3.1, 3.2
4	<b>QUIZ 1</b>  <b>Lecture 4</b> EIGENTHEORY AND DYNAMICAL SYSTEMS	<p style="text-align: center;"><b>QUIZ 1</b> (based on Weeks 1-3)</p> <ul style="list-style-type: none"> <li>• Diagonalization and powers of a matrix</li> <li>• Application: Dynamical Systems <ul style="list-style-type: none"> <li>◦ Systems with Real eigenvalues</li> <li>◦ The Predator-Prey simulation</li> <li>◦ Systems with Complex eigenvalues</li> </ul> </li> </ul>	Sections KN: 2.9, 3.1, 3.3 Appendix A
5	<b>Lecture 5</b> INNER PRODUCT SPACES	<ul style="list-style-type: none"> <li>• The concept of inner product space over real and complex numbers <ul style="list-style-type: none"> <li>◦ Definition</li> <li>◦ Properties</li> </ul> </li> <li>• The norm in an inner product space <ul style="list-style-type: none"> <li>◦ The Cauchy-Schwartz Inequality</li> <li>◦ The Triangle Inequality</li> <li>◦ The Pythagorean Theorem</li> </ul> </li> </ul>	Sections KN: 8.7, 10.1
6	<b>Lecture 6</b> ORTHOGONAL PROJECTION	<ul style="list-style-type: none"> <li>• Orthogonality of vectors</li> <li>• Orthogonal Projection</li> <li>• Angle</li> <li>• Orthogonal complements</li> </ul>	Sections KN: 5.3, 8.1
7	<b>Lecture 7</b> LEAST SQUARE SOLUTIONS	<p style="text-align: center;"><b>MIDTERM TEST</b> (on the material of <b>Lectures 1-6</b>)</p> <ul style="list-style-type: none"> <li>• Normal Equations and the Least Squares solutions to an inconsistent system.</li> </ul>	Sections KN: 5.6
8	<b>Lecture 8</b> ORTHOGONALIZATION	<ul style="list-style-type: none"> <li>• The Gram-Schmidt Process</li> <li>• Gram Matrix</li> </ul>	Sections KN: 8.1, 10.2
9	<b>Lecture 9</b> UNITARY MATRICES QUADRATIC FORMS	<ul style="list-style-type: none"> <li>• Orthonormal bases</li> <li>• Orthogonal &amp; Unitary matrices</li> <li>• Quadratic Forms</li> </ul>	Sections KN: 8.1, 8.2
10	<b>Lecture 10</b> SELF-ADJOINT OPERATORS SPECTRAL THEOREM  <b>QUIZ 2</b>	<ul style="list-style-type: none"> <li>• Hermitian matrices, Self-adjoint mappings</li> <li>• Eigenvalues of Hermitian and symmetric matrices</li> <li>• Spectral Theorem</li> <li>• Cayley-Hamilton Theorem</li> </ul> <p style="text-align: center;"><b>QUIZ 2</b> (based on Weeks 7, 8, 9, 10)</p>	Sections KN: 8.7, 8.2

11	<b>Lecture 12</b> SVD FACTORIZATION	<ul style="list-style-type: none"> <li>• Singular Values and Vectors</li> <li>• Singular Value Decomposition</li> </ul>	<b>Sections</b> <b>KN: 8.6</b>
12	<b>REVIEW</b>	<b>Review classes</b>	

**Academic Integrity and the Academic Code of Conduct**

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: <https://www.concordia.ca/conduct/academic-integrity.html>" [*Undergraduate Calendar, Sec 17.10.2*]

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All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

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