Department of Mathematics & Statistics  
Concordia University

STAT 461  
Statistical Simulation  
Summer 2022

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Office Hours: Thursdays, 11:00-13:00.


Objectives: This course is an introduction to the methods of simulation and the Monte Carlo techniques. Simulation consists of formulating a suitable statistical model for a given system (in economy, industry, insurance, etc.) in terms of appropriate random variables and their (joint) distributions, and generating values of those variables on a computer to see how the system works. Monte Carlo techniques are statistical methods for estimating various quantities of interest for the system, based on repeated simulations, which are difficult to compute theoretically based on the model. In Part I of the course we shall review basic probability theory and study methods for generating (pseudo) random variables. In Part-II we shall study simulation of a few complex systems and their estimation using Monte Carlo methods.

Assignments: There will be 3 or 4 assignments. Most of the assignments will involve use of the software–R that will be demonstrated during one class. R is installed on the PCs in the Department’s Computer Lab, and a freely downloadable student version can be found at http://www.r-project.org. There will be a few separate questions for undergraduate and graduate students in the assignments and exams.

Midterm Test: There will be one midterm test, based on the material of lectures 1-6, which will contribute up to 25% to your final grade (see the Grading Scheme below). Midterm test will be held on Monday, July 18, 2022, in class. This exam, as well as the final, will be closed book exams.

NOTE: It is the Department’s policy that tests missed for any reason, including illness, cannot be made up. If you miss the midterm test because of illness (medical note required) the final exam will count for 85% of your final grade, and the assignments will count for the remaining 15%.
Final Exam: The final examination will be three hours long and will cover all the material in the course. In order to obtain a good grade, the student **MUST** show that she/he has a THOROUGH understanding of the subject.

**NOTE:** Students are responsible for finding out the date and time of the final exams once the schedule is posted by the Examinations Office. Conflicts or problems with the scheduling of the final exam must be reported directly to the Examinations Office, *not to your instructor*. It is the Department’s policy and the Examination Office’s policy that students must be available to take the final exam on the selected date and time. Conflicts due to travel plans will not be accommodated.

Final Grade: a) Assignments (15%)
b) Midterm test (25%)
c) Final examination (60%)

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

**IMPORTANT:** **PLEASE NOTE THAT THERE IS NO “100% FINAL EXAM” OPTION IN THIS COURSE.**

<table>
<thead>
<tr>
<th>Lectures</th>
<th>Chapters</th>
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| 1        | Chapter 2: Elements of Probability  
Sample Space and Events  
Axioms of Probability  
Conditional Probability and Independence  
Random Variables  
Expectations  
Variance  
Chebyshev’s Inequality and the Laws of Large Numbers  |
| 2 & 3    | Chapter 2: Elements of Probability  
Some Discrete Random Variables  
Continuous Random Variables  
Conditional Expectation and Conditional Variance  |
|          | Chapter 3: Random Numbers  
Introduction to R  
Pseudorandom Number Generation  |
|          | Using Random Numbers to Evaluate Integrals  |
| 4 & 5    | Chapter 4: Generating Discrete Random Variables  
The Inverse Transform Method  
Generating a Poisson Random Variable  
Generating Binomial Random Variables  
The Acceptance-Rejection Technique  
The Composition Approach  
The Alias Method of Generating Discrete Random Variables  |
|          | Generating Random Vectors  |
Chapter 5: Generating Continuous Random Variables
- Introduction
- The inverse Transform Algorithm
- The Rejection Method
- The Polar Method for Generating Normal Random Variables
- Generating a Poisson Process

Mid-Term Test

Chapter 5: Generating Continuous Random Variables
- Generating a Nonhomogeneous Poisson Process
- Simulating a Two-Dimensional Poisson Process

Chapter 7: The discrete Event Simulation Approach
- Introduction
- Simulation via Discrete Events
- A Single-Server Queueing System
- A queueing System with Two Servers in Series
- A queueing System with Two Parallel Servers
- An Inventory Model
- An Insurance Risk Model

Chapter 7: The discrete Event Simulation Approach
- A Repair Problem
- Exercising a Stock Option
- Verification of the Simulation Model

Chapter 8: Statistical Analysis of Simulated Data
- Introduction
- The Sample Mean and Sample Variance
- Interval Estimates of a Population Mean
- The Bootstrapping Technique for Estimating Mean Square Errors

Chapter 9: Variance Reduction Techniques
- Introduction
- The Use of Antithetic Variables
- The Use of Control Variates
- Variance Reduction by Conditioning
- Stratified Sampling
- Applications of Stratified Sampling
- Importance Sampling

Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia’s website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." [Undergraduate Calendar, Sec 17.10.2]

Use of Zoom
Zoom is included as an institutionally-approved technology. This means we have been assured of the privacy protections needed to use freely within the classroom)
Zoom may be used in this course to facilitate learning at a distance. It may be used to record some or all of the lectures and/or other activities in this course. If you wish to ensure that your image is not recorded, speak to your instructor as soon as possible.

Also, please note that you may not share recordings of your classes and that the instructor will only share class recordings for the purpose of course delivery and development. Any other sharing may be in violation of the law and applicable University policies, and may be subject to penalties.

**Behaviour**

All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Concordia students are subject to the [Code of Rights and Responsibilities](#) which applies both when students are physically and virtually engaged in any University activity, including classes, seminars, meetings, etc. Students engaged in University activities must respect this Code when engaging with any members of the Concordia community, including faculty, staff, and students, whether such interactions are verbal or in writing, face to face or online/virtual. Failing to comply with the Code may result in charges and sanctions, as outlined in the Code.

**Intellectual Property**

Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member. Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the [Academic Code of Conduct](#) and/or the [Code of Rights and Responsibilities](#). As specified in the [Policy on Intellectual Property](#), the University does not claim any ownership of or interest in any student IP. All university members retain copyright over their work.

**Extraordinary circumstances**

In the event of extraordinary circumstances and pursuant to the [Academic Regulations](#) the University may modify the delivery, content, structure, forum, location and/or evaluation scheme. In the event of such extraordinary circumstances, students will be informed of the change.