

MATH 494 (MAST 699/MAST 833), Sec. F
Topics in Pure & Applied Mathematics
Arithmetic Galois Groups
Winter 2023

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Office Hours: Tuesdays, 16:00-17:30, Zoom or in person.

Class Schedule: Tuesdays, 17:45-20:15. In person, LB 928 SGW.

Textbook: Neukirch & Schmidt & Wingberg , *Cohomology of number fields*, Springer GTM 228 (not compulsory).

Topics: This course will be an introduction to qualitative and quantitative aspects of the Galois inverse problems. One of the highlights will be the proof of the celebrated theorem of Shafarevich that every finite solvable group is a Galois group over the rationals. Along the way, we will examine the more approachable case of nilpotent groups; which will be examined also from the quantitative point of view of Malle's conjecture (roughly predicting how many such solutions) and other explicit families of groups. We will also focus on pro-nilpotent completion of global Galois groups determining the number of generator and relations and sometimes a presentation (this is also work of Shafarevich), along with an analogue theorem in the local case. Here we will also state and prove the famous Golod—Shafarevich inequality. Time allowing, we will present Liu and Wood's theory of random profinite groups and its applications to number theory and topology. We will build from scratch all the needed Galois cohomology to arrive at the main results, so this course can be taken also as a (goal-oriented) introduction to Galois cohomology. Despite the heavy inputs from class field theory, the course can be followed also without previous (or with parallel) exposure to the subject.

Evaluation: Assignments only, during the course, for Master and Undergraduate students;

Assignments during the course and a presentation for PhD students.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

Academic Integrity and the Academic Code of Conduct

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: <https://www.concordia.ca/conduct/academic-integrity.html>" [*Undergraduate Calendar, Sec 17.10.2*]

Behaviour

All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Concordia students are subject to the [Code of Rights and Responsibilities](#) which applies both when students are physically and virtually engaged in any University activity, including classes, seminars, meetings, etc. Students engaged in University activities must respect this Code when engaging with any members of the Concordia community, including faculty, staff, and students, whether such interactions are verbal or in writing, face to face or online/virtual. Failing to comply with the Code may result in charges and sanctions, as outlined in the Code.

Use of Zoom

Note: Zoom is included as an institutionally-approved technology. This means we have been assured of the privacy protections needed to use freely within the classroom.

Zoom might be used in this course to facilitate learning at a distance. It may be used to record some or all of the lectures and/or other activities in this course. If you wish to ensure that your image is not recorded, speak to your instructor as soon as possible.

Also, please note that you may not share recordings of your classes and that the instructor will only share class recordings for the purpose of course delivery and development. Any other sharing may be in violation of the law and applicable University policies, and may be subject to penalties.

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Extraordinary circumstances

In the event of extraordinary circumstances and pursuant to the [Academic Regulations](#) the University may modify the delivery, content, structure, forum, location and/or evaluation scheme. In the event of such extraordinary circumstances, students will be informed of the change.