# **Department of Mathematics & Statistics**

# **Concordia University**

# **MAST 235**

Linear Algebra and Applications II Fall 2022

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**Office Hours:** Thursdays, 13:30-15:00.

**Textbook:** There is no mandatory textbook for this course. All the material will be

available in the lecture and work files that will be posted on the MAST 234

Moodle site.

For additional reading and practice, most of the topics covered in this course

can be found in the following complementary texts:

Linear Algebra with Applications, by W. Keith Nicholson, Open Texts by Lyryx

lila1.lyryx.com/textbooks/OPEN\_LAWA\_1/marketing/Nicholson-OpenLAWA-2021A.pdf

Fred E. Szabo, Linear Algebra, An Introduction Using Maple: Academic Press.

This textbook will be posted on Moodle in PDF format at no cost to the student.

**Prerequisites:** Math 234 or equivalent is a prerequisite for this course.

**Objectives:** This course will focus on applications of the theory studied in Math 234 and its

> further development. There are two major concepts, on which this course is based: (a) Linear Systems & Operators, and their applications such as Economic Models or Dynamical systems, and (b) Inner Product Spaces and Self Adjoint Operators, leading to applications like Orthogonalization, Least Square solutions, and SVD diagonalization. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Algebra courses in your curriculum.

Pedagogy: Classes and all work in this course use the MAPLE as the tool, **not** object of

> study. The structure of classes includes lecture time on the theory, alternating with problem solving tasks done by students individually. Mathematical issues

that arise during problem solving are discussed in class.

Software: MAPLE (version 17 or higher) is mandatory for this course. The Waterloo's

Maplesoft is making MAPLE ("Student's edition", quite sufficient for the course)

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available to Concordia students at a special price. In this course the software is only used as a computational *tool*, *not* as an object of study in itself.

All the tests, the final examination and the assignments are done using *MAPLE*.

### **Assignments:**

Assignment are given and submitted online through Moodle. Late assignments will not be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in this course.

### **Midterm Test:**

There will be **one midterm test** based on the material of Lectures 1-6 which will contribute up to 30% to your final grade (see the Grading Scheme). It will be held in class on **Tuesday October 25, 2022**.

**NOTE:** It is the Department's policy that tests missed for any reason, **including illness**, cannot be made up. If you missed the midterm because of illness (<u>Short-Term Absence form</u> or valid medical note required); the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and quizzes (see the **Grading Scheme**).

# **Final Exam:**

The Final Examination will be 3 hour closed-book Maple examination. Access to resources such as lecture notes, class notes, and similar material is disabled Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, *not* to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans **will not** be accommodated.

NOTE: There are no supplemental exams for this course.

# **Grading Scheme:**

The final grade will be based on the higher of (a) and (b) below:

- (a) 10% for the assignments.5% for the best of 2 quizzes (written in class)30% for the midterm test55% for the final examination.
- (b) 10% for the assignments5% for the best of 2 quizzes15% for the midterm test.70% for the final examination.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

MPORTANT: NOTE that there is NO "100% FINAL EXAM" option in this course.

The term work contributes at least 30% to the final grade. Therefore, active participation in classes and continuous work on the course material **during** the semester is essential for success in this course.

# **CONTENTS**

Week	Lectures TOPIC	Sub-Topics considered	Complementary Reading
1	Lecture 1 LINEAR SYSTEMS: NETWORKS	<ul> <li>Systems of Linear Equations,         Homogeneous: an <i>Overview</i></li> <li>Network Problems</li> </ul>	Sections KN: 1.3, 1.4
2	Lecture 2 LINEAR SYSTEMS: ECONOMIC MODELS	<ul> <li>Economic Models</li> <li>Closed Leontieff Model</li> <li>Open Leontieff Model</li> </ul>	Sections KN: 2.8
3	Lecture 3 DETERMINANTS PROPERTIES & APPLICATIONS	<ul> <li>Determinants: overview and basic properties</li> <li>Applications:         <ul> <li>Polynomial Interpolation Problem</li> <li>Vandermonde matrix</li> </ul> </li> </ul>	Sections KN: 3.1, 3.2
4	QUIZ 1	QUIZ 1 (based on Weeks 1-3)	
	Lecture 4 EIGENTHEORY AND DYNAMICAL SYSTEMS	<ul> <li>Diagonalization and powers of a matrix</li> <li>Application: Dynamical Systems         <ul> <li>Systems with Real eigenvalues</li> <li>The Predator-Prey simulation</li> <li>Systems with Complex eigenvalues</li> </ul> </li> </ul>	Sections KN:2.9, 3.1, 3.3 Appendix A
5	Lecture 5 INNER PRODUCT SPACES	<ul> <li>The concept of inner product space over real and complex numbers         <ul> <li>Definition</li> <li>Properties</li> </ul> </li> <li>The norm in an inner product space         <ul> <li>The Cauchy-Schwartz Inequality</li> <li>The Triangle Inequality</li> <li>The Pythagorean Theorem</li> </ul> </li> </ul>	Sections KN: 8.7, 10.1
6	Lecture 6 ORTHOGONAL PROJECTION	<ul> <li>Orthogonality of vectors</li> <li>Orthogonal Projection</li> <li>Angle</li> <li>Orthogonal complements</li> </ul>	Sections KN: 5.3, 8.1
7	Lecture 7 UNITARY MATRICES	Orthonormal bases & subspaces , unitary matrices	Sections KN: 8.1, 8.2
		MIDTERM TEST based on the material of the Weeks 1-6.	
8	Lecture 8 APPLICATIONS OF ORTHOGONALITY	The Gram-Schmidt Process     Gram Magtrix	Sections KN: 8.1, 10.2
9	Lecture 9 LEAST SQUARES SOLUTION	Normal Equations and the Least Squares solutions to an inconsistent system.	Sections KN: 5.6

10	Lecture 10 HERMITIAN MATRICES & SELF-ADJOINT OPERATORS	<ul> <li>Hermitian matrices, symmetric matrices</li> <li>Self-adjoint mappings</li> <li>Eigenvalues of Hermitian and symmetric matrices</li> </ul>	Sections KN: 8.7
	QUIZ 2	<b>QUIZ 2</b> (based on Weeks 7, 8, 9, 10)	
11	Lecture 11 SPECTRAL THEOREM & APPLICATIONS. QUADRATIC FORMS	<ul> <li>Spectral Theorem (Theorems 6 and 7)</li> <li>Cayley-Hamilton Theorem and applications (pp. 470-472)</li> <li>Quadratic Forms</li> </ul>	Sections KN: 8.2, 8.7
12	Lecture 12 SVD FACTORIZTION	• Singular Value Decomposition (pp. 501-502)	Sections KN: 8.6
13	REVIEW	Review classes	

#### Academic Integrity and the Academic Code of Conduct

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: <a href="https://www.concordia.ca/conduct/academic-integrity.html">https://www.concordia.ca/conduct/academic-integrity.html</a>" [Undergraduate Calendar, Sec 17.10.2]

#### **Behaviour**

All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Concordia students are subject to the <u>Code of Rights and Responsibilities</u> which applies both when students are physically and virtually engaged in any University activity, including classes, seminars, meetings, etc. Students engaged in University activities must respect this Code when engaging with any members of the Concordia community, including faculty, staff, and students, whether such interactions are verbal or in writing, face to face or online/virtual. Failing to comply with the Code may result in charges and sanctions, as outlined in the Code.

#### Use of Zoom

Note: Zoom is included as an institutionally-approved technology. This means we have been assured of the privacy protections needed to use freely within the classroom.

Zoom might be used in this course to facilitate learning at a distance. It may be used to record some or all of the lectures and/or other activities in this course. If you wish to ensure that your image is not recorded, speak to your instructor as soon as possible.

Also, please note that you may not share recordings of your classes and that the instructor will only share class recordings for the purpose of course delivery and development. Any other sharing may be in violation of the law and applicable University policies, and may be subject to penalties.

# **Intellectual Property**

Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member. Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the <u>Academic Code of Conduct</u> and/or the <u>Code of</u>

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# **Extraordinary circumstances**

In the event of extraordinary circumstances and pursuant to the <u>Academic Regulations</u> the University may modify the delivery, content, structure, forum, location and/or evaluation scheme. In the event of such extraordinary circumstances, students will be informed of the change.