Department of Mathematics & Statistics
Concordia University

MAST 661 (MATH 494/MAST 837), Sec. B
Selected Topics in Analysis
Topic: "Fourier Analysis"
Fall 2021

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Schedule: The course will be given in a blended format. There will be in-person meetings on Fridays, 11:45-13:00, in LS 105, as well as live Zoom meetings, accessible through the course Moodle site, and recorded for later viewing. The online meetings will take place mostly on Mondays and Wednesdays, 11:45-13:00, with the schedule posted in advance on Moodle. Attendance at either the in-person or online meetings is recommended but not mandatory.

Office Hours: In person on Friday, 10:00 - 11:30, in LB 927-15. Online office hours via Zoom will be announced later, as well as by appointment.

Textbooks: The following textbooks are recommended:
Fourier Analysis: An Introduction, by E. M. Stein & R. Shakarchi.
An Introduction to Harmonic Analysis, by Y. Katznelson.

Other texts: These references will be put on reserve in the library, or are available online:
Fourier Series and Integrals, by H. Dym and H. P. McKean.
Fourier Analysis and its Applications, by G. B. Folland.
Real Analysis: Modern Techniques and Their Applications, by G. B. Folland.
An introduction to H^p spaces, by P. Koosis.
Fourier Analysis, by T. W. Korner.
Real and Complex Analysis, by W. Rudin.
Functional Analysis, by W. Rudin.
Functional Analysis: Introduction to Further Topics in Analysis, by E. M. Stein & R. Shakarchi.

Introduction to Fourier Analysis on Euclidean Spaces, by E. M. Stein & G. Weiss.
Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals, by E. M. Stein,
Singular integrals and differentiability properties of functions, by E. M. Stein.
Harmonic Analysis from Fourier to Wavelets, by C. Pereyra and L. Ward.
Trigonometric Series, by A. Zygmund.

Topics:

1. Fourier series on the circle
   - Introduction: the origins of Fourier analysis (independent reading: Stein & Shakarchi 1)
   - Convergence of Fourier series (basic: Stein & Shakarchi 2-3, extra: Katznelson I-II)
   - The Hardy-Littlewood maximal function (extra: Katznelson III.2. Rudin R&C 7, Stein SI I)
   - Applications of Fourier series (Stein & Shakarchi 4)
   - Harmonic functions, Poisson integrals and the conjugate function (Katznelson III, Rudin 11, Stein & Weiss II)
2. Fourier transforms on the line and on R^n
   - Definition and properties of the Fourier transform, the Schwartz space (basic: Stein & Shakarchi 5-6, extra: Katznelson VI, Rudin R&C 9, Stein & Weiss I)
   - Tempered distributions (extra: Stein & Weiss I, Stein & Shakarchi FA 3, Rudin FA 6)
   - The Poisson Summation Formula (Stein & Shakarchi 5, Katznelson VI)
3. Additional topics if time permits/ student presentations:
   - Interpolation (Katznelson IV, Stein & Weiss V)
   - Singular integrals (Stein SI II, Stein & Weiss II, VI)
   - Littlewood-Paley Theory
   - Hardy spaces, BMO (Stein & Shakarchi FA 2, Katznelson III.3, Koosis Rudin 17, Stein HA III, IV)
   - Weak/weak* topologies; topological vector spaces, districutions (Rudin FA 1,6)
   - Applications to PDE, Sobolev spaces (Stein & Shakarchi 5-6, extra: Rudin FA 8)
   - Spherical harmonics (Stein & Weiss IV)
   - Fourier analysis on groups, the discrete Fourier transform (Stein & Shakarchi 7)
   - Applications to number thory (Stein & Shakarchi 8)
   - Wavelets (Pereyra & Ward)

Undergraduates: Undergraduate students in MATH 494 will cover the same material, but the marking will be on a separate scale.

PhD students: More advanced material will be assigned to PhD students for independent study throughout the semester.

Pre-requisites: Previously or concurrently: real analysis/metric spaces (equivalent to MATH 464); measure theory (equivalent to MATH 467/669); basic complex analysis (equivalent to MATH 366).

Assignments: Homework will be assigned approximately once every two weeks, on Moodle, via Assignments, and submitted on Moodle. **Late homework will not be accepted.**

Exams: There will be no examinations. Instead, students will be required to complete an independent study project on a topic of their choice as approved by the instructor, related to the course material, and submit it as a written report or in the form of an oral presentation.

Evaluation: Homework assignments 60%, Final project/presentation 40%.
PhD students will be required to do additional work (to be determined) compared with their MA/MSc classmates.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

Communication: Communication between the students and the instructor will take place in person and online via Zoom meetings, Moodle announcements and email messages. **Students are responsible for reading and taking note of all electronic communication from the instructor and the University.**

Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*

Behaviour
All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Concordia students are subject to the Code of Rights and Responsibilities which applies both when students are physically and virtually engaged in any University activity, including classes, seminars, meetings, etc. Students engaged in University activities must respect this Code when engaging with any members of the Concordia community, including faculty, staff, and students, whether such interactions are verbal or in writing, face to face or online/virtual. Failing to comply with the Code may result in charges and sanctions, as outlined in the Code.

Use of Zoom
Note: Zoom is included as an institutionally-approved technology. This means we have been assured of the privacy protections needed to use freely within the classroom)

Zoom will be used in this course to facilitate learning at a distance. It may be used to record some or all of the lectures and/or other activities in this course. If you wish to ensure that your image is not recorded, speak to your instructor as soon as possible.

Also, please note that you may not share recordings of your classes and that the instructor will only share class recordings for the purpose of course delivery and development. Any other sharing may be in violation of the law and applicable University policies, and may be subject to penalties.

Intellectual Property
Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member. Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the Academic Code of Conduct and/or the Code of Rights and Responsibilities. As specified in the Policy on Intellectual Property, the University does not claim any ownership of or interest in any student IP. All university members retain copyright over their work.

Extraordinary circumstances
In the event of extraordinary circumstances and pursuant to the Academic Regulations the University may modify the delivery, content, structure, forum, location and/or evaluation scheme. In the event of such extraordinary circumstances, students will be informed of the change.