Department of Mathematics & Statistics
Concordia University

MAST 235
Linear Algebra and Applications II
Winter 2021

Preface: **Due to exceptional circumstances, this course will be taught and all assessments will be done completely ONLINE.**

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Office hours: Thursday: 13:30-15:00.

Textbook: *Linear Algebra, Theory and Applications*, by Ward Cheney & David Kincaid, 2nd Edition, Publisher: Jones & Bartlett.
The textbook will be available at:
https://www.bkstr.com/concordiastore/home
Note: Students should order textbooks as early as possible, especially for printed versions in case books are backordered or there are any shipping delays.

Prerequisites: Math 234 or equivalent is a prerequisite for this course.

Objectives: This course will focus on applications of the theory studied in Math 234 and its further development. There are two major concepts, on which this course is based: (*a*) *Linear Systems & Operators*, and their applications such as Economic Models or Dynamical systems, where also the *Eigentheory* is applied, and (*b*) *Inner Product Spaces*, leading to applications like Orthogonalization, Least Square solutions, and SVD diagonalization. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Algebra courses in your curriculum.

Pedagogy: Classes and all work in this course use the *MAPLE* as the tool, ***not*** object of study. The structure of classes includes lecture time on the theory, alternating with problem solving tasks done by students individually. Mathematical issues that arise during problem solving are discussed in class.

Software: *MAPLE (version 17 or higher)* is mandatory for this course. The Waterloo's *Maplesoft* is making MAPLE ("Student's edition", quite sufficient for the course) available to Concordia students at a special price. In this course the software is only used as a computational *tool*, ***not* as an object of study** in itself.

All the tests, the final examination and the assignments are done using *MAPLE.*

Assignments: Assignment are given and submitted online through Moodle. Late assignments **will not** be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in this course.

Midterm Test: There will be **one midterm test** based on the material of Lectures 1-6 which will contribute up to 30% to your final grade (see the Grading Scheme). It will be held online during class time on **Tuesday March 9, 2021**.

NOTE: It is the Department's policy that tests missed for any reason, **including illness,** cannot be made up. If you missed the midterm because of illness **(to be confirmed by a valid medical note)** the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and quizzes (see the **Grading Scheme**).

Final Exam: The Final Examination will be 2 hours long and will be written using Maple. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, ***not*** to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans **will not** be accommodated.

NOTE: There are **no supplemental exams** for this course.

Grading Scheme: The final grade will be based on the higher of (***a***) and (***b***) below:

(a) 10% for the assignments.
5% for the best of 2 quizzes (written in class)
30% for the midterm test
55% for the final examination.

(b) 10% for the assignments
5% for the best of 2 quizzes
15% for the midterm test.
70% for the final examination.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

IMPORTANT: **NOTE** that there is **NO "100% FINAL EXAM"** option in this course.
The term work contributes at least 30% to the final grade. Therefore, active participation in classes and continuous work on the course material **during** the semester is essential for success in this course.

CONTENTS

Week	Textbook	Topics	Recommended Problems
1	**Sections 1.3, 2.3, 2.3** LINEAR SYSTEMS: NETWORKS	• Systems of Linear Equations (Sec.1.3), and Linear Mapping (Sec. 2.3): an ***Overview*** • Network Problems (Sec. 2.1, pp.99-100)	G.Ex. 1.3 # 5, 13, 25, 47, 63 G.Ex 2.3: 1, 21,39 G.Ex. 2.1: # 6, 30, 42

2	**Sections 2.4, 6.1, 8.3**
	LINEAR SYSTEMS: ECONOMIC MODELS
		• Economic Models (pp.177-180, 385-388, 548-551)
		  ○ Theorem 14 (p.177), Theorem 4 (p.549)
		  ○ Closed Leontieff Model
		  ○ Open Leontieff Model
			Comp.Ex. 2.4
			# 1, 2, 3.

3	**Sections 4.1, 4.2/3.2**
	DETERMINANTS
	PROPERTIES & APPLICATIONS
		• Determinants: overview and basic properties
		• Applications:
		  ○ Interpolation Problem (pp. 231-233).
		  ○ Vandermonde matrix (pp. 279-280).
			Gen.Ex. 3.2
			# 19, 69, 73, 75
			Gen.Ex. 4.2
			# 7, 27, 33, 45

4	**QUIZ 1**
	
	Sections 6.1, 2.2
	EIGENTHEORY AND DYNAMICAL SYSTEMS
		QUIZ 1 (based on Weeks 1-3)
		
		• Diagonalization and powers of a matrix (pp. 371-380)
		• Application: Dynamical Systems (pp. 380-385).
		  ○ Systems with Real eigenvalues
		  ○ The Predator-Prey simulation (pp. 125-126)
		  ○ Systems with Complex eigenvalues
			Comp.Ex. 2.2
			# 1, 2
			Comp. Ex. 6.1
			# 1, 2, 7, 13

5	**Section 7.1**
	INNER PRODUCT SPACES
		• The concept of inner product space over real and complex numbers (pp. 403-408)
		  ○ Definition
		  ○ Properties
		• The norm in an inner product space (Theorem 1, p. 409)
		  ○ The Cauchy-Schwartz Inequality
		  ○ The Triangle Inequality
		  ○ The Pythagorean Theorem
			G.Ex. 7.1
			# 5, 15, 17, 19, 35, 67, 71, 77

6	**Section 7.1** ORTHOGONAL PROJECTION	• Orthogonality of vectors (Theorems 2, 4, pp.411, 412). • Orthogonal Projection (Theorem 5, p.413) • Angle (p.415). • Orthogonal complements (pp. 416-419)	G.Ex. 7.1 # 3, 11, 13, 51, 57
7	**Section 7.1** UNITARY MATRICES	• Orthonormal bases & subspaces , unitary matrices (pp. 419-423, Theorems 9-15) **MIDTERM TEST** based on the material of the **Weeks 1-6.**	G.Ex. 7.1 # 27, 31, 39
8	**Section 7.2** APPLICATIONS OF ORTHOGONALITY	• The Gram-Schmidt Process (pp.432-439), Theorems 1 & 2.	G.Ex. 7.2 # 7, 9, 23, 25, 27, 29
9	**Section 7.2** LEAST SQUARES SOLUTION	• Normal Equations and the Least Squares solutions to an inconsistent system; Theorem 3. (pp. 439-445) • Geometrical interpretation of the least-square approximations in terms of distance, projections and hyperplanes.	G.Ex. 7.2 # 1, 3, 5, 11, 13

10	**Section 8.1** HERMITIAN MATRICES & SELF-ADJOINT OPERATORS **QUIZ 2**	• Hermitian matrices, symmetric matrices • Self-adjoint mappings • Eigenvalues of Hermitian and symmetric matrices **QUIZ 2** (based on Weeks 7, 8, 9)	G.Ex. 8.1 # 1, 3, 9, 17, 25, 33
11	**Section 8.1** SPECTRAL THEOREM & APPLICATIONS. QUADRATIC FORMS	• Spectral Theorem (Theorems 6 and 7) • Cayley-Hamilton Theorem and applications (pp. 470-472) • Quadratic Forms	G.Ex. 8.1 # 27, 29, 39, 47 G.Ex. 8.1 # 11, 61
12	**Sections 8.2** SVD FACTORIZTION	• Singular Value Decomposition (pp. 501-502)	
13	**REVIEW**	**Review classes**	

Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*

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