Instructor: Dr. A. Atoyan, Office: LB 1041.24 (SGW)
Email: armen.atoyan@concordia.ca

Office hours: Thursday: 13:30-15:00.


Prerequisites: Math 234 or equivalent is a prerequisite for this course.

Objectives: This course will focus on applications of the theory studied in Math 234 and its further development. There are two major concepts, on which this course is based: (a) Linear Systems & Operators, and their applications such as Economic Models or Dynamical systems, where also the Eigentheory is applied, and (b) Inner Product Spaces, leading to applications like Orthogonalization, Least Square solutions, and SVD diagonalization. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Algebra courses in your curriculum.

Pedagogy: Classes take place in a computer lab equipped with Maple. The structure of the classes includes lecture time on the theory and examples, alternating with problem solving tasks done in class by students individually. Mathematical issues that arise during problem solving are discussed in class.

Software: Maple (version 15 or higher). The Waterloo Maplesoft is making Maple (“Student’s edition”, quite sufficient for the course) available to Concordia students at a special price. In this course the software is only used as a computational tool, not as an object of study in itself. All the tests, the final examination and the assignments are done using Maple.

Assignments: Assignment are done using Maple and are submitted online through Moodle. Late assignments will not be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in this course.

Midterm Test: There will be one midterm test (based on the material of weeks 1-7) which will contribute up to 25% to your final grade (see the Grading Scheme). It will be held in class on Thursday February 20, 2020 (the last class before the Break week).
NOTE: It is the Department’s policy that tests missed for any reason, including illness, cannot be made up. If you missed the midterm because of illness (to be confirmed by a valid medical note) the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and quizzes (see the Grading Scheme).

Final Exam: The Final Examination will be 3 hours long (closed-book exam, no notes are allowed) written using Maple. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, not to the Instructor. Students are to be available until the end of the final exam period. Conflicts due to travel plans will not be accommodated.

NOTE: There are no supplemental exams for this course.

Grading Scheme: The final grade will be based on the higher of (a) and (b) below:

(a) 10% for the assignments.
   5% for the best of 2 quizzes (written in class)
   25% for the class test
   60% for the final examination.

(b) 10% for the assignments
   5% for the best of 2 quizzes
   10% for the class test
   75% for the final examination.

The grading scheme for this course includes graded assignments of which a reasonable representative subset will be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

IMPORTANT: NOTE that there is NO “100% FINAL EXAM” option in this course.
The term work contributes at least 25% to the final grade. Therefore active participation in classes and continuous work on the course material during the semester is essential for success in this course.

Disclaimer: The instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons. Every effort will be made to minimize such changes.
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**Academic Integrity and the Academic Code of Conduct**

This course is governed by Concordia University’s policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia’s website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate studies: concordia.ca/students/academic-integrity." [Undergraduate Calendar, Sec 17.10.2]