Department of Mathematics & Statistics
Concordia University

MAST 661/MAST 837 (MATH 494)

Selected Topics in Analysis
Topic: "Fourier Analysis"
Winter 2018

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Lectures: M-W 16:15-17:30, LB 759-6. The schedule will be finalized during the first meeting of the course (January 8, 2018).

Office Hours: To be announced.

Textbooks: On order at the Concordia Bookstore:
Fourier Analysis: An Introduction, by E. M. Stein & R. Shakarchi.
An Introduction to Harmonic Analysis, by Y. Katznelson.

Other texts: These references will be put on reserve in the library, or are available online:
Fourier Series and Integral, by H. Dym and H. P. McKean.
Fourier Analysis and its Applications by G. B. Folland.
Real Analysis: Modern Techniques and Their Applications, by G. B. Folland.
An introduction to H^p spaces, by P. Koosis.
Fourier Analysis, by T. W. K orner.
Real and Complex Analysis, by W. Rudin.
Functional Analysis, by W. Rudin.
Functional Analysis: Introduction to Further Topics in Analysis, by E. M. Stein & R. Shakarchi.
Introduction to Fourier Analysis on Euclidean Spaces, by E. M. Stein & G. Weiss.
Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals, by E. M. Stein,
Singular integrals and differentiability properties of functions, by E. M. Stein.
Harmonic Analysis from Fourier to Wavelets, by C. Pereyra and L. Ward.
Trigonometric Series, by A. Zygmund.

Topics:

1. Fourier series on the circle
   - Introduction: the origins of Fourier analysis (independent reading: Stein & Shakarchi 1)
   - Convergence of Fourier series (basic: Stein & Shakarchi 2-3, extra: Katznelson I-II)
   - The Hardy-Littlewood maximal function (extra: Katznelson III.2, Rudin R&C 7, Stein SI I)
   - Applications of Fourier series (Stein & Shakarchi 4)

2. Fourier transforms on the line and on R^n
   - Definition and properties of the Fourier transform, the Schwartz space (basic: Stein & Shakarchi 5-6, extra: Katznelson VI, Rudin R&C 9, Stein & Weiss I)
   - Tempered distributions (extra: Stein & Weiss I, Stein & Shakarchi FA 3, Rudin FA 6)
   - The Poisson Summation Formula (Stein & Shakarchi 5, Katznelson VI)

3. Additional topics if time permits/ student presentations:
   - Harmonic functions, Poisson integrals and the conjugate function (Katznelson III, Rudin 11, Stein & Weiss II)
   - Interpolation (Katznelson IV, Stein & Weiss V)
   - Singular integrals (Stein SI II, Stein & Weiss II, VI)
   - Hardy spaces, BMO (Katznelson III.3, Koosis, Rudin 17, Stein HA III, IV)
   - Weak/weak* topologies, topological vector spaces, distributions (Rudin FA 1,6)
   - Applications to PDE, Sobolev spaces (Stein & Shakarchi 5-6, extra: Rudin FA 8)
   - Spherical harmonics (Stein & Weiss IV)
   - Fourier analysis on groups, the discrete Fourier transform (Stein & Shakarchi 7)
   - Applications to number theory (Stein & Shakarchi 8)
   - Wavelets (Pereyra & Ward)

Undergraduates: Undergraduate students in MATH 494 will cover the same material, but the marking will be on a separate scale.

PhD students: More advanced material will be assigned to PhD students for independent study throughout the semester. The results will be presented in a written or oral presentation and may also be included in the exams. The evaluation of the presentation will be incorporated in the final grade.

Pre-requisites: Previously: real analysis/metric spaces (equivalent to MATH 464); previously or concurrently: measure theory (equivalent to MATH 467/669), basic complex analysis (equivalent to MATH 366).

Assignments: Homework will be assigned approximately once every two weeks, during lecture. In the case of an absence, it is the student's responsibility to find out the homework assignment. **Late homework will not be accepted.**

You should submit your homework handwritten on paper, not electronically, and provide complete arguments. Some assigned problems may not be marked. Understanding of the homework is essential to success on the exams.
Students must follow the University's policy on Academic Integrity: http://www.concordia.ca/students/academic-integrity.html

Exams: To be announced.

Evaluation: Homework assignments 40%, Exams 60%.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

Academic Integrity and the Academic Code of Conduct
This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*