Department of Mathematics & Statistics
Concordia University

MAST 662(MAST 837), Sec. C

Functional Analysis

Fall 2017

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Topics: The course is devoted to the basics of the Functional Analysis. It covers the following topics:

1. Metric spaces. Completeness and the Baire theorem. Contractions and fixed points. Applications: existence/uniqueness theorem for the ODEs, Orlitcz theorem on the nonuniqueness of solution of ODE, condensation of singularities.
2. Compact metric spaces. Examples, criteria of compactness. Peano theorem on the existence of solution of ODE with continuous RHS. Lower semicontinuous functionals; existence of a minimal geodesic on a surface in $\boldsymbol{R}^3$.
3. Banach spaces. Examples. Compact sets in Banach spaces. Compact maps. Schauder fixed point theorem and its applications.
4. Linear functionals in Banach spaces. Dual space and reflexivity. The Hahn-Banach theorem and its applications. The Riesz-Markov representation theorem.
5. The weak and weak-* convergence in Banach spaces. The Alaoglou theorem.
6. Linear operators in Banach spaces. Weak, strong and uniform convergence. The Banach-Steinhaus theorem.
7. The Closed graph theorem. The open map theorem. Banach inverse operator theorem.

Some topics are devoted to applications and extensions of the theory. They are intended for the students' presentations. Here are some sample topics:

8. The Zorn lemma.
9. The Bogoliubov-Krylov Theorem.
10. The Monge-Kantorovich Problem.
11. The Runge theorem.
12. The Kolmogorov ε -entropy.

13. The existence of a closed geodesic on a convex surface.
14. Applications of the Close Graph Theorem.
15. The Dirichlet Principle and existence of solution of the Dirichlet Problem.
16. Uniform convexity and the reflexivity criterion.
17. The Krein-Milman Theorem.

Good presentations are counted as successful exams.

Academic Integrity and the Academic Code of Conduct

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*