## **Department of Mathematics & Statistics**

## Concordia University

## **MAST 234**

Linear Algebra and Applications I Fall 2014

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**Textbook:** Linear Algebra, Theory and Applications, by W. Cheney & D. Kincaid. 2nd Edition.

Publisher: Jones & Bartlett.

**Prerequisites:** MATH 204 or equivalent is a prerequisite for this course.

**Objective** This is the first part of the two connected *Linear Algebra and Applications* courses,

the second part being MAST 235. There are two major concepts, *Vector Spaces* and *Linear Transformations*, on which this first part is based. In learning these concepts we will use related constructs such as *vectors*, *matrices* and *systems of linear equations*. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further

Linear Algebra courses in your curriculum.

**Pedagogy**: Classes are interactive, and start with a lecture introducing the principal concepts

of the topic considered, followed by problem solving by students in the lab equipped with computers. Mathematical issues that arise during problem

solving are discussed in class.

**Software:** The software used in this course is *Maple (version 12 or higher)*. The Waterloo

*Maplesoft* is making "Maple 16 Student's edition" available to Concordia students at a special price. In this course the software is only used as a computational *tool*, **not** as an object of study in itself. All the quizzes, the test, the final examination

and the assignments are done using Maple.

**Assignments:** Assignment are given and submitted online through Moodle. Late assignments

*will not* be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in

this course.

**Midterm Test:** 

There will be **one midterm test**, based on the material of weeks 1-5, which will contribute up to 25% to your final grade (see the Grading Scheme). It will be held in the **second** class of the week 6.

**NOTE:** It is the Department's policy that tests missed for any reason, **including illness**, cannot be made up. If you missed the midterm because of illness (**to be confirmed by a valid medical note**) the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and the quizzes.

**Final Exam:** 

The Final Examination will be 3 hours long (*closed-book* exam, no notes are allowed) written using Maple. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, *not* to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans will not be accommodated.

**Note:** There are **no supplemental exams** for this course.

**Grading Scheme:** The final grade will be based on the higher of (a) and (b) below:

(a) 10% for the assignments.

5% for the best 2 of 3 quizzes (written in class)

25% for the class test

60% for the final examination.

(b) 10% for the assignments

5% for the best 2 of 3 quizzes

10% for the class test

75% for the final examination.

**IMPORTANT:** 

NOTE that there is NO "100% FINAL EXAM" option in this course.

The term work contributes at least 25% to the final grade. Therefore, active participation in classes and continuous work on the course material *during* the semester is essential for success in this course.

Disclaimer:

The instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons. Every effort will be made to minimize such changes.

## **CONTENTS**

Week	Textbook	Topics	Recommended Problems
1	Section 1.1 LINEAR SYSTEMS & MATRICES	<ul> <li>Review of systems of linear equations.</li> <li>Matrix form of a system</li> <li>Matrix of the system; augmented matrix</li> <li>Elementary Row Operations</li> <li>Row Echelon Form</li> <li>Row equivalent matrices</li> <li>Number of solutions for a system of linear equations; systems with parameters</li> </ul>	Gen.Ex. 1.1 # 5, 23, 35, 39
2	Section 1.2 VECTORS, MATRICES, SPANS	<ul> <li>Review of vectors and matrices         <ul> <li>Vectors in R<sup>n</sup>. Linear combination of vectors</li> <li>Matrix-vector products</li> </ul> </li> <li>Span of a set of vectors</li> <li>Existence of solutions for linear systems in terms of span of columns of the matrix</li> </ul>	Gen.Ex. 1.2 # 11, 15, 19, 25
3	Section 1.3 HOMOGENEOUS SYSTEMS	<ul> <li>Homogeneous systems of equations, and the         <ul> <li>Null Space of a matrix</li> </ul> </li> <li>The rank of a matrix</li> <li>Nontrivial solutions of a homogeneous system.</li> <li>Linear dependence/independence of vectors and         <ul> <li>homogeneous systems of equations</li> </ul> </li> <li>QUIZ 1 (on the material covered in previous weeks)</li> </ul>	<u>G.Ex. 1.3</u> # 5, 9, 13, 21, 25
4	Sections 2.1, 2.4 VECTOR SPACES	<ul> <li>Examples of Vector Spaces         <ul> <li>n-tuples (R<sup>n</sup>) &amp; Euclidean vector spaces</li> <li>Polynomials as vectors</li> <li>Other examples (Matrices, Continuous functions)</li> </ul> </li> <li>Generalization: the axioms of a vector space.</li> <li>Properties of vector spaces (Theorems 2.4: 1-5)</li> <li>Generalization of the notions of Span and linear Dependence -</li> <li>Independence for abstract vectors spaces.</li> </ul>	G.Ex. 2.1: # 1, 7, 11, 13, 23 G.Ex. 2.4 # 7,15,27,33, 35
6	Section 3.1 OPERATIONS ON MATRICES, DETERMINANTS  Section 3.2 MATRIX INVERSES	<ul> <li>Operations on matrices         <ul> <li>Matrix addition and scalar multiplication</li> <li>Multiplication of matrices: definition and properties</li> <li>Matrix-vector products</li> </ul> </li> <li>Special matrices, Diagonal and Triangular matrices</li> <li>Determinants (introductory overview)</li> <li>Left and Right inverses of matices.</li> <li>Square matrices: definition of an invertible matrix</li> <li>Properties of invertible matrice</li> <li>Invertible matrices and linear independence</li> <li>MIDTERM TEST (based on materials of Weeks 1-5)</li> </ul>	G.Ex. 3.1 #11, 19, 41, 47, 51 G.Ex. 3.2 # 9, 11, 19, 21
7	Sections 2.3, 5.1 LINEAR TRANS- FORMATIONS IN R <sup>n</sup>	<ul> <li>The notion of Subspaces: Definition, Properties, Examples</li> <li>Linear transformations (or linear mapping)         <ul> <li>Domain, Co-domain, Range: definition and examples</li> <li>The Linearity properties: definition and examples</li> <li>Matrices and linear maps: Theorems 1-3.</li> <li>Composition of linear mappings (Theorem 2.3.8)</li> </ul> </li> </ul>	G.Ex. 5.1 # 1,3,5,11,15,29 G.Ex. 2.3 # 3, 7, 9, 11, 21, 15, 25, 43

8	Sections 5.1, 5.2 BASIS, DIMENSION & COORDINATES	<ul> <li>Subspaces associated with matrices</li> <li>Basis for a vector space &amp; Dimension         <ul> <li>Unique representation of a vector in a given basis</li> <li>Theorems 5.2: 9, 10, 11, 12, 13.</li> </ul> </li> </ul>	G.Ex. 5.2: #1,5,7,15,17, 21,41
		QUIZ 2 (on material covered in Weeks 6 and 7)	
9	Sections 5.2, 5.3, COORDINATE SYSTEMS, LINEAR MAPS (GENERAL)	<ul> <li>Coordinate vector, Coordinatization</li> <li>Transition from one basis to another (Theorem 1, section 5.3)</li> <li>Linear transformations in abstract vector spaces.</li> </ul>	G.Ex. 5.3: # 5, 7, 9, 11, 17, 23, 32
10	Section 5.3 LINEAR MAPS 7 MATRICES	<ul> <li>Examples of linear transformations of spaces other than R<sup>n</sup></li> <li>Rank-Nullity Theorem (section 5.2, #17 &amp; #18)</li> <li>Linear transformations and basis in the domain and co-domain.         <ul> <li>Theorem 2 (section 5.3)</li> </ul> </li> <li>Matrix representation of a linear transformation</li> <li>Matrices of linear mappings of a vectors space into itself</li> <li>Similar matrices, Theorem 5 (sec. 5.3)</li> </ul>	G.Ex. 2.3: # 9, 11, 15, 43 G.Ex. 5.3 # 7, 11 15, 17, 31, 33.
11	Section 6.1 LINEAR OPERATORS & EIGENTHEORY	QUIZ 3 (on material covered in Weeks 8, 9, 10)  Introduction to Eigentheory  Eigenvectors and eigenvalues of matrices (sec. 6.1)  Definition of eigenvectors and eigenvalues of a linear operator (sec. 6.1)  Eigenvalues of a linear operator are the eigenvalues of any of its matrix representations	G.Ex. 6.1 # 15, 23, 29, 31, 37, 45
12	Section 6.1 EIGENTHEORY & DIAGONALI- ZATION	Diagonalizable matrices and diagonalizable linear operators     Definition (sec 6.1)     Conditions of diagonalizability: basis of eigenvectors.     Theorem 3 and its Corollaries	G.Ex. 6.1: # 33, 52, 66, 71
13	REVIEW	Review Classes for the Final Examination	