MATHEMATICS AND STATISTICS

Section 31.200

Faculty

Chair

CODY HYNDMAN, PhD University of Waterloo; Associate Professor

Associate Chair

HARALD W. PROPPE, PhD McGill University; Professor

Professors

MARCO BERTOLA, PhD SISSA-ISAS, Trieste, Italy ABRAHAM J. BOYARSKY, PhD McGill University YOGENDRA P. CHAUBEY, PhD University of Rochester, Provost's Distinction CHANTAL DAVID, PhD McGill University JOSÉ GARRIDO, PhD University of Waterloo PAWEL GORA, PhD DSc Warsaw University JOHN HARNAD, DPhil University of Oxford; Provost's Distinction ADRIAN IOVITA. PhD Boston University HERSHY KISILEVSKY, PhD Massachusetts Institute of Technology DMITRY KOROTKIN, PhD Steklov Mathematical Institute ROBERT RAPHAEL, PhD McGill University ALEXANDER SHNIRELMAN, PhD Moscow State University ANNA SIERPINSKA, PhD Krakow Higher School of Pedagogy RONALD J. STERN, PhD Northwestern University WEI SUN, PhD Chinese Academy of Sciences FRED E. SZABO, PhD McGill University

Associate Professors

JOSEF BRODY, PhD Carleton University
G. ELIE COHEN, PhD McGill University
GALIA DAFNI, PhD Princeton University
PATRICE GAILLARDETZ, PhD University of Toronto
NADIA HARDY, PhD Concordia University
ALEXEY KOKOTOV, PhD Steklov Mathematical Institute
LEA POPOVIC, PhD University of California, Berkeley
ARUSHARKA SEN, PhD Indian Statistical Institute
ALINA STANCU, PhD University of Rochester

XIAOWEN ZHOU, PhD University of California, Berkeley

Assistant Professors

FRÉDÉRIC GODIN, PhD HEC Montréal LISA KAKINAMI, PhD University of Rochester School of Medicine and Dentistry MÉLINA MAILHOT, PhD Université Laval GIOVANNI ROSSO, PhD KU Leuven and Université Paris 13

Lecturers

ARMEN ATOYAN, PhD Moscow Engineering-Physics Institute DEBARAJ SEN, PhD Concordia University

Affiliate Professors

JOHN DENTON, PhD Harvard University ANDREW GRANVILLE, PhD Queen's University HENRY HUNG, PhD McGill University

Affiliate Assistant Professors

FERENC BALOGH, PhD Concordia University
MARIO GHOSSOUB, PhD University of Waterloo
MANUELA GIROTTI, PhD Concordia University
MOHAMMED NAJAFI IVAKI, PhD Concordia University

For the complete list of faculty members, please consult the Department website.

Location

Sir George Williams Campus J.W. McConnell Building, Room: LB 901 514-848-2424, ext. 3223

Department Objectives

The Department of Mathematics and Statistics provides the general mathematical culture necessary for training those who will either be using the tools of mathematics or statistics in their work or who will become future mathematicians or statisticians. For students who are contemplating graduate work in mathematics or statistics, the Department has leading researchers in the fields of actuarial mathematics, applied probability, computational algebra, differential geometry, dynamical systems, mathematical physics, mathematics education, number theory and statistics.

In addition to its co-op program, alternating work and academic terms, the Department offers a program in Actuarial Mathematics and prepares students for the associateship examinations of the Society of Actuaries and the Casualty Actuarial Society. The Department also provides instruction at the remedial and introductory levels to enable students who have been out of school for some time to re-enter the academic stream; and offers special courses for teachers of mathematics who wish to keep abreast of recent ideas and applications.

Programs

Students are required to complete the appropriate entrance profile for entry into the program (see §31.002 — Programs and Admission Requirements — Profiles).

A student wishing to follow a program in the Department of Mathematics and Statistics but not meeting the entrance requirements should consult with one of the Department's academic advisors.

By careful choice of electives, students can select whether the emphasis of the program will be in the area of Actuarial Mathematics, Mathematical and Computational Finance, Pure and Applied Mathematics, or Statistics through specialization/honours programs in the respective areas.

The enrolment in specialization/honours will be on a selective basis, whereas the major will be open to all students. However, students with less than a 70% average in Cegep Mathematics courses will be required to take a six-credit "transition" Calculus and Linear Algebra course (MAST 214) upon entry into the MATH/STAT Major. The course will not count for credits in the major. Students in a Mathematics and Statistics program may not take any of the following courses for credit: EMAT 212, 232, 252, 271. Students wishing to take other Engineering Mathematics courses for credit must obtain prior approval of the Department. The Department of Mathematics and Statistics offers honours programs as a six-credit extension to its specialization programs in Pure and Applied Mathematics, Statistics, and Actuarial Mathematics. The extra six credits are earned in one of the Project Courses: MATH 496, STAT 499, or ACTU 493. The Department does not allow direct entry to honours programs upon admission to Concordia University. Students seeking entry to an honours program should speak to the Department's honours director after completing 30 credits in their specialization.

Students admitted to an honours program require an overall GPA of at least 3.30, and at least 3.50 in their program of specialization with no more than one grade below B-. Students must find a supervisor for their project and register for the appropriate Project Course.

For additional information concerning programs and courses, students should consult the Department.

Students are responsible for satisfying their particular degree requirements. The superscript indicates credit value.

66 BA or BSc Honours in Actuarial Mathematics

- 27 MATH 251³, 252³, 264³, 265³, 354³, 364³, 365³; STAT 249³, 250³
- 30 ACTU 256³, 257³, 357³, 457³, 458³, 459³; STAT 349³, 360³, 460³, 461³
- 3 Chosen from ACTU 2861, 3862, 4862; MAST 2323, 3323; STAT 2871, 3882
- 6 Honours project ACTU 493

60 BA or BSc Specialization in Actuarial Mathematics

- 27 MATH 251³, 252³, 264³, 265³, 354³, 364³, 365³; STAT 249³, 250³
- 30 ACTU 256³, 257³, 357³, 457³, 458³, 459³; STAT 349³, 360³, 460³, 461³
- 3 Chosen from ACTU 286¹, 386², 486²; MAST 232³, 332³; STAT 287¹, 388²

90 BA or BSc Specialization in Actuarial Mathematics/Finance

- 27 MATH 251³, 252³, 264³, 265³, 354³, 364³, 365³; STAT 249³, 250³
- 30 ACTU 256³, 257³, 357³, 457³, 458³, 459³; STAT 349³, 360³, 460³, 461³
- 3 Chosen from ACTU 286¹, 386², 486²; STAT 287¹
- 18 ECON 2013, 2033; COMM 2203; ACCO 2303; FINA 3853, 3953
- 12 Chosen from MACF 401³, 402³; 400-level Finance courses

90 BA or BSc Specialization in Mathematical and Computational Finance

- MACF 4013, 4023 6
- MATH 251³, 252³, 264³, 265³, 364³, 365³, 370³, 473³
- STAT 249³, 250³, 349³, 360³, 460³, 461³
- Chosen from MACF 491³, 492³; MATH 361³*, 464³, 467³, 478³, 479³; STAT 449³, 450³, 452³ ACCO 230³; COMM 220³, 308³; ECON 201³, 203³; FINA 385³, 395³, 412³ 12
- Chosen from FINA 4113, 4133, 4553**, 4653
- COMP 2183 or 2483.5

*NOTE: Students electing to include MATH 361 in their program are normally expected to plan to take MATH 478 and FINA 411 in order to focus on Portfolio Management and Optimization as an area of the Mathematical and Computational Finance discipline. **NOTE: FINA 455 may only be included with prior departmental approval. The topic must be related to an area of the Mathematical and Computational Finance discipline.

BA or BSc Honours in Pure and Applied Mathematics 66

- MATH 251³, 252³, 264³, 265³, 354³, 361³, 364³, 365³; STAT 249³, 250³ MATH 366³, 369³, 370³, 464³
- 12
- Chosen from MAST 2173, 2323
- Chosen from any other 400-level MATH/STAT courses
- MATH/STAT chosen with prior departmental approval
- Honours project MATH 496 6

60 **BA or BSc Specialization in Pure and Applied Mathematics**

- MATH 251³, 252³, 264³, 265³, 354³, 361³, 364³, 365³; STAT 249³, 250³
- MATH 3663, 3693, 3703, 4643 12
- Chosen from MAST 2173, 2323
- Chosen from any other 400-level MATH/STAT courses
- MATH/STAT chosen with prior departmental approval

66 **BA or BSc Honours in Statistics**

- MATH 2513, 2523, 2643, 2653, 3543, 3613, 3643, 3653; STAT 2493, 2503 30
- MATH 3703; STAT 3493, 3603, 4503
- Chosen from MATH 4643, 4673, 4783, 4793; STAT 4493, 4523, 4603, 4613, 4803 or BSTA 4453
- Chosen from MAST 2173, 2323, 3333*; MATH 3663; STAT 2871, 3882
- MATH/STAT chosen with prior departmental approval
- Honours project STAT 499

*NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Students taking a double Major or a Minor in Mathematics and Statistics and whose other program requires statistics courses should consult the Mathematics and Statistics undergraduate program advisor.

BA or BSc Specialization in Statistics

- MATH 2513, 2523, 2643, 2653, 3543, 3613, 3643, 3653; STAT 2493, 2503
- 12 MATH 370³; STAT 349³, 360³, 450³
- Chosen from MATH 4643, 4673, 4783, 4793; STAT 4493, 4523, 4603, 4613, 4803 or BSTA 4453
- Chosen from MAST 217³, 232³, 333^{3*}; MATH 366³; STAT 287¹, 388²
- MATH/STAT chosen with prior departmental approval

*NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Students taking a double Major or a Minor in Mathematics and Statistics and whose other program requires statistics courses should consult the Mathematics and Statistics undergraduate program advisor.

BA or BSc Major in Mathematics and Statistics

- COMP 218³ or 248³.5; MAST 217³ or COMP 232³; MAST 218³, 219³, 221³*, 232³, 234³, 235³, 324³, 331³, 333³* Chosen from MAST 330³, 332³
- Chosen from MAST 2233*, 3343, 3353, 3973, 3983
- Chosen with prior departmental approval**

*NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Students taking a double Major or a Minor in Mathematics and Statistics and whose other program requires statistics courses should consult the Mathematics and Statistics undergraduate program advisor.

**NOTE: This category may also include any other courses in the ACTU/MATH/STAT discipline, or certain courses in COMP or PHYS. It is not intended to include courses unrelated to the mathematical disciplines.

BA or BSc Joint Major in Mathematics and Statistics and Computer Applications

Mathematics and Statistics Component

- COMP 248^{3,5}; MAST 217³ or COMP 232³; MAST 218³, 219³, 221³ or COMP 233³; MAST 232³, 234³, 235³, 324³, 331³, 332³ or COMP 3673; MAST 3333*, 3343 or COMP 3613; MATH 3393 Computer Science Component (see §71.85)
- COMP 2283, 24935, 3353, 3464, 3483, 3523, 3544, 4653; ENCS 2823, 3933

4 Chosen from COMP courses with numbers 325 or higher; SOEN 287³, 321³, 387³, 422³, 423⁴, 487⁴

NOTE: The Faculty of Arts and Science and the Faculty of Engineering and Computer Science have created a program of study which combines a comprehensive education in computer science and mathematics. This program resides in both Faculties. In the Faculty of Engineering and Computer Science, it is offered under the aegis of the Bachelor of/Baccalaureate in Computer Science, Computer Applications Option. According to their preferences and aspirations, students may apply either for a Bachelor of/Baccalaureate in Science program, or Bachelor of/Baccalaureate in Arts program or a Bachelor of/Baccalaureate in Computer Science program. The Computer Science program is described in §71.85.

*NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Students taking a double Major or a Minor in Mathematics and Statistics and whose other program requires statistics courses should consult the Mathematics and Statistics undergraduate program advisor.

24 Minor in Mathematics and Statistics

- 18 MAST 217³, 218³, 219³, 221^{3*}, 324³, 333^{3*}
- 6 MATH/STAT chosen with prior departmental approval from MAST 223³*, 232³, 234³, 235³, 330³, 331³, 332³, 334³, 335³, 397³, 398³

*NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Students taking a double Major or a Minor in Mathematics and Statistics and whose other program requires statistics courses should consult the Mathematics and Statistics undergraduate program advisor.

Mathematics and Statistics Co-operative Program

Director

LISA KAKINAMI, Assistant Professor

The Mathematics and Statistics co-operative program is offered to students who are enrolled in the BA or BSc honours or specialization programs. Students interested in applying for the Mathematics and Statistics co-op should refer to §24 where a full description of the admission requirements is provided.

Academic content is identical to that of the regular program, but six or seven study terms are interspersed with three or four work terms

Students are supervised personally and must meet the requirements specified by the Faculty of Arts and Science and the Institute for Co-operative Education in order to continue their studies in the co-op format.

Liaison between the student, the employers, and the Institute for Co-operative Education is provided by the Mathematics and Statistics co-op committee, which includes the student's advisors.

Please refer to §24 for the schedule of study and work terms and the full description of admission requirements.

Courses

Actuarial Mathematics

ACTU 256 Mathematics of Finance (3 credits)

Prerequisite: MATH 264 previously or concurrently; and permission of the Department. Measurement of interest; annuities and perpetuities; amortization and sinking funds; rates of return; bonds and related securities; life insurance.

NOTE: Students who have received credit for MATH 326 may not take this course for credit.

NOTE: Only three credits will be awarded from ACTU 256; MAST 335.

ACTU 257 Actuarial Mathematics I (3 credits)

Prerequisite: ACTU 256. Measurement of mortality; pure endowments; life insurance; net single premiums; life annuities; net annual premiums; special topics.

NOTE: Students who have received credit for MATH 327 may not take this course for credit.

ACTU 286 Actuarial Mathematics Lab I (1 credit)

This lab features problem-solving sessions for the professional examination on financial mathematics of the Society of Actuaries and the Casualty Actuarial Society.

NOTE: Students who have received credit for MATH 229 may not take this course for credit.

ACTU 357 Actuarial Mathematics II (3 credits)

Prerequisite: ACTU 257. Net level premium reserves; multiple life functions; multiple decrements, the expense factor; special topics. NOTE: Students who have received credit for MATH 427 may not take this course for credit.

ACTU 386 Actuarial Mathematics Lab II (2 credits)

This lab will feature the use of programming languages and software applications.

NOTE: Students who have received credit for MATH 232 may not take this course for credit.

ACTU 456 Pension Mathematics (3 credits)

Prerequisite: ACTU 357. Valuation methods; gains and losses; dynamic control; special topics.

ACTU 457 Risk Theory (3 credits)

Prerequisite: ACTU 257. Applications of contingency theory in health insurance, individual and collective risk theory, ruin theory. NOTE: Students who have received credit for MATH 428 may not take this course for credit.

Credibility Theory (3 credits)

Prerequisite: ACTU 457; STAT 349 previously or concurrently. Credibility approach to inference for heterogeneous data; classical, regression and Bayesian models; illustrations with insurance data.

Loss Distributions (3 credits)

Prerequisite: ACTU 457; STAT 360. Probability model fitting to loss data; estimation and testing under variety of procedures and sampling designs.

ACTU 486 Actuarial Mathematics Lab III (2 credits)

This lab will be a workshop designed to prepare students for the Actuarial Models examination of the Society of Actuaries and the Casualty Actuarial Society.

NOTE: Students who have received credit for MATH 429 may not take this course for credit.

ACTU 491 Topics in Actuarial Mathematics (3 credits)

ACTU 492 Reading Course in Actuarial Mathematics (3 credits)

ACTU 493 Honours Project in Actuarial Mathematics (6 credits)

Specific topics for these courses, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.

Mathematical and Computational Finance

MACF 401 Mathematical and Computational Finance I (3 credits)

Prerequisite: FINA 385; MATH 265; STAT 349 previously or concurrently. This course is a rigorous introduction to the theory of mathematical and computational finance. Multi-period binomial model; state prices; change of measure; stopping times; European and American derivative securities; interest-rate models; interest-rate derivatives; hedging; convergence to the Black-Scholes model.

Mathematical and Computational Finance II (3 credits)

Prerequisite: MACF 401; MATH 473; STAT 461 previously or concurrently. This course is a continuation of MACF 401 and focuses on modelling and computational techniques beyond the binomial model. Simulation; Monte-Carlo methods in finance; option valuation; hedging; heat equation; finite difference techniques; stability and convergence; exotic derivatives; risk management; calibration and parameter estimation.

MACF 491 Topics in Mathematical and Computational Finance (3 credits)

Specific topics for this course, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.

Reading Course in Mathematical and Computational Finance (3 credits)

Specific topics for this course, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.

Mathematics and Statistics

Calculus and Linear Algebra (6 credits)

Prerequisite: Cegep Mathematics 105 or 201-NYC, 203 or 201-NYB. Functions; maxima and minima. Velocity and acceleration. Iterative solution of equations, parametric equation of curves. Integrals; change of variables, integration by parts, double integrals, numerical integration. Conic sections. Matrices, determinants, eigen-values, eigenvectors, system of equations. Series and their convergence. Introduction to vector space and complex numbers. Word problems.

NOTE: This course can be counted as an elective towards a 90-credit degree program, but must be taken before any other post-Cegep Mathematics course except for MAST 217, which may be taken concurrently. It must be taken, upon entry, by newly admitted students in the MATH/STAT Major who have less than 70% average in Cegep Mathematics courses.

Introduction to Mathematical Thinking (3 credits)

Prerequisite: MATH 203 and 204, or equivalent. This course aims to foster analytical thinking through a problem-solving approach. Topics include construction of proofs, number systems, ordinality and cardinality, role of examples and counter examples, role of generalizations and specializations; role of symbols, notations and definitions; styles of mathematical discourse.

NOTE: Students who have received credit for COMP 232 or COMP 238 may not take this course for credit.

NOTE: Students with more than 12 credits in post-Cegep Mathematics (excluding MAST 214) may not take this course for credit.

Multivariable Calculus I (3 credits) **MAST 218**

Prerequisite: MATH 204 and 205, or equivalent. Vector geometry; lines and planes; curves in Rn; vector functions; vector differential calculus; extrema and Lagrange multipliers. Introduction to multiple integrals and coordinate transformations. Problem solving with a symbolic computation system, e.g. MAPLE.

NOTE: Students who have received credit for MATH 264 may not take this course for credit.

MAST 219 Multivariable Calculus II (3 credits)

Prerequisite: MAST 218 or equivalent. Vector integral calculus; line and surface integrals; Green's, Stokes' and Gauss' theorems; coordinate transformations and Jacobians. Power series, applications. Problem solving with a symbolic computation system, e.g. MAPLE.

NOTE: Students who have received credit for MATH 265 may not take this course for credit.

MAST 221 Applied Probability (3 credits)

Prerequisite: MATH 204 and 205, or equivalent; MAST 218 or equivalent previously or concurrently. Counting rules, discrete probability distributions; random sampling; conditional probability; means and variances, normal and other continuous sampling distributions. Applications. Use of statistical software, e.g. MINITAB.

NOTE: Students who have received credit for STAT 249, COMP 233 or ECON 221 may not take this course for credit.

NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Please consult the Mathematics and Statistics undergraduate program advisor.

MAST 223 Introduction to Stochastic Methods of Operations Research (3 credits)

Prerequisite: MAST 221 or equivalent; MAST 219 or equivalent previously or concurrently. Markov chains; queuing theory; inventory theory; Markov decision processes; applications to reliablity.

NOTE: Students who have received credit for STAT 349 may not take this course for credit.

NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Please consult the Mathematics and Statistics undergraduate program advisor.

MAST 232 Mathematics with Computer Algebra (3 credits)

Prerequisite: MATH 204 and 205, or equivalent. An introduction to the use of a high-level mathematical programming language (MAPLE or MATHEMATICA) as a practical aid in doing mathematics. Most classes are given in an interactive way in the computer laboratory. The emphasis is on applications, not on general programming techniques or abstract structures. The aim is to arrive at a sufficient working familiarity with the computer algebra language to permit its regular use in subsequent studies and applications. The commands and online resources are introduced through a review of arithmetic, complex numbers, algebra, Euclidean geometry, trigonometry, coordinate systems and graphing, elementary functions and transformations, series, derivatives, integrals, vectors and matrices. There may be additional topics from domains such as number theory, differential equations, integral transforms, probability and statistics.

NOTE: Students who have received credit for MAST 332 or COMP 367 or 467 may not take this course for credit.

MAST 234 Linear Algebra and Applications I (3 credits)

Prerequisite: MATH 204 or equivalent. System of linear equations, matrix operations, echelon forms and LU-factorization; Rⁿ: subspaces, linear dependence, basis, dimension, matrix transformations; eigenvalues and eigenvectors in Rⁿ and applications (e.g. Markov chains, dynamical systems). A symbolic computation system, e.g. MAPLE, is extensively used. *NOTE: Students who have received credit for MATH 251 or ECON 325 may not take this course for credit.*

MAST 235 Linear Algebra and Applications II (3 credits)

Prerequisite: MAST 234 or equivalent. Rⁿ: Orthogonality, projections, Gram-Schmidt method and QR-factorization; applications to least square methods (data fitting, inconsistent systems). Symmetric matrices, principal axes theorem and applications. Special topics (e.g. coding theory, differential equations, error analysis). A symbolic computation system, e.g. MAPLE, is extensively used. *NOTE: Students who have received credit for MATH 252 may not take this course for credit.*

MAST 324 Introduction to Optimization (3 credits)

Prerequisite: MATH 205 or equivalent. Introduction to the theory of optimization; linear programming; the simplex method; duality and transportation problem. Introduction to graphs and networks; applications. Use of computing softwares.

NOTE: Students who have received credit for MAST 224 or MATH 361 may not take this course for credit.

MAST 330 Differential Equations (3 credits)

Prerequisite: MAST 219, 234 or equivalent. First order differential equations; second order differential equations; Laplace transform methods; mathematical models and numerical methods.

NOTE: Students who have received credit for MATH 370 may not take this course for credit.

MAST 331 Mathematical Modelling (3 credits)

Prerequisite: MAST 221, 324 previously or concurrently, 234 or equivalent. Introduction to mathematical modelling; predator-prey models in biology, game theory, decision analysis, stability theory; modelling electric circuits.

MAST 332 Techniques in Symbolic Computation (3 credits)

Prerequisite: MAST 217 or COMP 232 or equivalent; COMP 248 or equivalent; MAST 232 or permission of the Department. This course is an application-oriented introduction to symbolic computation, as it applies to algebra, number theory and combinatorics covering the following topics: capabilities of symbolic systems (e.g. MAPLE), modular methods, arithmetic mod p, arithmetic mod m, matrices mod p, Chinese remainder theorem, polynomial factorization mod p. Applications to coding theory and cryptography. Combinatorial algorithms.

NOTE: Students who have received credit for COMP 367 or 467 may not take this course for credit.

MAST 333 Applied Statistics (3 credits)

Prerequisite: MAST 221 or equivalent. Graphical and numerical descriptive methods; Estimation and hypothesis testing; linear regression and correlation; one way ANOVA; contingency and goodness of fit tests. Use of statistical software, e.g. MINITAB. NOTE: Students who have received credit for STAT 360, BIOL 322, COMM 215 or GEOG 362 may not take this course for credit. NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Please consult the Mathematics and Statistics undergraduate program advisor.

MAST 334 Numerical Analysis (3 credits)

Prerequisite: MAST 219 or equivalent; MAST 232 or equivalent; MAST 235 previously or concurrently. Introduction to computing softwares; numerical solution of non-linear equations; interpolations and approximations; quadrature and numerical integration. NOTE: Students who have received credit for MATH 354 may not take this course for credit.

MAST 335 Investment Mathematics (3 credits)

Prerequisite: MAST 218 or equivalent. Simple and compound interest; annuities; amortization and sinking funds; mortgage schemes; bonds and related securities; capital cost and depletion; spread-sheet implementation.

NOTE: Students who have received credit for MATH 326 may not take this course for credit.

NOTE: Only three credits will be awarded from MAST 335; ACTU 256.

MAST 397 Topics in Mathematics and Statistics (3 credits)

MAST 398 Reading Course in Mathematics and Statistics (3 credits)

Specific topics for these courses, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.

Mathematics

MATH 200 Fundamental Concepts of Algebra (3 credits)

This course is designed to give students the background necessary for MATH 201. Some previous exposure to algebra is assumed. Sets, algebraic techniques, inequalities, graphs of equations. Lectures and tutorials.

NOTE: Students who have received credit or exemption for a course at the level of MATH 201 or above may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 201 Elementary Functions (3 credits)

Sets, inequalities, graphs of functions, and relations. Trigonometric, exponential, and logarithmic functions. Lectures and tutorials. NOTE: Students who have received credit or exemption for MATH 203 or equivalent, or for a course having MATH 203 or equivalent in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 202 College Algebra (3 credits)

Prerequisite: MATH 201 or equivalent. Progressions, combinations, permutations, binomial theorem, mathematical induction, inequalities, polynomials, cartesian and polar forms of complex numbers, conics. Lectures and tutorials.

NOTE: Students who have received credit or an exemption for a course at the level of ACTU 256 or above; MAST 218 or above; MATH 251 or above; STAT 249 or above; or for a course having any of these courses in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 203 Differential and Integral Calculus I (3 credits)

Prerequisite: MATH 201 or equivalent. Functional notation. Differentiation of polynomials. The power, product, quotient, and chain rules. Differentiation of elementary functions. Implicit differentiation. Higher derivatives. Maxima and minima. Applications: tangents to plane curves, graphing, related rates. Approximations using the differential. Antiderivatives, definite integrals, area. Lectures and tutorials.

NOTE: Students who have received credit or an exemption for a course at the level of ACTU 256 or above; MAST 218 or above; MATH 251 or above; STAT 249 or above; or for a course having any of these courses in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 204 Vectors and Matrices (3 credits)

Prerequisite: MATH 201 or equivalent. Algebra and geometry of vectors, dot and cross products, lines and planes. System of equations, operations on matrices, rank, inverse, quadratic form, and rotation of axes. Lectures and tutorials.

NOTE: Students who have received credit or an exemption for a course at the level of ACTU 256 or above; MAST 218 or above;

MATH 251 or above; STAT 249 or above; or for a course having any of these courses in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 205 Differential and Integral Calculus II (3 credits)

Prerequisite: MATH 203 or equivalent. Techniques of integration: substitutions, integration by parts, partial fractions. Improper integrals. Physical applications of the definite integral. Infinite series: tests for convergence. Power series, Taylor's theorem. Lectures and tutorials.

NOTE: Students who have received credit or an exemption for a course at the level of ACTU 256 or above; MAST 218 or above; MATH 251 or above; STAT 249 or above; or for a course having any of these courses in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 206 Algebra and Functions (3 credits)

Coordinate systems. Radicals and distance formula. Polynomials, factoring, and graphing. Relations and functions. Linear and quadratic functions, equations, and systems. Exponents, exponential and logarithmic functions and equations. Lectures and tutorials. NOTE: Students who have received credit or exemption for a course at the level of MATH 201 or above may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 208 Fundamental Mathematics I (3 credits)

Prerequisite: MATH 206 or equivalent. This course is a prerequisite course for John Molson School of Business students*. Matrices, Gaussian elimination, input-output analysis, progressions, compound interest, annuities, permutations and combinations, probability, binomial theorem, exponential and logarithmic functions, inequalities, linear programming. Lectures and tutorials.

*NOTE: See §14.2.2 (Mature Entry) and 61.20 (Extended Credit).

NOTE: Students who have received credit or an exemption for a course at the level of ACTU 256 or above; MAST 218 or above; MATH 251 or above; STAT 249 or above; or for a course having any of these courses in its sequence of prerequisites, may not take this course for credit.

NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 209 Fundamental Mathematics II (3 credits)

Prerequisite: MATH 206 or equivalent. This course is a prerequisite course for John Molson School of Business students*. Limits; differentiation of rational, exponential, and logarithmic functions; theory of maxima and minima; integration. Lectures and tutorials. *NOTE: See §14.2.2 (Mature Entry) and 61.20 (Extended Credit).

NOTE: Students who have received credit or exemption for MATH 203 or equivalent may not take this course for credit. NOTE: Students in programs leading to the BSc degree or the BA programs in Mathematics and Statistics may not take this course for credit to be applied to their program of concentration.

MATH 212 The Fascinating World of Numbers (3 credits)

This course deals with a blend of fascinating mathematical themes in various contexts: historical, cultural, and practical. It is intended for non-mathematics students. One of the aims of the course is to demonstrate the presence of mathematics and mathematical ideas in many aspects of modern life. At a deeper level, it is also intended to explain what mathematics is all about and why some easily stated assertions, such as Fermat's last theorem, are so difficult to prove. Students who complete the course successfully should have enough understanding and knowledge of fundamental ideas and techniques of mathematics to appreciate its power, its beauty, and its relevance in so many different fields such as architecture, art, commerce, engineering, music, and all of the sciences.

NOTE: Students who have received credit for this topic under a MATH 298 number may not take this course for credit.

NOTE: Students enrolled in a Mathematics and Statistics program and students who have taken mathematics beyond the pre-calculus level may not take this course for credit.

MATH 215 Great Ideas in Mathematics (3 credits)

Mathematics is used to unravel the secrets of nature. This course introduces students to the world of mathematical ideas and mathematical thinking. Without being overly technical, that is, without requiring any formal background from the student other than high school mathematics, the course delves into some of the great ideas of mathematics. The topics discussed range from the geometric results of the Ancient Greeks to the notion of infinity to more modern developments.

NOTE: This course is designed as a suitable elective for students following an undergraduate program. It has no formal prerequisites and will not qualify students to enrol for any other Mathematics course, and cannot be used to satisfy a Mathematics requirement in any BSc or BA program.

NOTE: Students who have received credit for INTE 215 may not take this course for credit.

SELECTED TOPICS IN MATHEMATICS AND STATISTICS

The 200-level Topics and Reading courses (MATH 217; MATH 218) are designed as elective courses for students who are not registered in a Mathematics and Statistics program. The particular topic varies from one term to the next and the material is dealt with in a manner appropriate for students who have no background in university-level mathematics.

NOTE: Students registered in a Mathematics and Statistics program may not take these courses for credit.

MATH 217 Topics in Mathematics and Statistics (3 credits)

MATH 218 Reading Course in Mathematics and Statistics (3 credits)

MATH 220 Mathematical Methods in Chemistry (3 credits)

Prerequisite: Cegep Mathematics 203 or 201-NYB or MATH 205. Partial derivatives. First-order differential equations (first- and second-order chemical reactions). Hermite, Laguerre, and Legendre equations. Solutions by power series. Eigenfunctions and eigenvalues, Sturm-Liouville theory.

MATH 251 Linear Algebra I (3 credits)

Prerequisite: MATH 204 and 205 or equivalent. Matrices and linear equations: vector spaces; bases, dimension and rank; linear mappings and algebra of linear operators; matrix representation of linear operators; determinants; eigenvalues and eigenvectors; diagonalization.

NOTE: Students who have received credit for MAST 234 or ECON 325 may not take this course for credit.

MATH 252 Linear Algebra II (3 credits)

Prerequisite: MATH 251 or equivalent. Characteristic and minimum polynomials; invariant subspaces, invariant direct sums; nilpotent operators, Jordan canonical form; cyclic subspaces; rational canonical form; bilinear and quadratic forms; inner product; orthogonality; adjoint operators and orthogonal operators.

NOTE: Students who have received credit for MAST 235 may not take this course for credit.

MATH 264 Advanced Calculus I (3 credits)

Prerequisite: MATH 204 and 205 or equivalent. Introduction to limits and continuity in Rn. Multivariate calculus: the derivative as a linear approximation; matrix representation of derivatives; tangent spaces; gradients, extrema, including Lagrange multipliers, Taylor's formula and the classification of critical points.

NOTE: Students who have received credit for MAST 218 may not take this course for credit.

MATH 265 Advanced Calculus II (3 credits)

Prerequisite: MATH 264 or equivalent. Implicit functions and the implicit function theorem. Multiple integrals and change of variables. Curves, surfaces and vector calculus.

NOTE: Students who have received credit for MAST 219 may not take this course for credit.

Combinatorics (3 credits)

Prerequisite: 18 credits in post-Cegep Mathematics. General principles of counting, permutations, combinations, identities, partitions, generating functions, Fibonacci numbers, Stirling numbers, Catalan numbers, principle of inclusion-exclusion. Graphs, subgraphs, isomorphism, Euler graphs, Hamilton paths and cycles, planar graphs, Kuratowski's Theorem, trees, colouring, 5-colour theorem, matching, Hall's theorem.

NOTE: Students who have received credit for COMP 339 may not take this course for credit.

Linear Algebra III (3 credits)

Prerequisite: MATH 252. Matrices, linear transformations, determinants, metric concepts, inner-product spaces, dual spaces, spectral theorem, bilinear and quadratic forms, canonical forms for linear transformation, matrix functions, selected topics.

Numerical Analysis (3 credits)

Prerequisite: MATH 265 or equivalent: MATH 252 or equivalent, previously or concurrently. Error analysis in numerical algorithms; solution of non-linear equations; fixed point iterations, rate of convergence. Interpolations and approximations, Legendre polynomials. Numerical integration and quadrature.

NOTE: Students who have received credit for MAST 334 may not take this course for credit.

Operations Research (3 credits)

Prerequisite: MATH 251 or equivalent. Introduction to the theory of optimization; linear programming, simplex method; revised simplex method; transport and assignment problems; integer programming; introduction to graphs and networks. NOTE: Students who have received credit for MAST 224 or 324 may not take this course for credit.

Analysis I (3 credits)

Prerequisite: 12 credits in post-Cegep Mathematics or permission of the Department. Mathematical rigour: proofs and counterexamples; quantifiers; number systems; cardinality, decimal representation, density of the rationals, least upper bound. Sequences and series; review of functions, limits and continuity.

Analysis II (3 credits) **MATH 365**

Prerequisite: MATH 364 or equivalent. Connectedness and compactness in the reals. Intermediate value theorem; extreme values for continuous functions. Differential and integral calculus; fundamental theorem of calculus; power series.

MATH 366 Complex Analysis I (3 credits)

Prerequisite: MATH 265 or equivalent. Algebra and geometry of complex numbers, linear transformations, analytic functions, Laurent's series, calculus of residues, special functions.

MATH 369 Abstract Algebra I (3 credits)

Prerequisite: 12 credits in post-Cegep Mathematics or permission of the Department. Introduction to the ring of integers and the integers modulo N. Groups: definitions and examples; sub-groups, quotients and homomorphisms (including Lagrange's theorem, Cayley's theorem and the isomorphism theorems). Introduction to the Cauchy and Sylow theorems and applications.

MATH 370 Ordinary Differential Equations (3 credits)

Prerequisite: MATH 265, 251 or equivalent. Separable equations, exact equations, integrating factors, force fields, first order linear equations, input-output concept, second order equations, Sturm-Liouville problems, applications, series solutions, reduction of order, variation of parameters, nth-order linear equations with constant coefficients, Laplace transforms, block diagrams, and signal-flow graphs.

NOTE: Students who have received credit for MAST 330 may not take this course for credit.

MATH 387 Mathematics Lab III (2 credits)

This lab will demonstrate the use of MAPLE software for Calculus, Linear Algebra, and Statistics.

NOTE: Students who have received credit for MATH 232 may not take this course for credit.

MATH 392 Elementary Number Theory (3 credits)

Prerequisite: 18 credits in post-Cegep Mathematics. Number systems, division and factorization, number-theoretic functions, congruences, algebraic congruences and primitive roots, quadratic residues, diophantine equations.

MATH 397 History of Mathematics (3 credits)

Prerequisite: MATH 252 or permission of the Department; MATH 365. Early mathematics, Greek mathematics, European mathematics in the Middle Ages, the origin and development of analytic geometry and calculus, mathematics as free creation, the generality of mathematics in the 20th century.

MATH 433 Calculus of Variations (3 credits)

Prerequisite: MATH 265, 365, 370 or equivalent. Nature of problems, weak variations, the first variation, Euler's equation. The second variation, Jacobi's equation, Legendre's test, conjugate points. Relative maxima and minima, iso-perimetrical problems. Integrals with variable end points. Applications to problems in pure and applied mathematics; the principle of least action. Strong variations, the Weierstrass E-function.

MATH 464 Real Analysis (3 credits)

Prerequisite: MATH 265, 365 or equivalent. Metric spaces; function spaces; compactness, completeness, fixed-point theorems, Ascoli-Arzela theorem, Weierstrass approximation theorem.

MATH 466 Complex Analysis II (3 credits)

Prerequisite: MATH 265, 365, 366 or equivalent. Cauchy's theorem, singularities, maximum modulus principle, uniqueness theorem, normal families, Riemann mapping theorem.

MATH 467 *Measure Theory* (3 credits)

Prerequisite: MATH 265, 365; MATH 464 previously or concurrently, or equivalent. Lebesque measure and integration on the real line, convergence theorems, absolute continuity, completeness of L²[0,1].

MATH 470 Abstract Algebra II (3 credits)

Prerequisite: MATH 369 or equivalent. Group action and proof of the Sylow theorems. Introduction to rings, ideals, euclidean domains, principal ideal domains and unique factorization domains; polynomial rings.

MATH 471 Abstract Algebra III (3 credits)

Prerequisite: MATH 470 or equivalent. Rings and modules; structure theorem of modules over principal ideal domains. Noetherian rings and modules (including Hilbert basis theorem for rings and modules). Hilbert's Nullstellensatz.

NOTE: Students who have received credit for MATH 491 may not take this course for credit.

NOTE. Students who have received credit for WATT 431 may not take this course for cre

Prerequisite: MATH 470 or equivalent. Elements of field and Galois theory, including straight-edge-and-compass construction and unsolvability of equations of fifth degree by radicals.

NOTE: Students who have received credit for MATH 492 may not take this course for credit.

MATH 473 Partial Differential Equations (3 credits)

Abstract Algebra IV (3 credits)

Prerequisite: MATH 370 or equivalent. Canonical forms for second order linear equations with constant coefficients, classification of linear second order equations, method of separation of variables, first order PDE's, method of characteristics. Non-linear first order equations, complete integrals, Cauchy conditions, Cauchy-Kowalewski theorem, Fourier and Laplace transforms, Green's functions, integral representations, introduction to non-linear PDE's.

NOTE: Students who have received credit for MATH 371 may not take this course for credit.

MATH 474 Linear and Non-Linear Dynamical Systems (3 credits)

Prerequisite: MATH 265, 365 or equivalent, or permission of the Department. Systems of linear differential equations; fundamental matrices; non-homogeneous linear systems; non-linear systems; solutions and trajectories; the phase plane; stability concepts; Liapounov's second method; periodic solutions and limit cycles; introduction to boundary-value problems and Sturm-Liouville theory. NOTE: Students who have received credit for MATH 373 may not take this course for credit.

MATH 475 Discrete Dynamical Systems, Chaos and Fractals (3 credits)

Prerequisite: MATH 265, 365 or equivalent, or permission of the Department. Introduction to discrete dynamical modelling; periodic points; bifurcation; period three points; symbolic dynamics; chaos; transitivity; conjugacy; complex behaviour; introduction to fractals; computer simulations.

NOTE: Students who have received credit for MATH 379 may not take this course for credit.

MATH 478 Non-Linear Programming (3 credits)

Prerequisite: MATH 361 or permission of the Department. Classical methods of optimization, Lagrange multipliers, Kuhn-Tucker conditions; line search methods, quadratic programming, gradient methods, introduction to dynamic programming.

NOTE: Students who have received credit for MATH 436 may not take this course for credit.

MATH 479 Convex and Non-Linear Analysis (3 credits)

Prerequisite: MATH 365 or permission of the Department. Support and separation of convex sets, extreme point characterizations, convex and dual cones, Farkas' theorem; minimax theorem of Game Theory, Legendre-Fenchel conjugate, infimal convolution, subgradient calculus; Lagrangians, necessary and sufficient conditions for optimality in constrained minimization; the dual problem.

MATH 480 Geometry and Topology (3 credits)

Prerequisite: MATH 252, 365, 369. This is an introductory course in the geometric topology and differential geometry of surfaces. The topics covered will be selected from curvature, Theorema Egregium, Gauss-Bonnet theorem, Euler characteristic, cohomology, homotopy groups, the applications of ideas and techniques from geometry and topology in knot or graph theory and map colourings. NOTE: Students who have received credit for MATH 380 may not take this course for credit.

- MATH 494 Topics in Pure and Applied Mathematics (3 credits)
- MATH 495 Reading Course in Pure and Applied Mathematics (3 credits)
- MATH 496 Honours Project in Pure and Applied Mathematics (6 credits)

Specific topics for these courses, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.

Statistics

STAT 249 Probability I (3 credits)

Prerequisite: MATH 264 or equivalent previously or concurrently. Axiomatic approach to probability; combinatorial probability; discrete and continuous distributions; expectation; conditional expectation; random sampling and sampling distributions.

NOTE: Students who have received credit for COMP 233 or ECON 221 may not take this course for credit.

NOTE: Students who have received credit for MAST 221 may take STAT 249 for credit only with prior permission of the Department.

NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Please consult a Mathematics and Statistics undergraduate program advisor.

STAT 250 Statistics (3 credits)

Prerequisite: STAT 249 or equivalent or permission of the Department; MATH 265 or equivalent previously or concurrently. Point and interval estimation; hypothesis testing; Neyman Pearson Lemma and likelihood ratio tests; introduction to correlation and regression.

NOTE: Students enrolled in a Mathematics and Statistics program who take probability/statistics courses in other departments may not receive credit for this course. Please consult a Mathematics and Statistics undergraduate program advisor.

STAT 287 Statistics Lab I (1 credit)

This lab is associated with STAT 249 and 250 and features problem-solving sessions for the probability examination of the Society of Actuaries and the Casualty Actuarial Society.

NOTE: Students who have received credit for MATH 329 may not take this course for credit.

STAT 342 Industrial Statistics (3 credits)

Prerequisite: STAT 250 or MAST 333. Concepts of statistical quality control; X-bar, R, P, and C control charts, acceptance sampling, sampling inspection, continuous sampling plans.

NOTE: Students who have received credit for MATH 342 may not take this course for credit.

STAT 343 Sample Survey Theory and Applications (3 credits)

Prerequisite: STAT 250 or MAST 333. Basic sampling designs and estimators; simple random sampling, stratified, cluster and systematic sampling. Sampling with unequal probabilities; ratio and regression methods of estimation.

NOTE: Students who have received credit for MATH 343 may not take this course for credit.

STAT 347 Introduction to Non-Parametric Statistics (3 credits)

Prerequisite: STAT 250 or MAST 333. Theory of rank tests, sign test, Mann-Whitney and Wilcoxon one-sample and two-sample tests, Kruskal-Wallis test, goodness of fit tests, Kolmogorov-Smirnov test, Pearson chi-square test, rank correlation and Kendall's tau.

NOTE: Students who have received credit for MATH 347 may not take this course for credit.

STAT 349 Probability II (3 credits)

Prerequisite: STAT 249 or equivalent. Markov decision process and applications. Poisson process, queuing theory, inventory theory; applications.

NOTE: Students who have received credit for MAST 223 may not take this course for credit.

STAT 360 Linear Models (3 credits)

Prerequisite: STAT 250 or equivalent. Least-squares estimators and their properties. General linear model with full rank. Analysis of residuals; adequacy of model, lack of fit test, weighted least squares; stepwise regression, Durbin-Watson statistic; one way and two way analysis of variance.

NOTE: Students who have received credit for MATH 348, ECON 222 or PSYC 316 may not take this course for credit.

STAT 388 Statistics Lab II (2 credits)

This lab will use various softwares such as SYSTAT, SAS, SPLUS, MINITAB for data analysis.

NOTE: Students who have received credit for MATH 232 may not take this course for credit.

STAT 449 Advanced Probability (3 credits)

Prerequisite: STAT 250, 349. Central limit theorems and law of large numbers, convergence of random variables, characteristic function, moment generating function, probability generating functions, random walk and reflection principle.

NOTE: Students who have received credit for MATH 451 may not take this course for credit.

STAT 450 *Mathematical Statistics* (3 credits)

Prerequisite: STAT 250, 349 previously or concurrently, or permission of the Department. Derivation of standard sampling distributions; distribution of order-statistics; estimation, properties of estimators; Rao-Cramer inequality, Rao-Blackwell theorem, maximum likelihood and method of moments estimation, Neyman-Pearson theory, likelihood ratio tests and their properties. NOTE: Students who have received credit for MATH 454 may not take this course for credit.

STAT 452 Intoduction to Stochastic Processes (3 credits)

Prerequisite: STAT 449. Continuous stochastic processes. Poisson processes, continuous time Markov process, queuing models, birth and death processes, renewal theory.

NOTE: Students who have received credit for MATH 353 may not take this course for credit.

STAT 460 Time Series and Forecasting (3 credits)

Prerequisite: STAT 360. Time series, forecasting by trend and irregular components (using multiple regression analysis and exponential smoothing); forecasting seasonal time series, additive and multiplicative decomposition methods, Box-Jenkins methodology, moving average, autoregressive and mixed models.

NOTE: Students who have received credit for MATH 443 may not take this course for credit.

STAT 461 Statistical Simulation (3 credits)

Prerequisite: STAT 349. Simulation and Monte-Carlo techniques; selected topics in operations research.

NOTE: Students who have received credit for MATH 437 may not take this course for credit.

STAT 465 *Multivariate Statistics* (3 credits)

Prerequisite: MATH 252; STAT 360 or equivalent. Multivariate normal distribution; estimation and testing of hypothesis about mean vector; multiple and partial correlation; MANOVA; principal components analysis.

STAT 468 Design of Experiments (3 credits)

Prerequisite: STAT 360. Construction and analysis of standard designs, including balanced designs; block designs; orthogonal designs; response surface designs.

STAT 480 Statistical Data Analysis (3 credits)

Prerequisite: STAT 360 or permission of the Department. The analysis of real life data sets using statistical software packages in SAS or R. Topics involve techniques from generalized linear models, model selection, log-linear models for categorical data, logistic regression, survival models.

STAT 497 Topics in Statistics (3 credits)

STAT 498 Reading Course in Statistics (3 credits)

STAT 499 Honours Project in Statistics (6 credits)

Specific topics for these courses, and prerequisites relevant in each case, are stated in the Undergraduate Class Schedule.