Department of Mathematics & Statistics
Concordia University

> **MAST 235**
> Linear Algebra and Applications II
> *Winter 2022*

Preface:　　T**his course will be taught and all assessments will be done completely in person. We will try to stream the lecture via zoom as a convenience.**

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Office hours:　　Tuesdays, 16:00-17:30. Available In-Person and Online.

Textbook:　　*Linear Algebra, Theory and Applications*, by Ward Cheney & David Kincaid, 2nd Edition, Publisher: Jones & Bartlett.
The textbook will be available at:
https://www.bkstr.com/concordiastore/home
Note: Students should order textbooks as early as possible, especially for printed versions in case books are backordered or there are any shipping delays.

Prerequisites:　　Math 234 or equivalent is a prerequisite for this course.

Accessibility:　　This classroom strives to create a safe and accessible place for all students. Please email me confidentially if any safety concerns arise. All abilities and identities are encouraged and valued. Students in this classroom agree to the Behavioural Integrity policies of Concordia. Discrimination of any kind will not be tolerated, and will be delt with according to the *Code of Rights and Responsibilities*. I encourage students to connect with the Access Centre (ACSD: concordia.ca/students/accessibility), or email me privately for any other accessibility concerns, comments, or questions.

Objectives:　　This course will focus on applications of the theory studied in Math 234 and its further development. There are two major concepts, on which this course is based: (*a*) *Linear Systems & Operators,* and their applications such as Economic Models or Dynamical systems, where also the *Eigentheory* is applied**,** and (*b*) *Inner Product Spaces*, leading to applications like Orthogonalization, Least Square solutions, and SVD diagonalization. The general objective of the course is to master your understanding and skills in these key concepts of Linear Algebra that will be critical for further Algebra courses in your curriculum.

Pedagogy: Classes and all work in this course use the *MAPLE* as the tool, ***not*** object of study. The structure of classes includes lecture time on the theory, alternating with problem solving tasks done by students individually. Mathematical issues that arise during problem solving are discussed in class.

Software: *MAPLE (version 17 or higher)* is mandatory for this course. The Waterloo's *Maplesoft* is making MAPLE ("Student's edition", quite sufficient for the course) available to Concordia students at a special price. In this course the software is only used as a computational *tool*, ***not* as an object of study** in itself.
All the tests, the final examination and the assignments are done using *MAPLE.*

Assignments: Assignment are given and submitted online through Moodle. Late assignments **will not** be accepted. Assignments contribute 10% to your final grade (see the Grading Scheme). Working regularly on the assignments, as well as class attendance and working on the problems in the class, is essential for success in this course.

Midterm Test: There will be **one midterm test** based on the material of Lectures 1-6 which will contribute up to 30% to your final grade (see the Grading Scheme).

    NOTE: It is the Department's policy that tests missed for any reason, **including illness,** cannot be made up. If you missed the midterm because of illness (Short-Term Absence form or valid medical note required); the final exam can count for 85% of your final grade, and 15% will be contributed by the assignments and quizzes (see the **Grading Scheme**). *Please talk to me as soon as possible for any absence for any organization concerns, and for assistance in preparation therein.*

Final Exam: The Final Examination will be 2 hours long and will be written using Maple. Students are responsible for finding out the date and time of the final exam once the schedule is posted by the Examinations Office. Conflicts or problems with the schedule of the final exam must be reported directly to the Examinations Office, *not* to the Instructor. **Students are to be available until the end of the final exam period.** Conflicts due to travel plans **will not** be accommodated.

    NOTE: There are **no supplemental exams** for this course.

Grading Scheme: The final grade will be based on the higher of (*a*) and (*b*) below:

    (a) 10% for the assignments.
        5% for the best of 2 quizzes (written in class)
        30% for the midterm test
        55% for the final examination.

(b) 10% for the assignments
5% for the best of 2 quizzes
15% for the midterm test.
70% for the final examination.

If the grading scheme for this course includes graded assignments, a reasonable and representative subset of each assignment may be graded. Students will not be told in advance which subset of the assigned problems will be marked and should therefore attempt all assigned problems.

IMPORTANT: **NOTE** that there is **NO "100% FINAL EXAM"** option in this course.
The term work contributes at least 30% to the final grade. Therefore, active participation in classes and continuous work on the course material **during** the semester is essential for success in this course. *If you have any concerns about your participation, please email me as soon as possible to discuss options and make a participation plan moving forward.*

CONTENTS

Week	Textbook	Topics	Recommended Problems
1	**Sections 1.3, 2.3, 2.3** LINEAR SYSTEMS: NETWORKS	• Systems of Linear Equations (Sec.1.3), and Linear Mapping (Sec. 2.3): an ***Overview*** • Network Problems (Sec. 2.1, pp.99-100)	G.Ex. 1.3 # 5, 13, 25, 47, 63 G.Ex 2.3: 1, 21,39 G.Ex. 2.1: # 6, 30, 42

2	**Sections 2.4, 6.1, 8.3** LINEAR SYSTEMS: ECONOMIC MODELS	• Economic Models (pp.177-180, 385-388, 548-551)
		    ○ Theorem 14 (p.177), Theorem 4 (p.549)
		    ○ Closed Leontieff Model
		    ○ Open Leontieff Model
			Comp.Ex. 2.4 # 1, 2, 3.

3	**Sections 4.1, 4.2/3.2** *DETERMINANTS* PROPERTIES & APPLICATIONS	• Determinants: overview and basic properties
		• Applications:
		    ○ Interpolation Problem (pp. 231-233).
		    ○ Vandermonde matrix (pp. 279-280).
			Gen.Ex. 3.2 # 19, 69, 73, 75 Gen.Ex. 4.2 # 7, 27, 33, 45

4	**QUIZ 1**
	
	Sections 6.1, 2.2 EIGENTHEORY AND DYNAMICAL SYSTEMS
		QUIZ 1 (based on Weeks 1-3)
		
		• Diagonalization and powers of a matrix (pp. 371-380)
		• Application: Dynamical Systems (pp. 380-385).
		    ○ Systems with Real eigenvalues
		    ○ The Predator-Prey simulation (pp. 125-126)
		    ○ Systems with Complex eigenvalues
			Comp.Ex. 2.2 # 1, 2 Comp. Ex. 6.1 # 1, 2, 7, 13

5	**Section 7.1** INNER PRODUCT SPACES	• The concept of inner product space over real and complex numbers (pp. 403-408)
		    ○ Definition
		    ○ Properties
		• The norm in an inner product space (Theorem 1, p. 409)
		    ○ The Cauchy-Schwartz Inequality
		    ○ The Triangle Inequality
		    ○ The Pythagorean Theorem
			G.Ex. 7.1 # 5, 15, 17, 19, 35, 67, 71, 77

6	**Section 7.1** ORTHOGONAL PROJECTION	• Orthogonality of vectors (Theorems 2, 4, pp.411, 412). • Orthogonal Projection (Theorem 5, p.413) • Angle (p.415). • Orthogonal complements (pp. 416-419)	G.Ex. 7.1 # 3, 11, 13, 51, 57

7	**Section 7.1** UNITARY MATRICES	• Orthonormal bases & subspaces , unitary matrices (pp. 419-423, Theorems 9-15) **MIDTERM TEST** based on the material of the **Weeks 1-6.**	G.Ex. 7.1 # 27, 31, 39
8	**Section 7.2** APPLICATIONS OF ORTHOGONALITY	• The Gram-Schmidt Process (pp.432-439), Theorems 1 & 2.	G.Ex. 7.2 # 7, 9, 23, 25, 27, 29
9	**Section 7.2** LEAST SQUARES SOLUTION	• Normal Equations and the Least Squares solutions to an inconsistent system; Theorem 3. (pp. 439-445) • Geometrical interpretation of the least-square approximations in terms of distance, projections and hyperplanes.	G.Ex. 7.2 # 1, 3, 5, 11, 13
10	**Section 8.1** HERMITIAN MATRICES & SELF-ADJOINT OPERATORS **QUIZ 2**	• Hermitian matrices, symmetric matrices • Self-adjoint mappings • Eigenvalues of Hermitian and symmetric matrices **QUIZ 2** (based on Weeks 7, 8, 9)	G.Ex. 8.1 # 1, 3, 9, 17, 25, 33
11	**Section 8.1** SPECTRAL THEOREM & APPLICATIONS. QUADRATIC FORMS	• Spectral Theorem (Theorems 6 and 7) • Cayley-Hamilton Theorem and applications (pp. 470-472) • Quadratic Forms	G.Ex. 8.1 # 27, 29, 39, 47 G.Ex. 8.1 # 11, 61
12	**Sections 8.2** SVD FACTORIZTION	• Singular Value Decomposition (pp. 501-502)	
13	**REVIEW**	**Review classes**	

Academic Integrity and the Academic Code of Conduct

This course is governed by Concordia University's policies on Academic Integrity and the Academic Code of Conduct as set forth in the Undergraduate Calendar and the Graduate Calendar. Students are expected to familiarize themselves with these policies and conduct themselves accordingly. "Concordia University has several resources available to students to better understand and uphold academic integrity. Concordia's website on academic integrity can be found at the following address, which also includes links to each Faculty and the School of Graduate Studies: concordia.ca/students/academic-integrity." *[Undergraduate Calendar, Sec 17.10.2]*

Behaviour

All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Concordia students are subject to the Code of Rights and Responsibilities which applies both when students are physically and virtually engaged in any University activity, including classes, seminars, meetings, etc. Students engaged in University activities must respect this Code when engaging with any members of the Concordia community, including faculty, staff, and students, whether such interactions are verbal or in writing, face to face or online/virtual. Failing to comply with the Code may result in charges and sanctions, as outlined in the Code.

Use of Zoom
Note: Zoom is included as an institutionally-approved technology. This means we have been assured of the privacy protections needed to use freely within the classroom)

Zoom will be used in this course to facilitate learning at a distance. It may be used to record some or all of the lectures and/or other activities in this course. If you wish to ensure that your image is not recorded, speak to your instructor as soon as possible.

Also, please note that you may not share recordings of your classes and that the instructor will only share class recordings for the purpose of course delivery and development. Any other sharing may be in violation of the law and applicable University policies, and may be subject to penalties.

Intellectual Property

Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member. Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the Academic Code of Conduct and/or the Code of Rights and Responsibilities. As specified in the Policy on Intellectual Property, the University does not claim any ownership of or interest in any student IP. All university members retain copyright over their work.

Extraordinary circumstances

In the event of extraordinary circumstances and pursuant to the Academic Regulations the University may modify the delivery, content, structure, forum, location and/or evaluation scheme. In the event of such extraordinary circumstances, students will be informed of the change.