A primer in Random Matrices

Marco Bertola

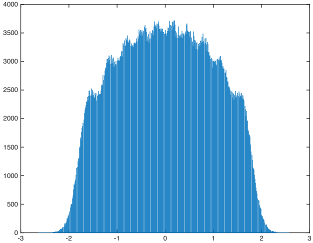
Mathematics and Statistics, *Concordia University*, Montréal, Canada

Math 680B (MAST 856B): 2017

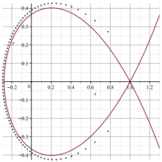
Short description

Random Matrices were introduced by Wigner in the '50 to model the energy levels of absorption/emission of heavy nuclei. They have found a wide range of applications that include; orthogonal polynomials, cellular networks, number theory, enumerative geometry.

The course is an introduction to the problems and techniques; an example of question that we will answer is how to explain the following histogram of the eigenvalue distribution of 100000 Hermitean 10×10 random matrices with independent normally distributed entries: can we explain the clearly visible "ripples"? (yes)



Or for example explain the **Szegö** curve, i.e. why the roots of the Taylor polynomials $T_N(z) = \sum_{j=0}^{N} \frac{z^n}{n!}$ of e^z tend to the curve $|ze^{1-z}| = 1$. (up to a rescaling; here the zeroes of $T_{80}(80z)$)



Oddly, the answers to both questions use the same techniques!

The main references from which material will be lifted are [2], [1] and the notes [3]. The following is a rough list of topics, to be added (or subtracted!) as time permits.

1 Random Matrices

- Examples: introduction of GUE, GOE, GSE and β–ensembles
- Spectral asymptotics: Wigner Semicircle Law
- GUE's and Unitary ensembles
  - Weyl integration formula
  - Dyson theorem
  - Orthogonal Polynomials techniques
- Fluctuations of the largest eigenvalue
  - Determinantal Point processes and Fredholm determinants
  - Tracy–Widom distribution: statement and proof
- Universality of local asymptotic behaviour: Sine and Airy Kernels
- Multi-Matrix models
  - Determinantal Random Point Fields and Processes
  - Computations of Gap probabilities

Evaluation

There will be two take home assignments (50%) and either a take-home final or a presentation on a topic to be decided.

Greg W. Anderson, Alice Guionnet, and Ofer Zeitouni.
An introduction to random matrices, volume 118 of *Cambridge Studies in Advanced Mathematics*.
Cambridge University Press, Cambridge, 2010.

P. A. Deift.
Orthogonal polynomials and random matrices: a Riemann-Hilbert approach, volume 3 of *Courant Lecture Notes in Mathematics*.
New York University Courant Institute of Mathematical Sciences, New York, 1999.

Slava Kargin Elena Yudovina.
Random matrix theory.
2011.