A primer in Random Matrices

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Random Matrices were introduced by Wigner in the '50 to model the energy levels of absorption/emission of heavy nuclei. They have found a wide range of applications that include; orthogonal polynomials, cellular networks, number theory, enumerative geometry.

The course is an introduction to the problems and techniques; an example of question that we will answer is how to explain the following histogram of the eigenvalue distribution of 100000 Hermitean $10 \times 10$ random matrices with independent normally distributed entries: can we explain the clearly visible “ripples”? (yes)

Or for example explain the Szegö curve, i.e. why the roots of the Taylor polynomials $T_N(z) = \sum_{j=0}^{N} \frac{z^n}{n!}$ of $e^z$ tend to the curve $|ze^{1-z}| = 1$. (up to a rescaling; here the zeroes of $T_{80}(80z)$)

Oddly, the answers to both questions use the same techniques!
The main references from which material will be lifted are [2], [1] and the notes [3].
The following is a rough list of topics, to be added (or subtracted!) as time permits.

1 Random Matrices

- Examples: introduction of GUE, GOE, GSE and $\beta$–ensembles
- Spectral asymptotics: Wigner Semicircle Law
- GUE’s and Unitary ensembles
  - Weyl integration formula
  - Dyson theorem
  - Orthogonal Polynomials techniques
- Fluctuations of the largest eigenvalue
  - Determinantal Point processes and Fredholm determinants
  - Tracy–Widom distribution: statement and proof
- Universality of local asymptotic behaviour: Sine and Airy Kernels
- Multi-Matrix models
  - Determinantal Random Point Fields and Processes
  - Computations of Gap probabilities

EVALUATION

There will be two take home assignments (50%) and either a take-home final or a presentation on a topic to be decided.

Greg W. Anderson, Alice Guionnet, and Ofer Zeitouni.
P. A. Deift.  

Slava Kargin Elena Yudovina.  
Random matrix theory.  
2011.