Environmental taxation in a federal system of government with equalization payments

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Abstract

We evaluate the potential for a double dividend of environmental taxation in a federation. We study the two-region framework of Boadway and Flatters (1982) with fully mobile citizens. Regions are unequally endowed in non-renewable resources, which extraction generates a global externality. The presence of a fixed stock of resources creates unequal rents across regions, thus calling for a corrective equalization scheme. Ideally, these rents should be the sole property of the central government. When they are not, there is a potential for a weak double dividend (Goulder, 1995) if equalization payments must otherwise be funded through distortive labor taxation. We also find a possibility for a strong double dividend, which remains to be characterized numerically. Finally, we find that Pigouvian taxation by the federal government broadens the scope for equalization payments, and increases the equilibrium population in the resource-poor region.
1 Introduction

This paper seeks to provide a simple analysis of how carbon emissions should be taxed by the central government of a federation. We do so in a simple environment where the federation consists of two regions. The output in a region depends on two factors of production: an exhaustible natural resource and labor. Each region benefits from a fixed and immobile stock of an exhaustible resource (e.g., oil), whereas the labor force is mobile. Citizens can freely migrate across jurisdictions. The presence of a higher stock of natural resource in one region has two effects: it increases the productivity of labor and generates a higher rent. Thus, inequality in the ownership of the resources is a potential source of inter-jurisdictional inequality, which justifies the presence of equalization transfers by the central government (Hartwick, 1980; Boadway and Flatters, 1982).

The issue of rent ownership is crucial and goes at the heart of current policy debates. We allow the rents accruing from natural resources extraction to be the property of either the central government or of the local ones. In theories of equalization, it generally standard to assume that Ricardian rents are captured by local governments in a lump-sum way. The implicit assumption is therefore that each region have an access to a rent tax that is exploited at the fullest (Boadway et al., 2003). When one region is endowed with more fixed factor than another, it is then optimal to correct inter-jurisdictional inequalities by transferring resources from the rent-rich region to the rent-poor one in order equalize per capita rent differentials.\textsuperscript{1} The more specific question of how to properly allocate the property of non-renewable rent-generating natural resources across levels of government in a federation has recently gained much interest (Brosio, 2006). In practice, it has been suggested that a federal ownership of non-renewable resources would be desirable, so as to reduce the scope for such inter-jurisdictional disparities in tax capacities (Boadway, 2007).

\textsuperscript{1}This is in the simplest case where mobility is costless. Such policies eliminate migration inefficiencies due to individuals migrating according to their average product, but not on the basis of their marginal product.
We insert natural resources extraction in a baseline model of fiscal federalism in the simplest possible way. Regions are unequally endowed with stocks of natural resources. The model is static, and each region must decide how much of its resource it will extract. The resource-poor region has a higher marginal cost of extraction and will decide to extract less resource than the resource-rich region. Extraction generates an externality, for example air pollution, that affects all citizens in the federation. Thus, there is naturally room for purely Pigouvian taxation if the extraction decision is taken either by a profit-maximizing private sector or by regional governments who only care about their own citizens.

Our key argument is that, if resources themselves can only be taxed by a local government, taxing carbon may have more than simply environmental benefits when inequality in resources endowment generates wealth inequality across the federation. This is especially the case when the central government must equalize rents inequality across regions using equalization payments, and when the equalization program must be funded through distortive taxation. In this case, Pigouvian taxation mitigates inequality in rents and reduces the overall tax collection required to fund the equalization program. Second, because Pigouvian taxation raises revenue itself, it reduces the share of equalization payments that must be funded through distortive taxation.

Because Pigouvian taxation potentially has more benefits than simply improving the quality of the environment, our model is strongly linked to the literature on the double-dividend of environmental taxation. The double dividend hypothesis states that in addition to the correction of pollution externalities, environmental taxation can also reduce the inefficiencies brought about by other distortionary taxes if the revenues from environmental taxation are used to reduce such distortionary taxes. Even in its weak form – using environmental taxes revenues to reduce other taxes is preferable to returning the revenues lump sum – the hypothesis is controversial. Because of the tax interaction effect, several studies find no double dividend at all (see Bovenberg and de Mooij (1994), Bovenberg and Goulder (1996), Goulder et al. (1999) and Parry (1995)) . Despite these findings, the strong double
dividend remains of interest to policy makers. If it held, the financial burden brought about by the environmental tax would at least be compensated by the efficiency gains from the reduction in other distortionary taxes such that there would be no economy-wide financial costs (or even a financial gain) from the imposition of an environmental tax. This is of course a much stronger hypothesis for which the theoretical and empirical evidence is at best mixed (Goulder, 1995).

This problem has received surprisingly little attention in the literature. A related paper is Bento and Jacobsen (2007), which considers the possibility of a double dividend when the production of the polluting good depends on a fixed factor. That model is not however set in a multi-jurisdiction framework. Nevertheless, it is highly relevant from the standpoint of public policy. Indeed, most federations have attributed the ownership of such resources to their central government. Canada is one notable exception. The Canadian Constitutional act (1982) has allocated all the legislative power over natural resources to the provincial legislatures. Nevertheless, rent-seeking induced by the property of resources can be mitigated by establishing a system of equalization that compensates the poorly endowed provinces (Boadway and Flatters, 1982). Again, by the Canadian constitution, it is the duty of the federal government to “ensure that provincial governments have sufficient revenues to provide reasonably comparable levels of public services at reasonably comparable levels of taxation” (art. 36(2)).

The Canadian equalization scheme is an expenditure program that transfers money to the provinces with a smaller tax base than average. In 2012-2013, a total of $15.4 billion will be transferred to six provinces (Department of Finance Canada, 2011). These transfers are funded through general and distortionary taxation, and mostly by the personal income tax, corporate taxation and sales taxes. Natural resources, and especially oil, play a crucial role in the determination which provinces benefit and which contribute to the equalization scheme. The only three significant oil producing provinces, Alberta, Saskatchewan and Newfoundland-and-Labrador are all contributing to the equalization payments, while the
fourth contributing province, British-Columbia, is a significant natural gas producer (CAPP, 2013). Natural resources revenues play such a central role in the fiscal capacity of producing provinces that their inclusion in payment formula has receive a particular treatment.\footnote{The purpose of this particular treatment has been to reduce the impact of natural resource revenues on the equalization payment formula to lower the contribution of resource rich provinces. For a detailed discussion of this treatment see Feehan (2005).}

2 Framework

The economy consists of two regions, denoted by $i = 1, 2$ that are part of a federal system of government. The population of the whole federation is fixed to $N = 1$, and we denote by $N^1$ and $N^2$ the respective population of each region. An individual who lives in region $i$ supplies $\ell^i$ units of labor, consumes a quantity $X^i$ in private consumption, and benefits from a quantity $G^i$ of a local public good that is provided by the regions’ own governments.

The total output in region $i$ depends on two variable inputs: the total labor supply $L^i = N^i\ell^i$ which is mobile across regions, and the extraction of a non-renewable resource $e^i$. The aggregate production function, given by $F^i(L^i, e^i)$, is increasing and concave in both of its arguments.\footnote{Specifically the production function satisfies the following conditions: $F(L, 0) > 0$, $F_L(L, E) > 0$, $F_{LL}(L, E) < 0$, $F(0, E) = 0$, $F_E(L, E) > 0$, for $L > 0$, $F_{EE} < 0$ and $F_{LE} > 0$.} The non-renewable resource is immobile and can be extracted at cost $c^i(e^i)$. This strictly convex cost function is such that $c^1_\epsilon(e) \geq c^2_\epsilon(e) \forall e$ and $c^i_\epsilon(0) = 0$ for $i = 1, 2$. The higher marginal cost in region 1 represents the fact that this region is less well endowed with the non-renewable resource than region 2. The extraction of one unit of natural resources in either region generates $\alpha$ units of a global negative externality. The utility function of an individual in region $i$ is

$$U^i = U(X^i, G^i, \ell^i, e^i, e^{-i}).$$
For simplicity, we assume a quasi-linear form where

$$U^i = X^i + b(G^i) - v(e^i) - \alpha(e^i + e^{-i}).$$  \hspace{1cm} (1)

Finally, individuals can freely migrate across regions. The timing is as follows:

1. The central government announces its policy
2. Regional governments announce their policies
3. Individuals migrate across regions
4. Individuals choose their labor supply.

3 Lump-sump taxation with regionally-owned rents

Let us first derive the allocation that maximizes the social welfare of the federation in an unconstrained way. We assume that the federal government can use lump-sum head taxes and inter-regional transfers (equalization payments), that it can directly set the optimal level of extraction (accounting for the global externality), and that regional governments own the rents generated by the immobile factor in their jurisdiction. We assume that the production function is homogenous of degree one in \((L,e)\) and we denote the gross rents from non-renewable resources by

$$R^i = F(L^i, e^i) - L^i F_L(L^i, e^i).$$  \hspace{1cm} (2)

We denote the net rents by \(R^i - c^i(e^i)\).
Problem of individuals

Individuals are forward looking and before migrating, they anticipate their labor supply decision in each region. The resource constraint in the economy of region \( i \) is

\[
NX^i + G^i = F(L^i, e^i) - c^i(e^i) + S^i. \tag{3}
\]

Substituting (3) into (1), an individual who has already chosen his region of residence solves

\[
\max_{\ell^i} U^i = \frac{F(L^i, e^i) - c^i(e^i) + S^i - G^i}{N^i} + b(G^i) - v(\ell^i) - \alpha(e^i + e^{-i}). \tag{4}
\]

Individuals thus supply labor according to the first-order condition:

\[
\frac{\partial U^i}{\partial \ell^i} = F_L(L^i, e^i) - v'(\ell^i) = 0. \tag{5}
\]

4) yields an indirect utility function \( V^i(N^i, S^i, G^i, e^i, e^{-i}) \). Using the envelope theorem, we obtain that

\[
\begin{align*}
N^i V_{N^i} &= \ell F_L(L^i, e^i) - \frac{1}{N^i} \left( F(L^i, e^i) - c^i(e^i) + S^i - G^i \right) \\
&= - \left[ \frac{R(L^i, e^i) - c^i(e^i)}{N^i} + \frac{S^i}{N^i} - \frac{G^i}{N^i} \right]  \\
V_{S^i} &= \frac{1}{N^i}  \\
V_{G^i} &= b'(G^i) - \frac{1}{N^i}  \\
V_{e^i} &= \frac{F_e(L^i, e^i) - c_{e}^i(e^i)}{N^i} - \alpha  \\
V_{e^{-i}} &= -\alpha.
\end{align*}
\]

Each individual chooses the region in which to live according to
\[
\max_{1,2} \{V^1, V^2\}.
\]

It will be the case that either all the population moves to the same region or populations 
\(N^1\) and \(N^2\) are such that \(V^1 = V^2\). Stable, interior solutions are assumed.

**Problem of the central government**

The central government maximizes a Rawlsian social welfare function. Its problem amounts to maximize the utility of the resource-poor region. subject to the free-migration constraint. We use a Rawlsian objective for two main reasons. First, it is analytically tractable. More importantly, it is consistent with the tradition in fiscal federalism, according to which the principle of “equal treatment of equals” must be respected. Individuals themselves are all identical, and based on the principle of horizontal equity, there is no reason to allow citizens of one region to have more utility than those in the rest of the federation. Accordingly, the government maximizes the Lagrangian

\[
\max_{S, N^1} \mathcal{L} = V^1(N^1, S) - \lambda[V^1 - V^2].
\]  

where the multiplier \(\lambda\) is the marginal social cost of free-migration, and where \(N^2 = N - N^1\). Notice that \(N^1\) is used as an artificial control variable.

The first-order condition with respect to \(S\) is

\[
\frac{(1 - \lambda)}{N^1} - \frac{\lambda}{N^2} = 0
\]

which implies that we give to the individual welfare in each region a weight equal to the
population of the region, i.e. $\lambda^* = \frac{N^2}{N}$. The first-order condition with respect to $N_1$ is

$$(1 - \lambda) V^1_N - \lambda V^2_N = 0$$

which using the previous result on $\lambda^*$ can be rewritten with each region’s population as weights

$$N^1 V^1_N = N^2 V^2_N.$$ 

Using our previous envelope condition and the fact that transfers must sum to zero, we have

$$N^2 [R(L^1, e^1) - c^1(e^1) + S - G^1] = N^1 [R(L^2, e^2) - c^2(e^2) - S - G^2].$$

Hence equalization transfers satisfy

$$S^* = \frac{N^1 [R(L^2, e^2) - c^2(e^2) - G^2] - N^2 [R(L^1, e^1) - c^1(e^1) - G^1]}{N^1 + N^2}.$$ 

We define $T^i = G^i - [R(L^i, e^i) - c^i(e^i)]$ as the total tax revenues that region $i$ requires to finance its public good expenditures. That is the amount public good expenditures in excess of net rents. Note that the optimal provision of public goods satisfies the Samuelson condition in each region: $N^i b'(G^i) = 1$. Using this definition, we can rewrite the optimal transfer as a function of regional lump sum tax collection:

$$S^* = \frac{N^2 T^1 - N^1 T^2}{N}.$$ 

This optimal transfer implies that the less well endowed region, that is therefore less populated and more taxed, would receive a transfer from the other region. That transfer exactly compensates that region for its lower fiscal capacity. Indeed, if the transfer was paid directly to (by) the regional government(s) then each would have equal per capita taxes.

Using the population weighted indirect utility functions, one can easily show that the
optimal extraction in each region satisfies

\[ F_e(L_i, e^i) = c^i_e(e^i) + \alpha N. \]

That is the marginal product of the resource must be equal to its marginal private cost plus its marginal external cost per capita weighted by the total population.

4 Lump-sum and Pigouvian taxation with regionally-owned rents

In this section, we still allow lump sum taxation by both levels of government, but we relax the assumption of exogenous optimal extraction. Instead, the federal government is allowed to use taxation to incentivize a competitive extractive industry to internalize the negative externality. Hence the federal government can impose a Pigouvian tax \( \tau \) per unit of \( e \) of extraction. Regions own rents.

If the central government can use the proceeds of Pigouvian taxes to equalize, then its budget transfer is

\[ S^1 + S^2 - \tau(e^1 + e^2) = 0. \]

The competitive extractive industry in each region would set the extraction such that

\[ F_e(L_i, e^i) = c^i_e(e^i) + \tau. \]

Therefore, the optimum level of Pigouvian tax is \( \tau = \alpha N \). There is no double dividend.

\[ \text{We ignore the intermediate case in which both levels of government can use Pigouvian taxes. In such a setting, each region would only incentivize its industry to internalize a fraction } \frac{N^i}{N} \text{ of the externality, while the federal government would have to use region specific Pigouvian taxes to make up for the difference. Using such an approach would not change the results of our analysis. One could interpret the unique federal tax we propose as a consolidated regional and federal tax.} \]
because the Pigouvian tax is exactly set at the level of the marginal external cost. Indeed, we can rearrange the budget constraint of the federal government to be

\[ S^2 = -S^1 - \alpha N(e^1 + e^2). \]

The first-order condition with respect to \( S^1 \) and \( S^2 \) gives the same thing as before, and now that with respect to \( N_1 \) is

\[ R(L^1, e^1) - c^1(e^1) + S - G^1 = R(L^2, e^2) - c^2(e^2) - S - \alpha N(e^1 + e^2) - G^2. \]

We obtain that carbon taxation necessarily reduces the scope of equalization payments, but without generating a welfare gain. If \( \tau(e^1 + e_2) = \alpha N(e^1 + e^2) > S^1 \) then the Pigouvian taxes are sufficient to redistribute across regions.

**Lagrangian of the government**

When the regions do not internalize externalities at all, the Envelope theorem gives us that

\[ \frac{\partial V^i}{\partial \tau} = \frac{e^i}{N^i} - \alpha \frac{\partial e^1}{\partial \tau} - \alpha \frac{\partial e^2}{\partial \tau}. \]

The Lagrangian of the central government can be expressed as

\[ \mathcal{L} = (1 - \lambda)V^1 + \lambda V^2 - \gamma[S^1 + S^2 - \tau e^1 - \tau e^2]. \]

Taking the first-order condition with respect to \( S \) we obtain that, if the population is normalized to one, \( \lambda = 1 - N^1 \). The first-order condition with respect to \( \tau \) gives

\[ -e^1 - e^2 - \alpha \frac{\partial e^1}{\partial \tau} - \alpha \frac{\partial e^2}{\partial \tau} + \gamma \left( e^1 + e^2 + \tau \frac{\partial e^1}{\partial \tau} + \tau \frac{\partial e^2}{\partial \tau} \right) = 0. \]
Notice that $\gamma - 1$ represents the excess burden of taxation in the optimum. Unsurprisingly, the first-order conditions with respect to $S^1$ and $S^2$ imply that $\gamma = 1$ because lump-sum taxation is available. The first-order conditions with respect to $S^1$ and $S^2$ are

\[
\frac{1 - \lambda}{N_1} - \gamma = 0
\]

\[
\frac{\lambda}{N_2} - \gamma = 0
\]

which means that $\lambda^* = N^2 = 1 - N^1$. The only value for the marginal cost of public funds $\gamma$ that solves this problem is the unity. Thus, the only role of Pigouvian taxation is to correct externalities, and the first-order condition with respect to $\tau$ equals zero when $\tau = \alpha$. It is only used to the extent that its excess burden is smaller or equal than one. Afterwards, lump-sum taxation is used to collect the missing revenue, if necessary. Thus, as soon as the external effect per unit of extracted resource is not null, $\tau$ is strictly positive.

5 Distortive transfers, Pigouvian taxation and regionally-owned rents

In this section, we constrain the federal government to finance transfers with either Pigouvian taxes or distorting labor taxes, or both. Regions can still finance their public goods with lump sum taxes and they still own the rents of their non-renewable resources. The social welfare objective is then maximized subject to

\[
S^1 + S^2 \leq \tau(e^1 + e^2) + t(L^1 + L^2)
\]

where $t$ is the per unit of labor federal tax.
Modified individual problem

The resource constraint in the economy of region $i$ is

$$NX^i + G^i = F(L^i, e^i) - c^i(e^i) - \tau e^i - tL^i + S^i. \quad (7)$$

Substituting (7) into (1), an individual solves

$$\max_{\ell^i} U^i = \frac{F(L^i, e^i) - c^i(e^i) - \tau e^i - tL^i + S^i - G^i}{N^i} + b(G^i) - v(\ell^i) - \alpha(e^i + e^{-i}). \quad (8)$$

Individuals thus supply labor according to the first-order condition:

$$\frac{\partial U^i}{\partial \ell^i} = F_{L}(L^i, e^i) - t - v'(\ell^i) = 0 \quad (9)$$

This yields the individual labor supply function. The supply of resources is given by

$$F_{e}(L^i, e^i) - c_{e}^{i}(e^i) - \tau = 0. \quad (10)$$

Envelope conditions

Using the envelope theorem, we obtain that
\[ \frac{N^i V}{N^i} = \ell F_L(L^i, e^i) - \frac{1}{N^i} \left( F(L^i, e^i) - c^i(e^i) - \tau e^i + S^i - G^i \right) \]
\[ = - \left[ \frac{R(L^i, e^i) - c^i(e^i)}{N^i} - \frac{\tau e^i}{N^i} + \frac{S^i}{N^i} - \frac{G^i}{N^i} \right] \]
\[ V_{\tau} = - \frac{e^i}{N^i} \]
\[ V_t = - \ell^i \]
\[ V_{S^i} = \frac{1}{N^i} \]
\[ V_{G^i} = b'(G^i) - \frac{1}{N^i} \]
\[ V_{e^i} = \frac{F_e(L^i, e^i) - c^i(e^i) - \tau}{N^i} - \alpha \]
\[ V_{e^{-i}} = -\alpha. \]

**Statics comparative with distortive taxation**

Individuals choose their labor supplies and regions decide how much resources they will extract. The two first-order conditions are

\[ F_L - t - v'(\ell) = 0 \]  \hspace{1cm} (11)
\[ F_e - c'(e) - \tau = 0. \]  \hspace{1cm} (12)

Total differentiation of the system yields

\[ N F_{LL} d\ell - v''(\ell)d\ell + F_{Le} de + \ell F_{LL} dN - dt = 0 \]  \hspace{1cm} (13)
\[ F_{ee} de - c''(e)de + N F_{Le} d\ell + \ell F_{Le} dN - d\tau = 0. \]  \hspace{1cm} (14)
Under matrix notation $Ax = b$, it becomes

$$
\begin{pmatrix}
NF_{LL} - v''(\ell) & F_{Le} \\
NF_{Le} & F_{ee} - c''(e)
\end{pmatrix}
\begin{pmatrix}
d\ell \\
dc
\end{pmatrix}
= 
\begin{pmatrix}
dt - \ell F_{LL} dN \\
d\tau - \ell F_{Le} dN
\end{pmatrix}
$$

where we assume that $\text{det}(A) > 0$. By Cramer’s rule, we have that

$$
\frac{\partial \ell}{\partial \tau} = \frac{-NF_{Le}}{\text{det}(A)} < 0
$$

$$
\frac{\partial e}{\partial \tau} = \frac{NF_{LL} - v''(\ell)}{\text{det}(A)} < 0
$$

$$
\frac{\partial \ell}{\partial t} = \frac{F_{ee} - c''(e)}{\text{det}(A)} < 0
$$

$$
\frac{\partial e}{\partial t} = \frac{-F_{Le}}{\text{det}(A)} < 0.
$$

**Problem of the central government**

Again, we can define the objective of the central government as maximizing the indirect utility in region 1 subject to it being equal to that of region 2. We however need to add extra constraints for the non-negativity of the transfers.

$$
\mathcal{L} = V^1(N^1, S) - \lambda[V^1 - V^2] - \mu S^1 - \gamma S^2
$$

We first consider the situation in which the revenues from the Pigouvian taxation are not going to be sufficient to finance the transfer to region 1, presumed to be having a higher marginal cost of extraction. Hence the budget constraint is going to hold with equality and
we can assume that $S^2$ is going to be zero.

\[
\mathcal{L} = V^1(N^1, \tau, t, S) - \lambda[V^1(N^1, \tau, t, S) - V^2(N - N^1, \tau, t, 0)] - \mu[S - \tau(e^1 + e^2) - t(L^1 + L^2)].
\]

We interpret $\mu$ as the marginal cost of not being able to collect revenue in a lump-sum way. This measures, equivalently, the marginal welfare cost of having to use distorting taxation. If $\mu = \frac{1}{N}$ it means that the revenues from Pigouvian taxation are sufficient to fund the equalization scheme, and thus the non-negativity constraints generate no distortions. When the revenues from purely Pigouvian taxation are not sufficient to fund the equalization program, the government has to either increase $\tau$ above its optimal level (let us call it $\tau^*$).

The first-order conditions are

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial S} &= (1 - \lambda)V^1_S - \mu = 0 \\
\frac{\partial \mathcal{L}}{\partial N^1} &= (1 - \lambda)V^1_{N^1} + \lambda V^2_{N^2} + \mu \left(\tau \frac{\partial e^1}{\partial N^1} - \tau \frac{\partial e^2}{\partial N^1} + tN^1 \frac{\partial \ell^1}{\partial N^1} + \ell^1 - tN^2 \frac{\partial \ell^2}{\partial N^1} - \ell^2\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial \tau} &= (1 - \lambda)V^1_{\tau} + \lambda V^2_{\tau} + \mu \left(e^1 + \tau \frac{\partial e^1}{\partial \tau} + e^2 + \tau \frac{\partial e^2}{\partial \tau} + tN^1 \frac{\partial \ell^1}{\partial \tau} + tN^2 \frac{\partial \ell^2}{\partial \tau}\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial t} &= (1 - \lambda)V^1_t + \lambda V^2_t + \mu \left(L^1 + t \frac{\partial L^1}{\partial t} + L^2 + t \frac{\partial L^2}{\partial t} + \tau \frac{\partial e^1}{\partial t} + \tau \frac{\partial e^2}{\partial t}\right) = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= V^1 - V^2 = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu} &= S - \tau(e^1 + e^2) - t(L^1 + L^2) = 0
\end{align*}
\]

The envelope conditions of interest are

\[
\begin{align*}
V^1_S &= \frac{1}{N^1} \\
V^i_{\tau} &= -\frac{1}{N_i}e^i - \alpha \frac{\partial e^i}{\partial \tau} - \alpha \frac{\partial e^{-i}}{\partial \tau} \\
V^i_t &= -\ell^i - \alpha \frac{\partial e^i}{\partial t} - \alpha \frac{\partial e^{-i}}{\partial t}.
\end{align*}
\]
Evaluating the potential for a double dividend

To evaluate the potential for a double dividend in this setting, we focus on the marginal cost of public funds. If the environmental tax reform lowers the marginal cost of public funds, we argue that this is indicative of a double dividend. To do this, we perform a few transformations on the previous first order conditions. We can re-express the first-order condition with respect to $\tau$ as

$$-(1 - \lambda) \frac{e_1}{N_1} - \lambda \frac{e_2}{N_2} + \mu \left( e_1 + \tau \frac{\partial e_1}{\partial \tau} + e_2 + \tau \frac{\partial e_2}{\partial \tau} + tN_1 \frac{\partial \ell_1}{\partial \tau} + tN_2 \frac{\partial \ell_2}{\partial \tau} \right) - \alpha \frac{\partial e_1}{\partial \tau} - \alpha \frac{\partial e_2}{\partial \tau} = 0;$$

$$\left( \mu - \frac{1 - \lambda}{N_1} \right) e_1 + \left( \mu - \lambda \frac{N_1}{N_2} \right) e_2 + \mu \left( \tau \frac{\partial e_1}{\partial \tau} + \tau \frac{\partial e_2}{\partial \tau} + tN_1 \frac{\partial \ell_1}{\partial \tau} + tN_2 \frac{\partial \ell_2}{\partial \tau} \right) - \alpha \frac{\partial e_1}{\partial \tau} - \alpha \frac{\partial e_2}{\partial \tau} = 0.$$  

Using $\mu N_1 = 1 - \lambda$ the first term vanishes. We therefore obtain

$$\left( \mu - \frac{\lambda}{N_2} \right) e_2 + \mu \left( \tau \frac{\partial e_1}{\partial \tau} + \tau \frac{\partial e_2}{\partial \tau} + tN_1 \frac{\partial \ell_1}{\partial \tau} + tN_2 \frac{\partial \ell_2}{\partial \tau} \right) - \alpha \frac{\partial e_1}{\partial \tau} - \alpha \frac{\partial e_2}{\partial \tau} = 0.$$

Changing the first term in the last equation

$$\left( \frac{1 - \lambda}{N_1} - \frac{\lambda}{N_2} \right) e_2 + \mu \left( \tau \frac{\partial e_1}{\partial \tau} + \tau \frac{\partial e_2}{\partial \tau} + tN_1 \frac{\partial \ell_1}{\partial \tau} + tN_2 \frac{\partial \ell_2}{\partial \tau} \right) - \alpha \frac{\partial e_1}{\partial \tau} - \alpha \frac{\partial e_2}{\partial \tau} = 0.$$  

Therefore,

$$\left( \frac{1 - \lambda}{N_1} - \frac{\lambda}{N_2} \right) e_2 + \mu \left( \tau \frac{\partial e_1}{\partial \tau} + \tau \frac{\partial e_2}{\partial \tau} + tN_1 \frac{\partial \ell_1}{\partial \tau} + tN_2 \frac{\partial \ell_2}{\partial \tau} \right) - \alpha \frac{\partial e_1}{\partial \tau} - \alpha \frac{\partial e_2}{\partial \tau} = 0.$$  

If we denote by $\tau^0 = \frac{\alpha}{\mu}$ the purely Pigouvian carbon tax in a Ramsey setting, this becomes

$$\left( \frac{1 - \lambda}{N_1} - \frac{\lambda}{N_2} \right) e_2 + \mu \sum_{i=1}^{2} \left[ (\tau - \tau^0) \frac{\partial e_i}{\partial \tau} + tN_i \frac{\partial \ell_i}{\partial \tau} \right] = 0.$$  

A similar condition holds for the labor tax.
\[
\left( \frac{1 - \lambda}{N^1} - \frac{\lambda}{N^2} \right) L^2 + \mu \sum_{i=1}^{2} \left[ (\tau - \tau_o) \frac{\partial e^i}{\partial t} + t N^i \frac{\partial \ell^i}{\partial t} \right] = 0.
\]

We can use the first order condition on the optimal transfer with that on the optimal carbon tax to solve for the marginal cost of public funds, \( \mu \). We get the following expression

\[
\mu = \frac{1}{N + \frac{N^2}{\bar{e}^i} \sum_{i=1}^{2} \left[ (\tau - \tau_o) \frac{\partial e^i}{\partial \tau} + t N^i \frac{\partial \ell^i}{\partial \tau} \right]}.
\]

Note that if the revenues of the carbon tax are sufficient to entirely fund the transfer, we can set \( t = 0 \) and \( \tau = \tau_o \), such that the summation term in the denominator of the expression for \( \mu \) becomes zero. In that case, the marginal cost of public fund is just \( \mu = \frac{1}{N} \) as in the lump sum tax case. There are no tax distortions and the optimal carbon tax is \( \tau = \alpha N \) just as in a partial equilibrium setting. In such a case, there is no double dividend.

What happens however, when the revenues from the carbon tax are not sufficient to entirely fund equalization payments? In this a case, there will be a positive labor tax. It is useful to consider the situation where no Pigouvian tax is imposed to evaluate the potential for a double dividend. We therefore rewrite the marginal cost of public funds as

\[
\mu = \frac{1}{N + \frac{N^2}{\bar{e}^i} \sum_{i=1}^{2} \left[ -\tau_o \frac{\partial e^i}{\partial \tau} + t N^i \frac{\partial \ell^i}{\partial \tau} \right]}.
\]

Since there is distortionary taxation, it must be the case that \( \mu > \frac{1}{N} \), which implies \( \sum_{i=1}^{2} \left[ -\tau_o \frac{\partial e^i}{\partial \tau} + t N^i \frac{\partial \ell^i}{\partial \tau} \right] < 0 \).

Let \( \hat{\mu} \) be the marginal cost of public funds defined by equation (16) and the optimal tax level, \( \hat{t} \), emissions levels, \( \hat{e}^i \), population levels, \( \hat{N}^i \), and labor supplies, \( \hat{\ell}^i \), of the constrained problem without Pigouvian taxation. Then it follows immediately from equation (15) that there is a weak double dividend. Indeed, introducing Pigouvian taxation without using the revenues generated to reduce \( t \) would unambiguously reduce the summation in the denom-
inator of $\mu$, pushing the marginal cost of public funds above $\hat{\mu}$. Recycling the Pigouvian tax revenues in reducing $t$ below $\hat{t}$ would mitigate that increase in $\mu$, providing the second dividend.\(^5\)

For the double dividend to be strong, $\mu$ needs to be pushed below $\hat{\mu}$. That can only happen if the tax interaction effect is outweighed by the revenue recycling effect. From equations (15) and (16), using tilde to denote the optimal value of variables from the problem with Pigouvian taxation, the condition for $\tilde{\mu} < \hat{\mu}$ can be expressed as

$$\frac{\tilde{N}^2}{\tilde{e}^2} \left\{ \sum_{i=1}^{2} \left[ (\tilde{\tau} - \tilde{\tau}^o) \frac{\partial e^i}{\partial \tau} \bigg| \tilde{e}^i \right] + \tilde{t} \tilde{N}^i \frac{\partial \ell^i}{\partial \tau} \bigg| \tilde{\ell}^i \right\} > \frac{\hat{N}^2}{\hat{e}^2} \left\{ \sum_{i=1}^{2} \left[ -\hat{\tau}^o \frac{\partial e^i}{\partial \tau} \bigg| \hat{e}^i \right] + \hat{t} \hat{N}^i \frac{\partial \ell^i}{\partial \tau} \bigg| \hat{\ell}^i \right\}.$$

Because of the opposing effect of increasing $\tau$ and decreasing $t$, we are not able to confirm that this expression always holds. Thus the potential for a strong double dividend must be assessed empirically.

We would like to point out an important feature of the double dividend that this model adds. It is related to migration. In this model, the presence of at least a weak double dividend allows for a greater scope of equalization payments than without the use of Pigouvian taxes. That is, the optimal transfer is larger when Pigouvian taxation can be used than when it is not available. That has an important consequence on the equilibrium population in each region. Indeed, with a higher transfer, region 1 will have a higher population. Given that this is the less well endowed region, the equilibrium marginal product of labor will be higher in this region. Hence the introduction of a Pigouvian tax would increase the population of the region for which the labor supply will be most responsive to the decrease in the tax. This migration effect increases the likelihood of finding a strong double dividend as it makes the broadening of the distortionary tax base more likely.

\(^5\)The first dividend being the internalization of the externality
6 Concluding comments

To our knowledge, the issue of optimal nonrenewable resources extraction in a federation with Pigouvian taxation has not been tackled yet in the economic literature. The goal of this paper is to provide a first pass on the subject. As our analysis reveals, in a federal context, Pigouvian taxation generates a weak double dividend. However, determining whether a strong double dividend may be present remains an empirical or a numerical issue. As a natural next step, we intend to numerically verify under what circumstances our model generates a strong double dividend.
References


