MATH 205 Self-Assessment ■ Duration: 1Hr 30Mins

Student Success Centre

Concordia University

1. Given the following function:

$$y = \begin{cases} \sqrt{4 - x^2} + 1, & -2 \le x < 0 \\ |3x - 3|, & 0 \le x \le 3 \end{cases}$$

   a. Sketch the graph of y on the interval $[-2, 3]$.

   b. Deduce the definite integral $\int_{-2}^{3} y\, dx$ in terms of signed area (do not anti-differentiate).

2. Consider the following function and its Riemann sum, with partitioning of the interval $[0, 4]$ into n-subintervals of equal length:

$$f(x) = x^2 - 2x - 1 \text{ and } R_n = \sum_{k=0}^{n} \left[\left(\frac{4}{n}k\right)^2 - 2\left(\frac{4}{n}k\right) - 1 \right] \frac{4}{n} \text{ respectively.}$$

   a. Calculate the area under y with the use of R_n.

   b. Calculate the area under y by integrating over $[0, 4]$.

3. Assume that:

$$H(x) = \int_{2}^{x} \frac{t}{t + \sin t}\, dt$$

Is $H(x)$ increasing or decreasing at $x = 2\pi$?

4. Find the following indefinite integrals:

   a. $\int x \arcsin(x^2)\, dx$

   b. $\int \sin^3 x\, dx$

   c. $\int \frac{x^2}{\sqrt{9 - x^2}}\, dx$

   d. $\int \frac{5x}{x^2 + 3x - 4}\, dx$

   e. $\int \frac{e^x}{e^{2x} + 1}\, dx$

5. Find the limit of sequence a_n at $n \to \infty$ or prove that it does not exist.

   a. $a_n = \sqrt{n+1} - \sqrt{n}$

   b. $a_n = \frac{(2n+1)(3n+4)}{\sqrt{100+15n^2+4n^4}}$

6. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally:

   a. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n+1}}$

   b. $\sum_{n=2}^{\infty} \frac{\sin n}{n^2}$

7. Find the interval and the radius of convergence of the following series:

   a. $\sum_{n=0}^{\infty} \frac{1}{3^{2n}} (6x+3)^n$

   b. $\sum_{n=0}^{\infty} \frac{2^n}{n!} (x-2)^n$

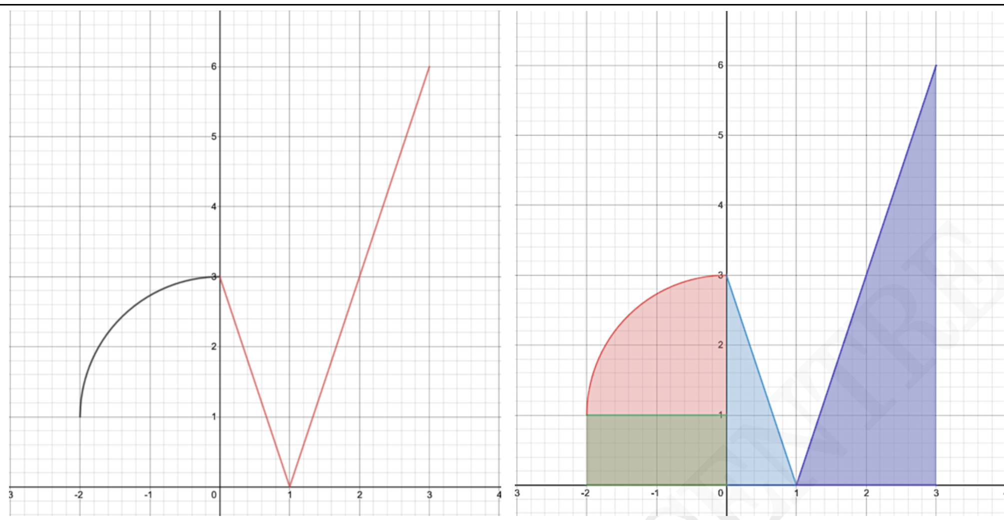
8. Determine the Taylor Series of $f(x) = e^{-6x}$ at $a = -6$.

NOTE (REFERENCES):

Some questions in this document have been selected from final exams and midterms at Concordia University.

ANSWER KEY:

1.

a.	 View 1 View 2
b.	$Area = \pi + 2 + \frac{3}{2} + 6 = \boldsymbol{\pi + \frac{19}{2}}$

2.

a.	4/3
b.	4/3

3. $H(x)$ is increasing, since $H'(2\pi) = 1$ which is positive.

4.

a.	$ans = \frac{1}{2}x^2 \arcsin(x^2) + \frac{1}{2}\sqrt{1 - x^4} + C$
b.	$ans = -\cos x + \frac{\cos^3 x}{3} + C$
c.	$ans = \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) - \frac{1}{2}\sin\left(2\arcsin\left(\frac{x}{3}\right)\right)\right) + C$
d.	$ans = \ln\lvert x - 1\rvert + 4\ln\lvert x - 4\rvert + C$
e.	$ans = \arctan(e^x) + C$

5.

a.	0
b.	3

6.

a.	***Converges absolutely*** – by Ratio Test
b.	***Converges absolutely*** – by Absolute Convergence Test

7.

a.	***interval***: $-2 < x < 1$ ***radius***: $3/2$
b.	***interval***: $-\infty < x < \infty$ ***radius***: ∞

8.

$$f(x) = e^{36} - 6e^{36}(x+6) + 18e^{36}(x+6)^2 + \cdots \quad \textbf{OR} \quad f(x) = \sum_{n=0}^{\infty} \frac{e^{36}(-6)^n}{n!}(x+6)^n$$